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Essays on Macroeconomic Dynamics
A Complex System Perspective

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ONE

INTRODUCTION

> *The hedgehog is captivated by a single big idea, which he applies unremittingly. The fox, by contrast, lacks a grand vision and holds many different views about the world – some of them even contradictory. [...] Foxes carry competing, possibly incompatible theories in their heads. They are not attached to a particular ideology and find it easier to think contextually. Scholars who are able to navigate from one explanatory framework to another as circumstances require are more likely to point us in the right direction. The world needs fewer hedgehogs and more foxes.*[1]
>
> Rodrick, (2014)

Questions on the origins of business cycles and economic fluctuations have historically attracted the attention of many economists. Also, economists have been questioning the presence and the characteristics of somehow stable, aggregate and dynamic macroeconomic relations. The answers to these questions have been among the most debated topics between the different macroeconomic schools of thought (see Snowdon and Vane, (2005) and table 1.1). In addition, the methods adopted to investigate

[1]Rodrick, (2014) uses this rhetorical sentence recalling an excerpt originally written by the Greek lyric poet Archilochus (around 650 B.C.) and then already adapted by Berlin, (1953).

on these questions have been extremely heterogeneous and have been the motive for fierce debates.

Following the distinctions between the explanations of business cycles put forward in the seminal paper by Zarnowitz, (1985) and by Snowdon and Vane, (2005), I here differentiate the macroeconomic schools of thought of the past century according to three different features. The first is about the *nature of the propagation mechanisms* of business cycles. Namely, a binary distinction between endogenous and exogenous nature. The second feature refers to the *level of analysis* adopted for the description of the business cycle in the model of a specific school of thought. This feature specifies the accuracy of the lens used to observe and describe the business cycle in the models adopted by a particular doctrine. The third feature distinguishes the different schools of thought according to the *source* of the business cycle; these are the economic forces that ignite the fluctuations and that are considered to be the most important generating factors of the observed co-movements between aggregate variables.

Referring to the *nature of the propagation mechanisms* of business cycles, I adopt the dichotomous distinction already characterized by Zarnowitz, (1985) between endogenous and exogenous explanations of business cycles. The endogenous explanations are those that attribute the business cycles to the normal *modus operandi* of all the industrialized private-enterprise economies and those that consider the business cycle to be self-sustained, without the need of a sequence of external shocks (see as examples Kaldor, 1940; Goodwin, 1951; Deissenberg et al., 2008; Dosi et al., 2010; DelliGatti et al., 2011). According to Zarnowitz, (1985) therefore: “a nonlinear model that requires only a single initial disturbance to produce self-sustaining cycles has maximum endogeneity”.[2] The exogenous explanations are, oppositely, those that attempt at describing business cycles as the cyclical response to external, monetary or real stochastic disturbances. That is exactly the nature of the propagation mechanisms that is typically applied in most of the stochastic and dynamically stable general equilibrium models (see as examples Frisch, 1933; Smets and Wouters, 2007) and which, according to Louca, (2007), is strongly linked with the Frisch-Slutsky econometric approach.

[2]Also the model developed by Guerini et al., (2016) and presented in chapter 3 of this thesis is in line with such a description of a endogenous model.

Concerning the *level of analysis* adopted in the models for the investigation of a business cycle, I will distinguish between "aggregate" or "micro founded" analyses. The "aggregate" models consider business cycles as *per se* existing macro entities and such a view is fully compatible with the traditional textbook separation between microeconomics and macroeconomics, according to which in order to understand the latter, investigation on the former is not required. The "micro founded" models instead, consider aggregate macroeconomic relations as stemming from the interplay of micro fundamental entities; in this case therefore macroeconomics is meaningful only in relation with the micro foundation. In this respect, Lucas, (1987) longed for a way of doing economics that do not need the prefixes "micro" or "macro"; he claimed that good economics needs micro foundations and macroeconomics without them is just bad economics. But such an extreme approach – together with the representative agent assumption – would represent the euthanasia of macroeconomics according to Hoover, (2001). In addition, many authorities in the macroeconomics profession such as Kirman, (2010b), Romer, (2015), and DeLong, (2015) are recently claiming that rather than adopting surely mistaken assumptions for building micro founded models, it is better to analyse only aggregate relations. In this thesis therefore a milder view is maintained. It is here argued that a distinction between "micro" and "macro" can be made, at least with respect to the phenomenon that needs to be explained, even when studying business cycles by means of fully-fledged micro founded models.

For what concerns the *source* of the business cycle, we take into consideration the main ones that have been analyzed historically following the guidelines put forward by Snowdon and Vane, (2005). In particular, in a first approximation we separate between real or financial sources and in a second approximation between demand, supply or monetary origins of the business cycles.

Starting from this threefold characterization of the previous business cycles schools and conscious about the different interpretations historically provided, in this thesis I focus on economic questions that are mostly concerned on business cycles issues. But in the attempt of providing new answers to these old questions, all along the thesis I have tried to adopt a pluralist approach. I have tried not to adhere a priori to any single school of thought and not to a priori reject any of them. In a way, I have tried

School of Thought	*Nature*	*Level*	*Source*
Orthodox Keynesian	Endogenous	Aggregate	Real (Demand)
Orthodox Monetarist	Exogenous	Aggregate	Financial (Monetary)
New Classical	Exogenous	Aggregate	Financial (Monetary)
Real Business Cycle	Exogenous	Micro founded	Real (Supply)
New Keynesian	Exogenous	Micro founded	Financial (Monetary)
New Neoclassical Synthesis	Exogenous	Micro founded	All
Post Keynesian	Endogenous	Aggregate	Real (Demand)

Table 1.1: The description of business cycles in competing school of thoughts.

to follow the suggestion put forward by Rodrick, (2014) and quoted at the beginning of this chapter. Such a pluralist view is required by the fact that the economy has a complex system nature. In fact, given the high degree of complexity, also the different economic models of business cycles developed by different schools of thought can be seen as different perspectives on phenomena that are complex in nature and therefore lack of a unique, all-embracing explanation (see Kirman, 2014, 2016b). In this thesis, I therefore investigate on business cycle by means of different econometric and economic modeling techniques which might appear as irreconcilable substitutes but that in a complexity framework might be seen as complementaries. Indeed, as Arthur, (2014) puts it:

> *The economy is a vast and complicated set of arrangements and actions wherein agents - consumers, firms, banks, investors, government agencies - buy and sell, speculate, trade, oversee, bring products into being, offer services, invest in companies, strategize, explore, forecast, compete, learn, innovate, and adapt. In modern parlance we would say it is a massively parallel system of concurrent behavior. And from all this concurrent behavior markets form, prices form, trading arrangements form, institutions and industries form. Aggregate patterns form. [...] Complexity is about formation – the formation of structures – and how this formation affects the objects causing it.*

Hence complexity is not a standalone theory taking a position on particular economic events or on particular methods. And therefore does not need to be considered as a new and alternative school of thought on business cycles. Complexity does not add a new line in table 1.1 as all the other doctrines. As already mentioned, all of the different business cycle school of thought are grounded on the idea that the economy possessed the characteristics of a complex system. But the assumptions that have been made to simplify such a grand view and to represent the economic system into mathematically tractable models (i.e. models with an analytical solution), led to the different approaches (see also the constructionist hypothesis and the hierarchical approach to scientific research of Anderson, 1972). The complex system paradigm therefore, goes deeper than any single doctrine (see Dosi, 2012b). Complexity could then be concisely defined as:

> *The study of the phenomena which emerge from a collection of interacting objects*[3]

From such a perspective therefore the economy is considered as a structural system under continuous evolution, as well explained in the introduction of Dosi, (2000). And in such a system, the decisions taken by individual agents might appear mutually independent, but they all share a common fate: they together determine the aggregate outcomes – i.e. the emerging macroeconomic patterns. Moreover, the complexity paradigm does not stop here, as a unique proposition on the problem of aggregation. It also adds the complementary claim about the fact that the aggregate outcome in turn, hits back the single microeconomic entities and affect them in their decision processes. Hence the economy is better described as a collection of feedback mechanisms between the micro and the macro level (see Hommes, 2014).

The complexity perspective therefore possesses the characteristics of an ampler viewpoint on the economic system, from which it is possible to suggest indications on how to better study it and on how to evaluate where and how previous doctrines have been correct or mistaken. In this sense, interpreting *complexity economics* as an innovative broad perspective rather than another economic doctrine, it can be said that it comprehends many different schools of thought, different research groups and any possible integrations that occurs between them. In fact agent-based modeling, nonlinear economic dynamics, causal search, applications of network theory, experimental

[3] This definition is due to Johnson, (2009).

economics, behavioral economics, are only a bunch of the possible economic research fields that might contribute to the further development of *complexity economics* as a whole. All the people working in these possibly separated fields – which are meaningful and can find useful applications also as standalone doctrines – are all different features of the same *kerass*, intended as a group of people who are working together toward some common goal fostered by a larger cosmic influence (see Akerlof, 2002).

When related to economics and to business cycle in particular, the great innovation of the complexity perspective is the fact that differently from many of the economic models present in the literature, such a characterization of the economic system allows for models in which the full evolution of the system itself might occur. The economic structure is itself a dynamic and adaptive environment. In general, a complexity framework permits to study four different scenarios:

1. scenarios with no system change;
2. scenarios with system change at the micro level but not at the macro level;
3. scenarios with system change at the macro level but not at the micro level;
4. scenarios with system change at the micro and at the macro levels.

Given the fact that aggregate macroeconomic relationships have been fairly stable for relatively long periods – e.g. most relevant macro variables share long-run common trends and are typically cointegrated Forni and Lippi, (1997) – even when microeconomic relations had changed – e.g. variations in the distributions of micro variables – it is hardly a surprise that most of the economic models following the ideas of complexity, display results that reproduce fairly stable aggregate relations between the macroeconomic variables. In more technical terms, the dynamics of the model reaches a state of *statistical equilibrium* – typically one in which endogenous business cycles is produced.

A critical reader might argue that: since in many of these complex system models, a statistical equilibrium is reached, then such a complex system approach is not necessary for better understanding business cycles; in fact (s)he might also argue that a researcher should get back to the adoption of the methods and the models belonging to one of the alternative doctrines proposed in table 1.1. In order to reply to such a possible critique, two points are worth to be raised concerning the presence of an aggregate statistical equilibrium and the idea of the economy as a complex system. Hopefully, these two points allows also the critical reader to get convinced about the importance of a complex system perspective.

First, the two concepts of statistical equilibrium and complex system are non-exclusive. The presence of a statistical equilibrium at the aggregate level, in fact does not exclude the presence of a complex system environment and the presence of the two-sided feedback system between micro and macro levels. The opposite statement holds true as well: the presence of a complex system type of economic environment does not rule out the possibility of the emergence of somehow stable aggregate relations and statistical equilibrium. Most of the complex system models indeed generate such an aggregate behavior and converge to some statistical equilibrium. But typically this convergence is reached with a peculiarity: the non-uniqueness of the statistical equilibrium (see Gualdi et al., 2015, 2016). Complex system models therefore, might help in detecting how and under which conditions one particular statistical equilibrium might be reached and how another, undesirable outcome, might be avoided.

Second, complexity is a priori non contrasting with all the previously existing business cycle schools of thought and goes deeper than all of them, being a ampler perspective rather than a school of thought. Indeed on one side the complexity tools (and in particular laboratory experiments with human subjects) might be – and have been – used to test for the underlying assumptions of many of the doctrines presented in table 1.1. In fact these tools have been useful for detecting economic approaches that were "ill description" of the economy, following Popperian-like ideals. On the other side, a complexity explanation of business cycle co-movements is perfectly integrable also with previous theories. The interaction between heterogeneous and myopic (non-rational) agents which fail to coordinate, has indeed proven useful in explaining – typically in an endogenous fashion – a great deal of business cycle co-movements (see the long list of stylized facts replicated by Dosi et al., 2015).

In this thesis a link between the complexity nature of the economic system, the existing business cycles theories and the standard business cycles tools is hence maintained tight. Complexity pushes toward the usage of different tools in order to better understand the different aspects of the fluctuations and the origins of co-movements between aggregate macroeconomic variables. Possible integrations between different tools are also considered in this thesis with the aim of better understanding the relevant features of business cycles that are described by complex system models.

Hence, the first paper of this thesis employs a structural vector autoregressive (SVAR) estimation to study the dynamic relations between public debt, private debt and output:

> In this paper we investigate on the causal nexus between public debt, private debt and output in the United States. Using data driven identification strategies for detecting causal effects in structural vector autoregressive estimations, we study whether the debt owned by different types of borrowing agents – i.e. public or private institutions – have positive or negative effects on GDP. The results suggest that both public debt and private debt shocks have a positive influence on GDP in the short run; but while positive effects brought about by public debt persist also in the long run, the effects of private debt are decreasing and eventually become negative in the long run. Disaggregation of private debt between corporate and household debt, suggest that the long-run negative effects of private debt are mostly driven by the latter type of liability. Finally, we also find that public debt crowds-in private consumption and private investment; this last result casts doubts on the crowding-out effects of public expenditure hypothesis.

The second paper employs an agent-based model (ABM) to investigate on the stability of the full-employment equilibrium and on the plausibility of the representative agent hypothesis:

> We develop an agent-based model in which heterogeneous firms and households interact in labor and good markets according to centralized or decentralized search and matching protocols. As the model has a deterministic

> backbone and a full-employment equilibrium, it can be directly compared to Dynamic Stochastic General Equilibrium (DSGE) models. We study the effects of negative productivity shocks by way of impulse-response functions (IRF). Simulation results show that when search and matching are centralized, the economy is always able to return to the full employment equilibrium and IRFs are similar to those generated by DSGE models. However, when search and matching are local, coordination failures emerge and the economy persistently deviates from full employment. Moreover, agents display persistent heterogeneity. Our results suggest that macroeconomic models should explicitly account for agents' heterogeneity and direct interactions.

Finally, the third paper integrates the two approaches of SVAR and ABM to understand how much of the business cycle features are well represented by means of a indirectly calibrated evolutionary model:

> This paper proposes a new method for empirically validate simulation models that generate artificial time series data comparable with real-world data. The approach is based on comparing structures of vector autoregression models which are estimated from both artificial and real-world data by means of causal search algorithms. This relatively simple procedure is able to tackle both the problem of confronting theoretical simulation models with the data and the problem of comparing different models in terms of their empirical reliability. Moreover the paper provides an application of the validation procedure to the Dosi et al., (2015) macro-model.

Moreover, also the concept of statistical equilibrium is an important feature that is preserved all along the thesis. In fact, in the first paper, a statistical equilibrium is found under the form of a long-run cointegration relation, which is then estimated. Such a long-run relation has to be interpreted then as an aggregate stylized fact. In this paper therefore I do not investigate on the possible source of this stylized fact, but I simply focus on its implications. In the second paper the concept of statistical equilibrium emerges again, this time in an economic system with persistent disequilibrium at the micro level. In the presented model in fact, I show how the two forces of heterogeneity and local interactions might lead to persistent market non-clearing and to different underemployment statistical equilibria. To conclude, in the last paper the statistical equilibrium is more a requirement than a result. As a matter of fact, in order to apply the method for validating and comparing causal structures of a VAR estimated on agent-based models data with the causal structures of a VAR estimated on real-world data, it is implicitly required that there exist a long-run fairly stable relation between the variables of interest.

CHAPTER

TWO

THE ECONOMIC EFFECTS OF PUBLIC AND PRIVATE DEBT

Debt is a claim on future wealth: lenders expect to be paid back. The stock of debt accordingly tends to expand at moments of economic optimism. Borrowers hope that their incomes are set to rise, or that the assets they are buying with borrowed money will increase in price; lenders share that enthusiasm. But if wealth does not rise sufficiently to justify the optimism, lenders will be disappointed. Debtors will default. This causes creditors to cut back on further lending, creating a liquidity problem even for solvent borrowers. Governments then step in, as they did in 2008 and 2009.

The Economist, (2015)

2.1 Introduction

The financial and economic crises of 2008 created strong imbalances in many borrowing-lending relations, inverting a growing trend in private and household debt that was continuing without interruption since the second half of the 90's. The problem of debt has since then spilled out also at the public sector level. Indeed, the US treasury reacted to such a situation with expansionary fiscal policies aimed at reducing the economic turmoil, at restoring economic growth and at reducing unemployment. But such an expansionary maneuver also had the effect of increasing the level of public debt, bringing the Debt-to-GDP ratio to a level never reached in the last 50 years (see figure 2.1, top-left panel). As a reaction to that situation, a vast empirical economic literature had emerged with the aim of studying how an increase in government debt might hinder economic growth, transforming a wishful good policy into a possibly harmful one. In this paper we add to this literature in two aspects. First, we are among the first studies that jointly analyze public and private debt. Second, we are the first doing this in a data-driven approach that – by means of machine learning algorithms which allow the data "to speak freely" – investigates this problem in a structural VAR framework.

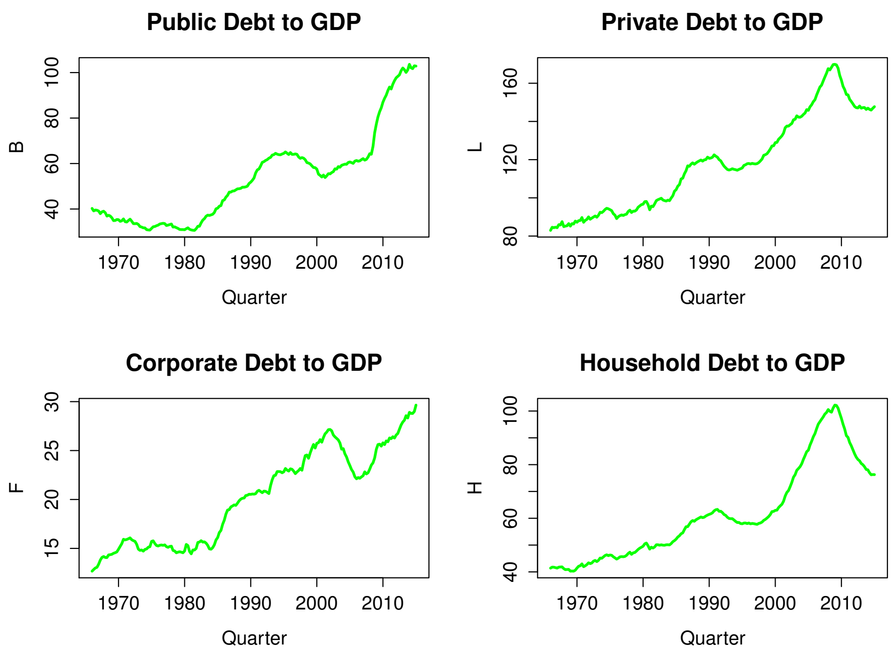

Figure 2.1: Various measures of debt related to GDP. There has been a clear trend since the 70's meaning that the US economic system has become more debt-based.

The first aim of the paper is indeed that of understanding and quantifying, by means of time series regressions, the effects that different forms of debt have on aggregate output in the United States. The decades preceding the recent financial crisis – starting from the seventies – have been characterized by strong debt expansions. In figure 2.1 indeed, it is possible to see that several forms of debt-to-GDP ratios have increased substantially in the United States; this implies that the growth rate of debt has been higher than the growth rate of output. Whether this stylized fact has to be considered an issue for the US or it is a characteristics of a new and evolving form of capitalism it is still a debated issue (see Schularick and Taylor, 2012; Palley, 2013; Akerlof et al., 2014; Turner, 2015). With this paper we investigate on a more technical question which can be written as: *"Do private and public debt shocks bear similar implications on economic growth or one of the two is more harmful and more prone to set-up the conditions for a fragile economic system?"* In the attempt to find a quantitative robust and economically meaningful answer to such a question, we therefore contribute on the discussion among the causes of what have been dubbed the "great recession".

The second aim of the paper is that of providing some policy implications. Bad regulations policies (see Pasinetti, 1998; Crotty, 2009) for what concerns debt contracts – both at the public level and at the private level – have indeed been pointed as crucial factors for the generation of the 2008 global financial crisis and for the 2012 European debt crisis. Convinced that good regulatory policies are among the most important factors for the settlement of a stable economic system and for avoiding deep and prolonged recessions, with this paper we also provide some indication on where the focus of regulatory policy should stand.

As already anticipated, in this paper we work with time series data and we estimate cointegrated vector autoregressive (CVAR) models, following the Johansen, (1995) procedure. We also identify the causal structure of these models by means of data driven causal search algorithms (see Moneta et al., 2013b). Then, by means of impulse response functions, we can detect the effects that a shock on one variables (g.e. government debt or private debt) have on another (g.e. output). In the study, we also differentiate between the type of private debt, differentiating between mortgage debt and corporate debt. The results suggest that private debt shocks, and in particular mortgage debt shocks, are the ones which bear the most negative effects on output in the long run. Public debt shocks instead are found to bear persistently positive effects on output; this is so because of some form of crowding-in effects that public debt have on private investment and private consumption. Therefore, as a policy implication we believe that regulation should be more focused on private debt contracts rather than on public debt ones. Our results cast serious doubts on some previous findings, such as the ones by Reinhart and Rogoff, (2010a) and Reinhart et al., (2012), which estimate a negative effect of public debt on economic growth in advanced economies.

The paper is organized as follows. Section 2 reviews the existing related literature. Section 3 describes the used data and estimation technique. Section 4 presents the main results. Finally section 5 concludes. Three appendixes add to the paper with additional technical explanations, robustness checks and corollary results.

2.2 Literature Review

The most important mechanisms that have been found for describing the generation of the financial crisis and of its consequences seem to confirm the ideas originally brought to the fore by Fisher, (1933), Minsky, (1986), Bernanke and Gertler, (1989) and Kiyotaki and Moore, (1997). It all begins with a large upsurge in private debt which, together with a loss of confidence, also called the *Minsky moment* (after Minsky, 1986), drives toward a positive-feedback and self-reinforcing mechanisms in which a sharp fall in asset prices causes an unexpected drop in the value of the assets used as collateral. This leads toward a situation of fear within credit institutions and also between credit institutions and other borrowers. Credit constraints become binding for many economic agents like households and firms. This situation characterized by credit constraint, decreases the demand for consumption and investment goods as well and pushes toward deflationary pressures which further increase the real value of debt (both private and public), worsening the financial conditions of the borrowers, conducting to default cascades and eventually to bank runs and to the collapse of financial institutions (interlinked via network borrowing-lending relations).[1] The situation also evolves into a liquidity drain and to an increased government debt. Moreover, the upsurge of government debt is even stronger if we consider an economy in which savers' deposits are insured by the government and where bank bailouts are also performed. In all these

[1]For the network analysis of the inter-linkages between credit institutions see Battiston et al., (2012) and the survey by Chinazzi and Fagiolo, (2013).

mechanisms therefore debt – private and public – plays a key role.

It is therefore no surprise that in the last decade a huge amount of papers and books that support the presence of many of such mechanisms (see Mian and Sufi, 2009, 2011; Geanakoplos et al., 2012; Schularick and Taylor, 2012; Jordà et al., 2013; Mian et al., 2015; Turner, 2015; Jordà et al., 2016) are mostly based on regressions that contain and evaluates the effects brought about by some measure of debt. Most of them indeed, provided robust empirical evidence, using panel-data estimations, for the fact that *(i)* high level of households debt, *(ii)* upsurge in house ownerships and *(iii)* increases in house prices, are all good features for predicting the occurrence of financial crises. The direct conclusion is that mortgage debt leads to a more fragile and more vulnerable economic system. In addition a closely related work by Jordà et al., (2014) argues that government driven exit strategies – namely expansionary fiscal policies - lose their grip if the economy enters into a financial crisis with a level of public debt which is already high. This last effect is most likely attributed to the lower government bargaining power in the choices about the type of fiscal policy that a government has in such a situation.

As another confirmation about the working of these mechanisms during the recent crisis, public debt had surged as well during the last decades (see in top-left panel in figure 2.1 how public Debt-to-GDP increased from a value around 60% in 2007 to a value around 100% in 2014) and in the attempt of understanding the effects and the consequences of such public debt overhang, a number of economists studied the relationships between public debt levels and economic growth. Beginning with the series of seminal papers and by the influential book by Reinhart and Rogoff, (2010a,b) and Reinhart et al., (2012) contrasting evidence had emerged (see the survey by Panizza and Presbitero, 2013) keeping the question still open to debate. This stream of literature aimed at answering two related questions: *(i)* is there a causal effect between public debt and economic growth? *(ii)* is there a threshold above which public debt becomes detrimental for economic growth?

There are several tentative answers to both the first and second question in a panel data context with several different multi-country, yearly, historical datasets and with different estimation techniques.

Reinhart and Rogoff, (2010a), using their own built historical dataset with both advanced and emerging economies, estimate the correlations between public Debt-to-GDP ratios and economic growth. They define three exogenous threshold at 30%, 60% and 90% of Debt-to-GDP and they find that when this ratio is higher than the 90% threshold, this correlation might become negative. Reinhart and Rogoff, (2010a) however, have been proven mistaken by Herndon et al., (2013) because of three different issues: *(i)* coding errors, *(ii)* selective exclusion of available data and *(iii)* unconventional weighting of summary statistics. Herndon et al., (2013) indeed use the same dataset and find milder negative results. After having estimated the effect of Debt-to-GDP on economic growth with kernel regressions, their results suggest that a positive (but declining) effect of debt on economic growth is present for any level of Debt-to-GDP ratio, contrasting therefore the Reinhart and Rogoff, (2010a) claim.[2] Cecchetti et al., (2011) estimate growth equations using a BIS dataset (1980-2010) that comprehends 18 OECD advanced economies while Checherita-Westphal and Rother, (2010) using the AMECO data (1970-2008) – built by the European Commission – use similar estimations with a focus on advanced European countries. Both the works adopt also endogenous threshold models and they find a inversely U-shaped relation between public debt and economic growth. They respectively position the thresholds for the negative effect to appear at 85% and 95% of Debt-to-GDP. Panizza and Presbitero, (2014) use the same dataset of Cecchetti et al., (2011) but they investigate on the causal effect of public debt on growth by using an IV approach. To instrument public debt, they use a combination of external public debt and exchange rate. They contrast with previous results and find that no causal effect is present since the estimation of the debt parameter are non significant. Also Kumar and Woo, (2010) use growth equations, but on a own built dataset that covers the period 1970-2007 and is created by merging data from the Penn World Tables, from the IMF datasets and from the World Bank datasets. This work differs from the previous because it includes both advanced and emerging economies. The authors find a negative U-shaped relations, with a threshold (exogenously settled) at 90%. Minea and Parent, (2012) use a longer historical dataset, available from the IMF which includes yearly data about 174 countries; they replicate

[2]They find that this positive effect disappears only when Debt-to-GDP is well above 150%.

the analysis of Reinhart and Rogoff, (2010a) finding milder results as found by Herndon et al., (2013) and then they use panel smooth threshold regressions, finding that an increase in public debt, when Debt-to-GDP is above 115% boosts economic growth. Baum et al., (2013) using the AMECO dataset, focusing on EU countries and applying dynamic panel threshold models find a positive short run effect of debt on growth; but they also find that this effect disappears and becomes insignificant if Debt-to-GDP ratio is higher than 67%.[3] Finally, Égert, (2015) which extends the Reinhart and Rogoff, (2010a) dataset and estimates different econometric models in different subsamples – of countries and years – find that estimating a negative nonlinear relation between public Debt-to-GDP ratio and economic growth is extremely difficult and typically the results are very sensitive to the modeling choices and to the data coverage.

A possible technical problem of the vast majority of the above mentioned papers relating public debt and economic growth is the fact that they are all based on panel data regressions (static or dynamic) and are therefore implicitly assuming that the estimates of the causal effects of public debt on economic growth is the same in all the countries considered in the datasets apart from controlling, by means of fixed effects, for country specific factors. To our knowledge, the unique work emphasizing the possibility of country heterogeneous effects brought about by debt on economic growth is the one by Eberhardt and Presbitero, 2015. There, the authors show by means of kernel estimations that country heterogeneity is an important feature which is completely missed by the current literature: aggregation and pooling of the different country datasets might suggest the presence of an inversely U-shaped curve, while actually for most of the countries this curve might be U-shaped (see figure 2 in their paper).

Finally, economists have studied also possible policy reactions to the problem of high debt and low growth. It is interesting to note how, in a world where monetary policy seem to have become the unique game in town, the recent economic literature relating economic growth, public debt and private credit have debated mostly on the role for fiscal policy and on the effects it might bring. The debate has been charac-

[3] It has to be noticed that by the way dynamic panel regression estimations are consistent for large N, while in their dataset the number of countries is N = 12, posing a serious problem to their estimation strategy.

terized by the division between two different macro-groups: the "austerians" and the "Keynesians". The first group (see Mountford and Uhlig, 2009; Alesina et al., 2015) finds that austerity and budget surpluses – especially when driven by public expenditure cuts rather than increases in taxation – aimed at reducing public debt might as well be expansionary. The second group is more heterogeneous but in general their findings suggest that as an exit strategy, sound Keynesian expansionary fiscal policies are needed (see also Krugman, 2013, for a general discussion). As a matter of fact, Auerbach and Gorodnichenko, (2012) have found, by means of regime switching time series models, that fiscal multipliers are larger during periods of recessions than during periods of booms; also, they have found that if the fiscal expansion is predictable, the multiplier is larger. Thus, they suggest that the first best is a well planned and declared countercyclical fiscal policy. Blanchard and Leigh, (2013) and Guajardo et al., (2014) have also found very similar results. Ferraresi et al., (2015) estimating threshold VAR models and differentiating the regimes according to the credit conditions, found that fiscal multipliers are higher during "tight credit regimes"; since credit regime has been tight after the 2007 crisis, also their policy suggestion is for expansionary fiscal policies. Finally, Jordà and Taylor, (2016) identify the causal effects of fiscal policy by means of new propensity-score based methods for time series data and show that austerity always hinder economic growth; even more so when the economy is in depression. Finally, also Bernardini and Peersman, (2015) find high fiscal multipliers, in periods of private debt overhangs.

2.3 Estimation Method

We estimate our multivariate time series model by means of a Vector Error Correction Model in its *transitory* formulation, using the Johansen and Juselius, (1990) procedure (see Lütkepohl, 1991; Johansen, 1995).

The model of interest is specified as:

$$\mathbf{\Delta Y_t} = \mathbf{\Pi Y_{t-1}} + \mathbf{\Theta_1 \Delta Y_{t-1}} + \cdots + \mathbf{\Theta_p \Delta Y_{t-p+1}} + \mathbf{u_t}. \tag{2.1}$$

with $\mathbf{\Theta_i} = -(\mathbf{A_{i+1}} + \cdots + \mathbf{A_p})$ and $\mathbf{\Pi} = \boldsymbol{\alpha\beta^t} = -(\mathbf{I} - \mathbf{A_1} - \cdots - \mathbf{A_p})$ where all the $\mathbf{A_i}$ matrices represent the lagged effects of the equivalent VAR model and $\mathbf{\Pi}$ represents the error correction term composed by the loading matrix $\boldsymbol{\alpha}$ and by the cointegration vector $\boldsymbol{\beta^t}$.

This modeling strategy allows the estimation of the possible existing cointegrating relations which are contained in $\boldsymbol{\beta}^{\mathbf{t}}\mathbf{Y_{t-1}}$ and also allows to cope with the possible common trends that our variables of interest might exhibit. Moreover, from this estimation is possible to recover the equivalent level-VAR model

$$\mathbf{Y_t} = \mathbf{A_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{A_p}\mathbf{Y_{t-p}} + \mathbf{u_t}. \tag{2.2}$$

With the aim of identifying the SVAR model, finally, it is important to notice that the residuals of the two models in equations 2.1 and 2.2 are equivalent. Therefore the SVAR model

$$\mathbf{\Gamma_0}\mathbf{Y_t} = \mathbf{\Gamma_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{\Gamma_p}\mathbf{Y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{2.3}$$

where the p relations $\mathbf{A_i} = \mathbf{\Gamma_0^{-1}}\mathbf{\Gamma_i}$ hold, can be recovered by means of Independent Component Analysis (ICA) on the reduced form residuals.

The ICA model for SVAR identification, has been put forward Hyvarinen et al., (2010) and allows to identify the SVAR in a more agnostic and data-driven fashion, allowing one to avoid as much as possible the subjective choices and the theory driven considerations which are typically done with the standard Cholesky decomposition.[4] Before explaining the ICA procedure, let us notice that the VAR reduced form disturbances $\mathbf{u_t}$ and the SVAR structural shocks $\boldsymbol{\varepsilon}_\mathbf{t}$ are linearly related as

$$\mathbf{u_t} = \mathbf{\Gamma_0^{-1}}\boldsymbol{\varepsilon}_\mathbf{t}. \tag{2.4}$$

Therefore the VECM (or VAR) residuals $\mathbf{u_t}$ might be interpreted as the linear combination of the structural, non-Gaussian and independent shocks which have been combined by the *mixing matrix* $\mathbf{\Gamma_0^{-1}}$. Independent Component analysis allows the estimation of the mixing matrix $\mathbf{\Gamma_0^{-1}}$ and of the independent components $\boldsymbol{\varepsilon}_\mathbf{t}$ by searching among all the possible linear combinations of $\mathbf{u_t}$, the one that minimizes mutual statistical dependence.

[4]This identification strategy builds on the previous works by by Swanson and Granger, (1997), Bessler and Lee, (2002), Demiralp and Hoover, (2003), and Moneta, (2008).

One among the advantages of ICA is the fact that it does not require any specific distribution of the structural residuals ε_t but only requires that they are independent and non-Gaussian.[5] Following Hyvarinen et al., (2010) we also assume that the VAR residuals can be represented as a *Linear Non-Gaussian Acyclic Model* (LiNGAM) so that the contemporaneous causal structure can be represented as a Directed Acyclic Graph (DAG). On the basis of this assumption, it is possible to apply a causal search algorithm, such as the one presented in appendix A, which draws on the original contributions by Shimizu et al., (2006), Hyvarinen et al., (2010), and Moneta et al., (2013b).

The outcome of our identification procedure is a particular selection of all the possible Cholesky contemporaneous causal order. In particular the algorithm allows to select the one Cholesky causal order which is more in line with the data, without the need of any *ad hoc* theoretical economic assumption about the contemporaneous causal structure of the variables of interest.

2.4 Data

We employ US quarterly data, downloaded from the FRED database released by the Federal Reserve of St Louis.[6] We focus our attention only on US because our estimation and identification strategies requires sufficiently long time series (at least $T > 150$) which – for what concerns quarterly private debt and quarterly mortgage debt, and to our knowledge – are publicly available only for this country. The used variables and their summary statistics are presented respectively in table 2.1 and in figure 2.2.

Even if interested mainly on the responses of GDP to different debt shocks, we have included in our dataset also consumption, investment and the 3-months T-Bill rate as controls which allow us to understand the presence and direction of possibly relevant transmission mechanisms and that also allow us to account for possible omitted variables which would bias our estimates. On the other side, we have decided not to include too many of these control variables, in order to keep the number of estimated

[5] At most one Gaussian ε_j it is allowed; for more details about Independent Component Analysis see Hyvarinen et al., (2001).

[6] `https://research.stlouisfed.org/fred2/`.

Label	Variable Description
Y	Real Gross Domestic Product
B	Real Federal Debt: Total Public Debt
L	Real Total Credit to Private Non-Financial Sector
F	Real Non-Financial Corporate Business Debt Securities
H	Real Mortgage Debt Outstanding
I	Real Gross Fixed Capital Formation
C	Real Personal Consumption Expenditures
R	3-Month Treasury Bill: Secondary Market Rate
P	Gross Domestic Product: Implicit Price Deflator, Index 2009=100

Table 2.1: Data description

parameters as low as possible – indeed the number of parameters to be estimated in VAR regressions is a squared function of the number of variables.

Looking at the cross correlations between the log-differences of the variables of interest (in figure 2.2) we get a first glimpse at the relations among our variables. It is interesting to note how public debt is negatively correlated, even if only slightly, with GDP and with all the other variables apart from consumption, with which public debt is basically uncorrelated. Private debt instead, which is measured by the total amount of credit to the private non-financial sector, is positively correlated with all the variables. Also when disaggregate into corporate and household debt, this positive correlation still holds true with the unique exception happening in the negative correlation between corporate debt and interest rate, which has a theoretical justification for the fact that interest rates measure the cost of borrowing for a firm.

Before performing our time series analysis we look at the integration order of all the variables of interest. We use the Phillips-Perron test which – since it augments the Dickey-Fuller test – is robust also with respect to possible unspecified autocorrelation and heteroskedasticity. The results, presented in table 2.2, do not reject the null hypothesis ($H_0 =$ "Presence of unit root") for all the variables in level. On the contrary the test always rejects the null for all the variables in difference. This suggests that all the variables of interest are integrated of order one – $I(1)$.

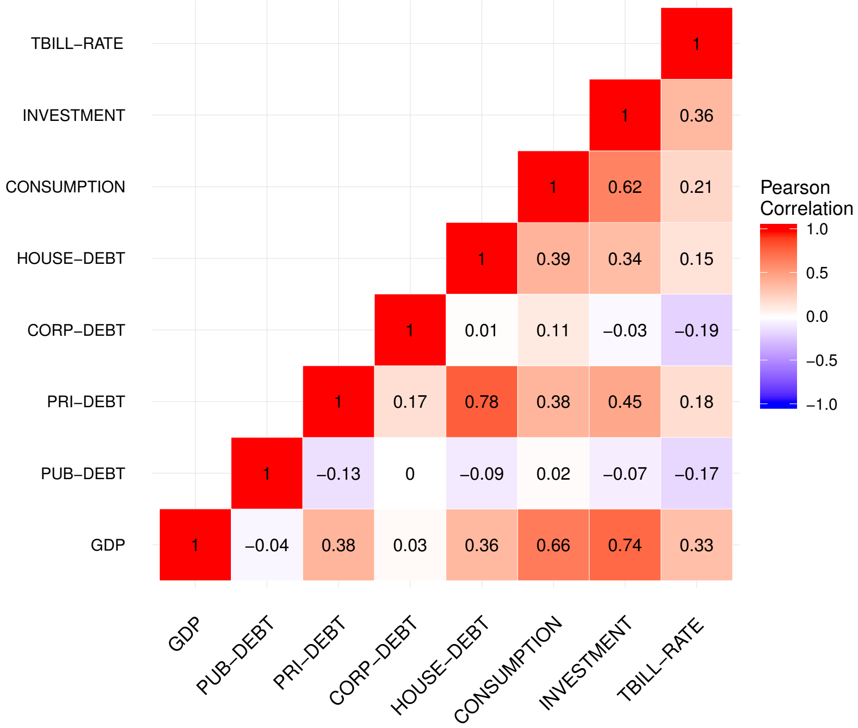

Figure 2.2: Cross correlations of the investigated variables.

2.5 Results

In this section we present the results of the regressions that we have run in order to understand the effects that public and private debt shocks have on the economy. All the regressions here presented are estimated with the variables in level – without differentiation or filtering – in order to extract all the information stemming from possible cointegration relations. Table 2.3 reports the variables contained in each regression, the suggested numbers of lags according to the standard criteria and the number of cointegration relations to be included according to the Johansen and Juselius, (1990) procedure.[7]

[7]The bold in the lag selection corresponds to our choice. Where possible, we have adopted the Bayes-Schwartz Criterion (BIC). In the cases in which the adoption of such a criterion was a poor one (because residuals did not display standard properties of a white noise) we have selected the more parsimonious between the Akaike Information Criterion (AIC) and the Hannan–Quinn Criterion (HQ). All the cointegration relations are estimated with a constant. All the estimations, unless clearly specified,

	Level		Difference	
Variable	*PP-test*	*p-Value*	*PP-test*	*p-Value*
Y	-1.1056	0.9193	-10.2842	0.01
B	-2.3184	0.4428	-9.3683	0.01
L	-0.5299	0.9796	-8.4035	0.01
F	-1.3161	0.8625	-8.8052	0.01
H	-0.1825	0.9998	-4.0346	0.01
I	-1.7969	0.6611	-8.0843	0.01
C	-0.8974	0.9514	-10.4661	0.01
R	-2.7281	0.2713	-11.0075	0.01

Table 2.2: Phillips-Perron test for stationarity.

ID	*Variables*	*Lags (p)*	*Cointegration order (r)*	*Period*
1	Y, B, L, R	AIC: 10, HQ: 6, **BIC: 5**	1	1966 - 2015
2	Y, B/Y, L/Y, R	AIC: 10, HQ: 6, **BIC: 5**	1	1966 - 2015
3	Y, B, L, I, R	AIC: 10, **HQ: 6**, BIC: 1	2	1966 - 2015
4	Y, B, L, C, R	AIC: 6, **HQ: 5**, BIC: 1	2	1966 - 2015
5	Y, B, F, I, R	AIC: 8, **HQ: 5**, BIC: 2	2	1966 - 2015
6	Y, B, H, C, R	AIC: 6, **HQ: 4**, BIC: 2	1	1966 - 2015

Table 2.3: Regressions settings.

Before proceeding further and discussing about the structural and causal information that we have extracted from the identification procedure, we here present the estimated cointegration relations in table 2.4. Even if here we are interested in causal, structural properties between the variables, and cointegration is a property of the reduced form model, it is interesting to observe the estimated cointegration relations in order because they suggest possible long-run relations between our variables of interest.

In what follows we describe our results in detail. We first present the results of the baseline models (the regressions with ID numbers 1 and 2), we then proceed by presenting the results of the augmented models (with ID numbers 3 and 4), finally we

are estimated for the whole time period: from 1966 (Q1) to 2015 (Q1).

Variable	*ID 1*	*ID 2*	*ID 3 (r=2)*		*ID 4 (r=2)*		*ID 5 (r=2)*		*ID 6*
Y	1	1	1	0	1	0	1	0	1
B	-0.0052	-0.0184	0	1	0	1	0	1	-0.1763
L	-0.7092	-2.4835	-3.3690	-21.2617	-0.5813	-22.9709			
F							-0.3506	-2.1589	
H									-0.3714
I			3.6784	27.6748			-0.4433	1.7958	
C					-0.1832	29.1448			-0.1634
R	-1.2553	-4.3960	-10.7382	-71.2981	-1.1836	-25.0376	0.3161	2.6357	-1.8142
const.	-5.7968	2.7435	1.0895	54.1615	-4.8187	-151.8507	-7.4530	-0.1093	-5.5472

Table 2.4: Estimated cointegration relations (β^t).

describe the findings of the disaggregated regressions (the models with ID numbers 5 and 6). We conclude the section by presenting some corollary results which study the crowding-out or crowding-in effects of public debt shocks on private consumption and on private investment and by presenting robustness checks related to the estimation of a model in a subsample without the crisis period.

2.5.1 Baseline models

The baseline regressions we consider here are the two four-dimensional VECMs that are presented in the first two rows of table 2.3. The first model is a very simple specification that contains GDP, government debt, non-financial firms debt ant the 3-months T-Bill interest rate that allows to control for the effects brought about by monetary policy. The second model employs the same variables, but with public and private debt which are measured as ratios with respect to GDP. This latter specification is included because the measurement of the variable is the closest to the typical panel data estimations which attempt at measuring the effects of public Debt-to-GDP on economic growth (see Panizza and Presbitero, 2013, and most of the literature presented in section 2.2). In spite of their simplicity these two models are able to capture the bulk of the effects that we also find on richer specifications.

Figure 2.3 contains the matrices of the structural VARS: even if the contemporaneous causal structure differ between the two models (Lag 0 matrices), the results stemming from the estimation and identification of these models are pretty much similar (Lag 1 and Lag 2 matrices).[8] The first two matrices on the left (Lag 0) represents the matrix $B = I - \Gamma_0$ – which has been identified with our data-driven causal search procedure. It is interesting to note that there is quite a difference between the two and this might be due to the fact that the measure of the two debt variables are different; in the second model indeed, GDP also appear in the denominator of the debt variables, creating some additional endogenous relation that our algorithm captures.

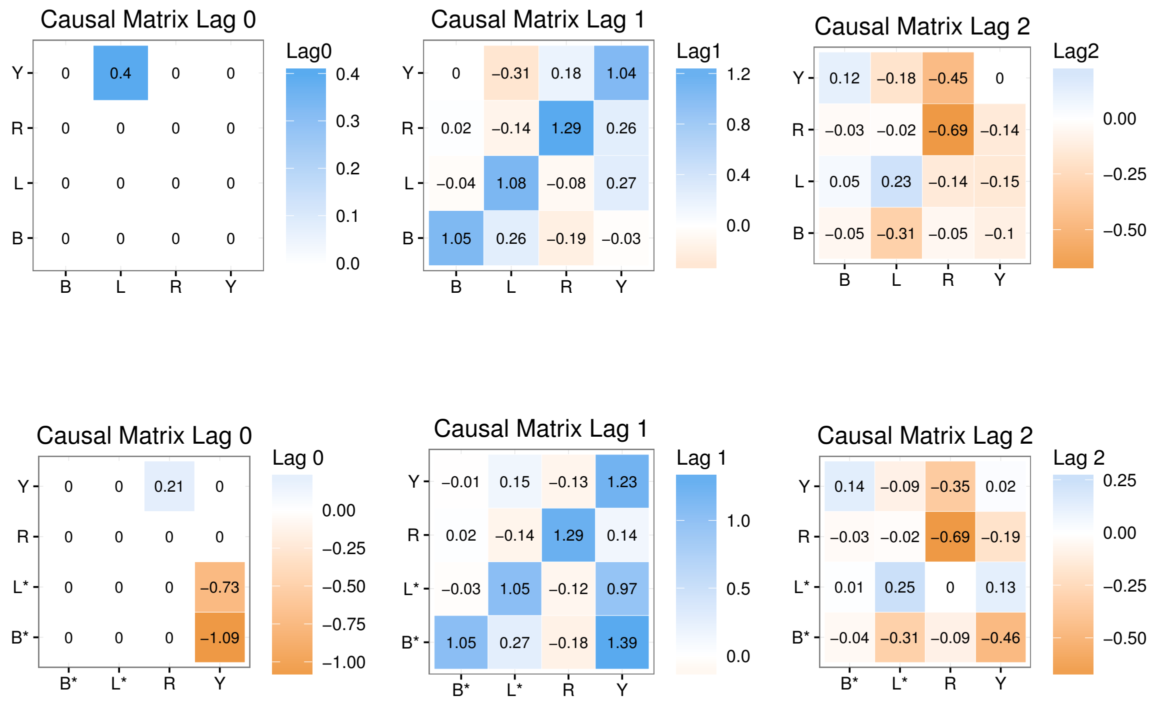

Figure 2.3: SVAR causal matrices up to the 2^{nd} lag for the baseline models 1 (top panels) and 2 (bottom panels). The entry below the table causes the entry on the left of the table.

The two contemporaneous causal structures are also depicted in figure 2.4, in a standard Directed Acyclic Graph (DAG) form. Concerning the first model, the con-

[8]Note that $B^* = B/Y$ and $L^* = L/Y$.

temporaneous causal structure suggests that output is positively caused by private debt while the other variables does not have contemporaneous impact. The second graph, representing model 2, contains the information entailed in the bottom-left matrix in figure 2.3. Identification suggests that interest rate slightly affects output which, in turn, negatively affects both private and public debt to GDP. This difference might be due to the fact that macroeconomic variables are all intrinsically endogenous, and the identification of a causal order that does not allows for loops produces uncertainty in the direction of contemporaneous causality, or also to the fact that the debt variables are measured in different ways in the two models. Notwithstanding the contemporaneous differences, it has to be noticed, that in both the cases the lagged causal structure is very similar (see figure 2.3) and this is particularly important for dynamic causal consideration.

$$L \xrightarrow{+} Y \qquad\qquad \begin{tikzcd} & & L^* \\ R \arrow[r, "+"] & Y \arrow[ur, "-"] \arrow[dr, "-"'] & \\ & & B^* \end{tikzcd}$$

Figure 2.4: Contemporaneous causal structure of models 1 (left) and 2 (right).

As we move to dynamic causal considerations, the natural tool that we adopt for estimating the causal effects that a shock to one variable has on the other variables is the “Impulse Response Function” (IRF). The estimated IRF for the two baseline models are depicted in figure 2.5 and are robust also to different model estimation strategy, such as the level-VAR estimated with OLS. The fact that the IRFs do not converge to the zero level is due to the fact that we estimate the model in level, without differentiating the variables, which is consistent with the Johansen, (1995) procedure.[9]

The results provide a new piece of evidence for what concerns the effects of public debt shocks and corroborate existing evidence presented in section 2.2 for what concerns the effects stemming from private debt shocks. As a matter of fact we find that a positive shock to public debt is persistently beneficial for economic growth, causing

[9]The represented confidence intervals are those calculated by means of bootstrapping techniques at the 5% and at the 95% levels.

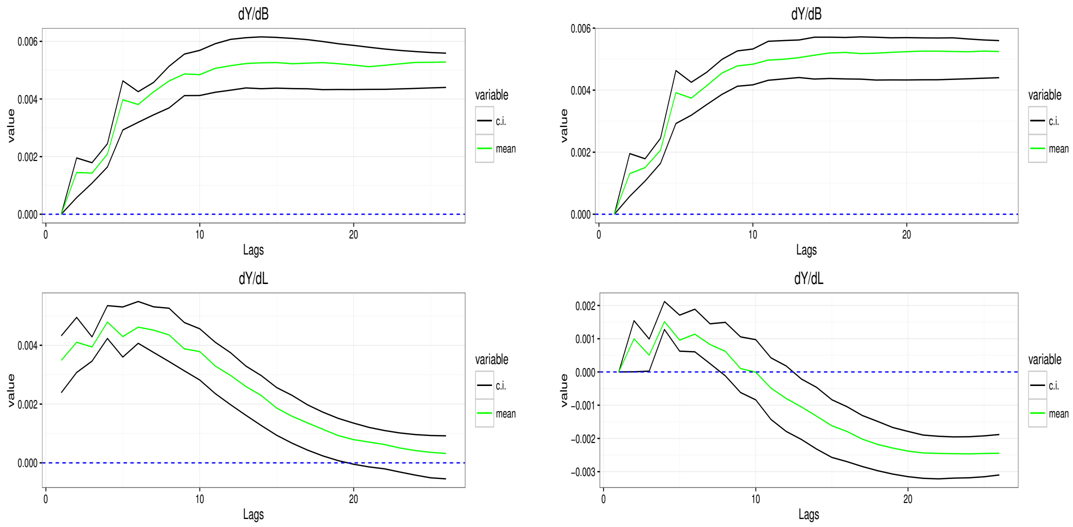

Figure 2.5: IRF of baseline models 1 (left) and 2 (right).

output to increase. A private debt shock instead, while being beneficial for GDP in the short-run – during the first two and a half years (10 quarters) after the shock hits – has negligible or even negative effects on output on longer horizons. We argue that the statistical significance of a negative effect is brought about by the higher likelihood of financial crisis that a higher private debt brings about, as stated in the series of papers by Jordà et al., (2013, 2014, 2016).

2.5.2 Augmented models

Concerned by the fact that in our two baseline specifications, some omitted variables might mediate and have relevant effects, we also estimate two richer models including possibly mediating features; these are dubbed the augmented models and in table 2.3 have respectively the ID 3 and 4. In both cases, we augment the first baseline model including aggregate investment (in model 3) and aggregate private consumption (in model 4).

Figure 2.6 represents the matrices of the structural VARs for the models 3 (top row) and 4 (bottom row). Again, the contemporaneous causal structure is different between the two models: this time the difference might be due to the change of variable. Still,

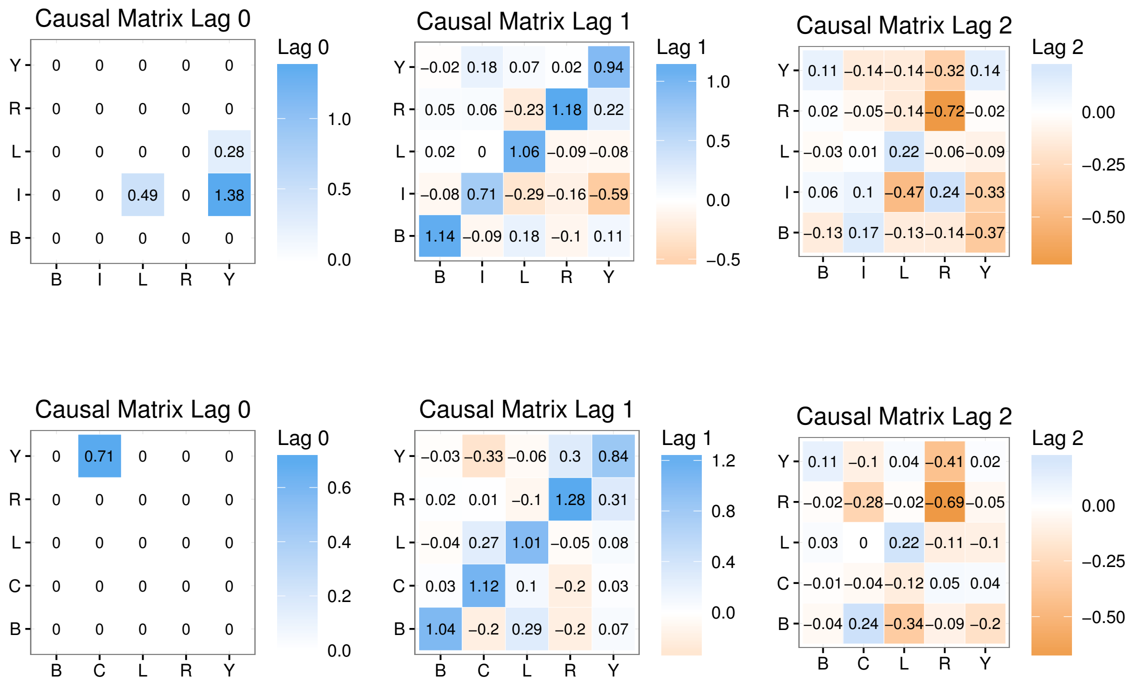

Figure 2.6: SVAR causal matrices up to the 2^{nd} lag for the baseline models 1 (top panels) and 2 (bottom panels). The entry below the table causes the entry on the left of the table.

the lagged components contains pretty much the same effects; moreover, the bulk of these effects is consistent with the two baseline models presented above.

The contemporaneous causal structure is also represented in DAG form in figure 2.7. In model 3 it is possible to observe the double contemporaneous role played by GDP as a booster for both private debt and private investment; private debt also has a positive effect on investment. Concerning model 4 instead, the unique significant contemporaneous causal effect is played by consumption on output.

The dynamic causal relations are represented by means of IRF in figure 2.8. The results confirm, and even reinforce, the claims stemming from the two baseline estimations. Public debt shocks do cause higher output while private debt shocks have positive and mild effects in the short-run, but negative effects in the long-run. IRF are

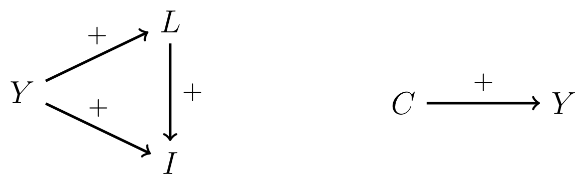

Figure 2.7: Contemporaneous causal structure of models 3 (left) and 4 (right).

the typical tools used also for policy analysis and in our models they suggest that public debt cannot be harmful for the US economic system, but instead might be a good device for restoring economic growth.

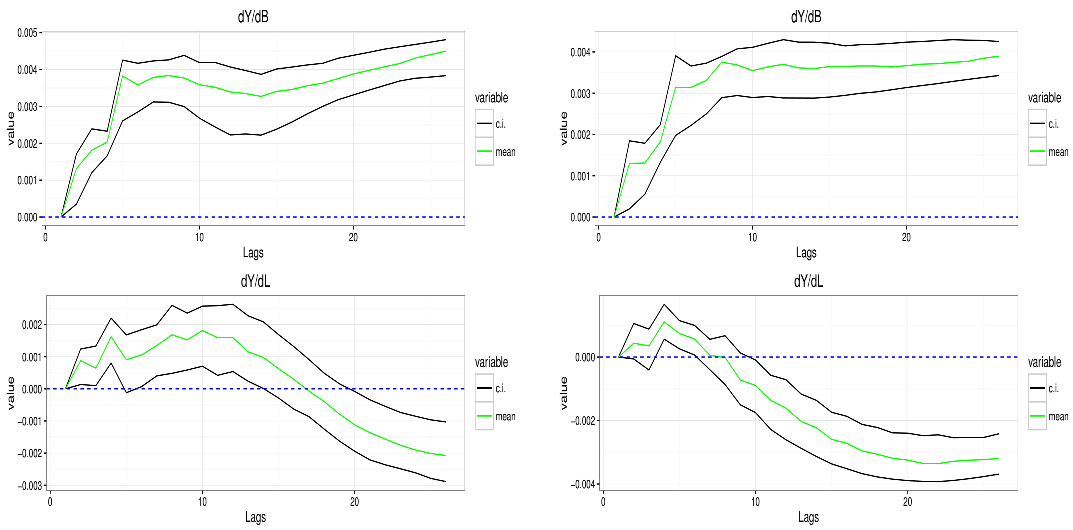

Figure 2.8: IRF of augmented models 3 (left) and 4 (right).

Across the four different specifications presented up to now, the three difference that shall be noticed are with respect to *(i)* the contemporaneous causal structure, *(ii)* the size of the effects in the IRFs and *(iii)* the number of lags after which the private debt begins to affect output negatively in the IRFs. Concerning the first point, we argue that the changes in measurement (level or ratio) and the changes in variables (investment or consumption) might be the motives behind the observed differences. With respect to the second difference, estimations seem to be consistent one with the other between 0.3% and 0.6% for what concerns public debt; for private debt instead, the support varies between a maximum of 0.5% and a minimum of -0.4% reflecting partly the higher variability of the effect over time and partly the higher variability of the estimation across the four specifications. About the third relevant point of difference instead – i.e. the number of lags for the effects of private debt to change its sign – we note that in the estimations this value lays between a minimum of 10 lags to a maximum of 20 lags in the best case scenario. However, it shall be noticed that in one scenario, the negative effect of private debt does not appear and instead, private debt only becomes insignificant after 20 lags.

2.5.3 Disaggregated models

We then proceed with our analysis by decomposing the total private debt into two smaller components: mortgage and corporate debts. Such a procedure allows us to address the possible issue of having selected a wrong/bad proxy for aggregate private debt – even if the variable that we use as a proxy for private debt is the "total credit to private non-financial sector", and has been already extensively used in the literature, also in the seminal contribution by Jordà et al., 2014. Moreover, such a decomposition allows us to better understand at a more disaggregated level how debt to different microeconomic entities – namely households for what concerns mortgages and firms for what concerns corporate debt – might differently impact on output.

The causal SVAR matrices for these disaggregated scenarios are presented in figure 2.9 while the implied contemporaneous causal structure is plotted in figure 2.10. Two points are worth to be noticed here. First, the causal structure of the two models are very similar with respect to the augmented models: this is so because we did not had a full change in the measurement or in the variables as we did before, since in both cases, disaggregated private debt remain proxies of the aggregated private debt. Hence it is with no surprise that we observe output causing investment in model 5 – even if here it is interesting to notice that corporate debt plays no role – and consumption causing output in model 6. Second, the fact that the lagged effects are again consistent with all the previous regressions, confirming that the baseline models are able to capture most of the effects.

Again, the dynamic results are presented in figure 2.11 by means of the IRFs. The results suggest that, while the effects of public debt remain positive both in the short

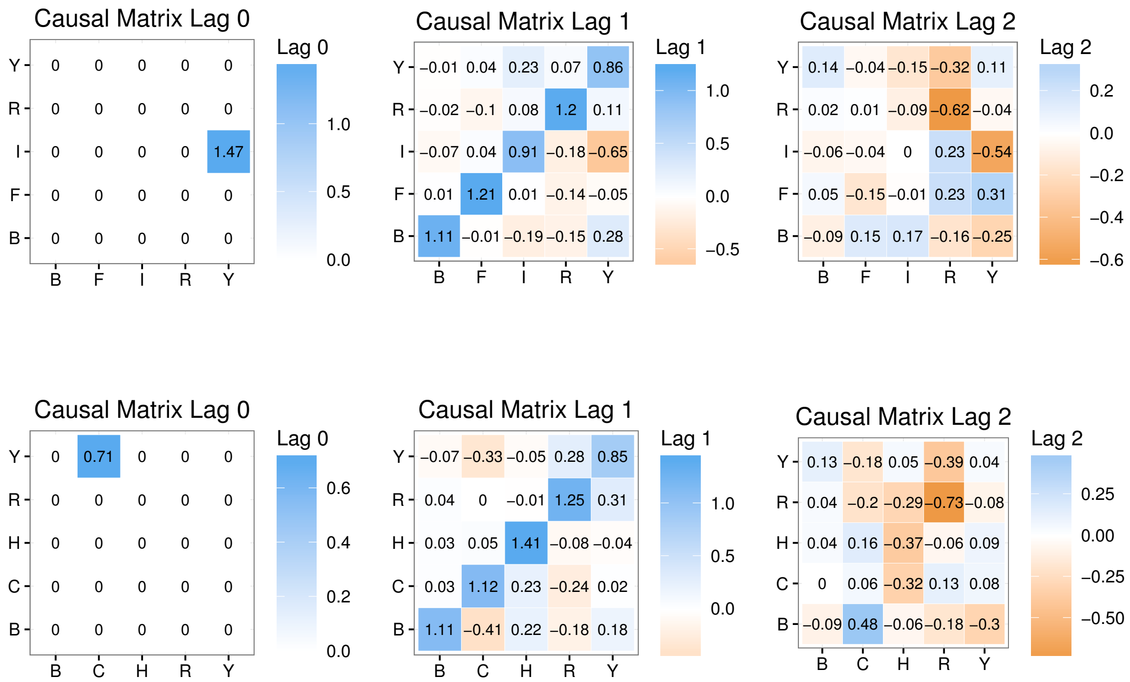

Figure 2.9: SVAR causal matrices up to the 2^{nd} lag for the baseline models 5 (top panels) and 6 (bottom panels). The entry below the table causes the entry on the left of the table.

and in the long-run (and converging to a value between 0.3% and 0.6%, consistently with the previously presented estimations), decomposing the private debt into mortgage and corporate debts allows a better understanding of the origin of the long-term negative effects caused by private debt in the all the previous figures. These new results indeed, which are shown in figure 2.11, suggest that not every form of credit to private entities has the same effect on output: corporate debt indeed (left panel) is mostly beneficial and has effects which are very similar to the public debt ones (also in size, even if in the long run it decreases); mortgage debt is instead the type of debt mostly harmful, which generates positive effect in the short-run, but strong negative effects in the long-run. Mortgage debt hence, we conclude, is the type of debt that also drives the dynamic of aggregate private debt in the previous results.

$$Y \xrightarrow{+} I \qquad\qquad C \xrightarrow{+} Y$$

Figure 2.10: Contemporaneous causal structure of models 5 (left) and 6 (right).

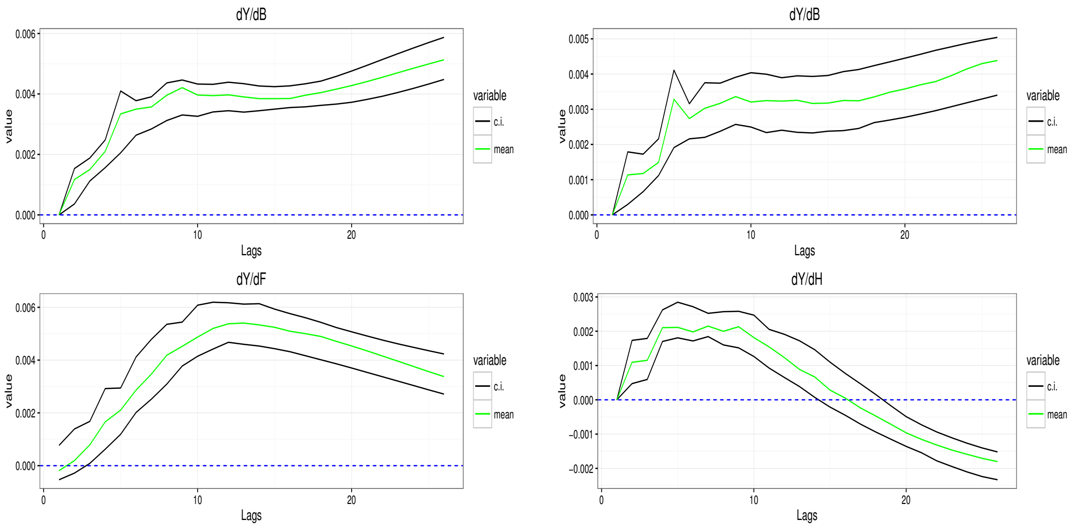

Figure 2.11: IRF of disaggregated models 5 (left) and 6 (right).

2.5.4 Crowding-out or crowding-in?

As an additional exercise we investigate the role that public and private debt have on driving other aggregate variables apart from aggregate output. In particular, a debate is still open concerning the crowding-out or crowding-in effects of public expenditure and public debt. Crowding-out hypothesis suggest that if government runs deficit and increases its debt by means of increased expenditures, this will not have any effect on output since the government "steals" opportunities – for consumption or for investment – that otherwise the private alone would have caught. The crowding-in hypothesis instead supports an opposite view; government running deficit, not only does not "steal" any opportunity for the private sector, but also contributes in creating a set of new opportunities which the private sector might catch.

Starting from the estimations of the two "disaggregated" scenarios, we control for two additional IRFs in figure 2.12, showing the effects of public and private disaggregated debts on aggregate private consumption and on aggregate private investment. The IRFs suggest that public debt shocks has positively persistent effects both on consumption and on investment (even if on the latter variable these effects are decreasing in the medium and long run). Corporate debt also bear positive effects, on investment. Household debt instead boosts consumption only in the very short run, while the effects become negative in the long-run, supporting once again the transmission mechanism already outlined before: mortgage debt is the most harmful among the debts and its negative effects goes all along from consumption to output.

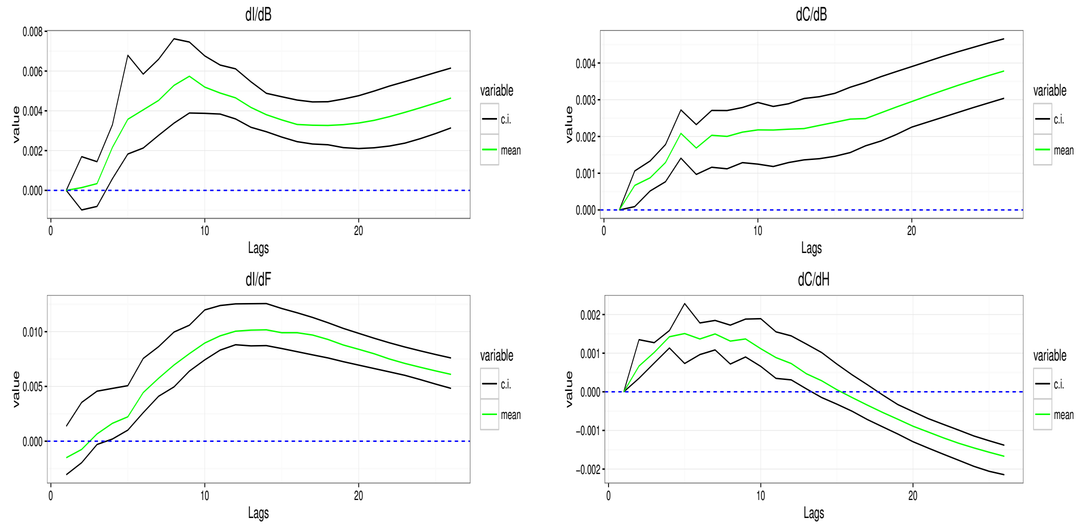

Figure 2.12: IRF of disaggregated models 5 (left) and 6 (right) related to the crowding-in effects.

2.5.5 Robustness check without the crisis period

As a final additional exercise we want to control for the fact that our results are not generate only by the last decade, characterized by turmoils and stagnation. Therefore we re-estimate the baseline regression (ID 1), but restricted to a sub sample which contains only data from 1966 (Q1) to 2008 (Q3) up until when the bankruptcy of the *Lehman Brothers Holdings Inc.* officially gave the start to the financial crisis.

In figure 2.13 it is possible to observe that the relations entailed in the full-sample models are kept without variations and only mild changes in the magnitudes of the effects can be found. Also the contemporaneous causal structure implied by the matrix $B = I - \Gamma_0$, also labeled "Lag 0" in the figure, is the same as the contemporaneous causal matrix presented for the baseline model 1 in figure 2.3.

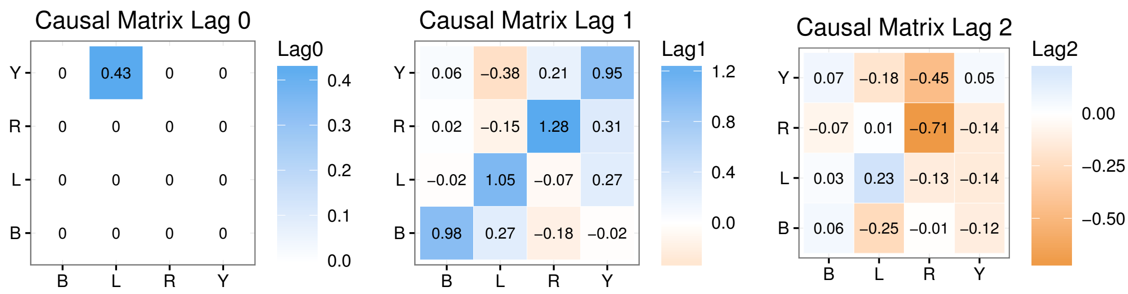

Figure 2.13: SVAR causal matrices up to the 2^{nd} lag for the baseline models 1 (top panels) and 2 (bottom panels). The entry below the table causes the entry on the left of the table.

Also concerning the dynamic causal relations entailed by the IRFs we can see in figure 2.14 confirms the results previously obtained with the same model, but estimated with the whole sample. The unique difference is that the right-tail of the IRF related to private debt is still non-significant, implying that private debt has no effect in the long run, but the mean of the IRF is shifted upward. This suggests that, even if the direction of the causality and of the effects are unchanged by the crisis, the great recession might have had some (negative) magnification effect on the IRF presented above.

2.6 Conclusions

In this paper we have employed modern multivariate time-series econometric techniques – that allow us to avoid the selection of *ad hoc* theory-driven restrictions – in order to study the effects that different forms of debt shocks have on aggregate output. We have found four types of evidence which we here collect under different nicknames:

- *The trivial: correlation is not causation.* As it can be seen in figure 2.2 and in

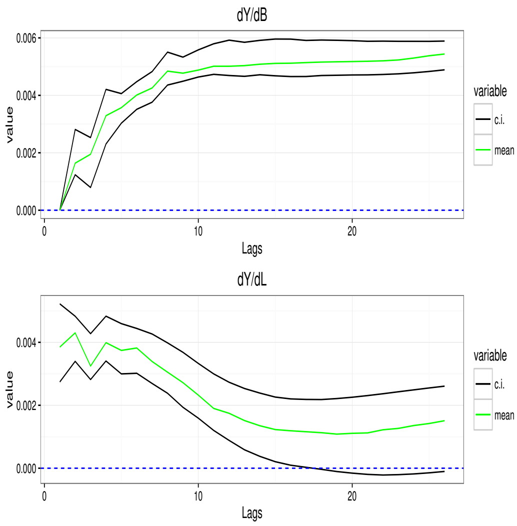

Figure 2.14: IRF of baseline model 1 related to the subsample without the crisis period.

figure 2.5, even if the cross-correlation (spurious effects / total derivative) between government debt and real GDP is mildly negative and the one between private debts (in its different forms) is positive, the causal effects (pure effects / partial derivative) has been found to be respectively positive and robust across all the analysed multivariate specifications for public debt while positive in the short-run but negative in the long-run for private debt.

- *The new: positive effects of public debt.* Notwithstanding the huge literature on the role of public debt on growth (see Reinhart and Rogoff, 2010b; Cecchetti et al., 2011; Minea and Parent, 2012; Herndon et al., 2013; Égert, 2015), we here introduce an innovation in terms of the estimation method and in terms of the included control variables, and we provide some new evidence on the effects of public debt on GDP. These effects, as already anticipated, are estimated to be positive and persistent.

- *The corroborating: negative effects of mortgage debt.* As in the recent growing literature relating mortgage debt and economic aggregate growth (see Mian and Sufi, 2009; Jordà et al., 2013; Batini et al., 2016), we find a strong negative association between household debt and long-run output level. This is due to the fact that booms in mortgage debt creates asset prices bubbles and is more prone in generating financial crisis, that "bites-back" the economy.

- *The unexpected: crowding-in effects of public debt.* We have also considered the effects of public debt on other macroeconomic aggregates and we found some results which might add on the open debate on whether the public sector, by investing and spending out of its own budget (i.e. creating deficits and eventually debt) crowds-out the private one. Our results suggest that this is not the case and that instead, the public debt bears positive feedbacks on the private sector, stimulating both aggregate consumption and investment.

Further research is still needed in order to verify whether the results that we have obtained are robust with respect to possible non-linearities in the level of debt (TVAR estimation) and possible heterogeneous effects in other countries. In particular it would be useful to apply the methodology also on other datasets, in order to control for the robustness of the findings with respect to other countries; two studies might be of particular interest: the first will compare the relation between the different types of debt and the economic performance in different European countries (e.g. northerns VS southerns); the second will analyze this relation in another country with a debt history very different from the US one (e.g. Japan). Moreover, with these datasets and in this framework, it might be possible to extend the causal search also to non-linear models – or at least to piecewise linear models such as the Threshold VAR (TVAR) – in order to better test the Reinhart and Rogoff, (2010a) conjecture of a non-linear relation between public debt and economic growth. After these empirical investigation will be done, a fully fledged micro founded agent-based model can be written and simulated, in order to investigate which are the microeconomic mechanisms that are able to reproduce the detected aggregate behaviors.

Notwithstanding these possible future improvements, we attempt at drawing some policy implication based on the above mentioned results and relating also to the continuously increasing literature providing empirical support for large fiscal multipliers and for the negative effects of fiscal austerity (Auerbach and Gorodnichenko, 2012; Blanchard and Leigh, 2013; Ferraresi et al., 2015; Jordà and Taylor, 2016).

We argue that recent restrictive fiscal policies aimed at the reduction of the debt-to-GDP ratios by means of increased taxation and reduced government expenditure are more harmful then beneficial since they “switch-off” all the positive feedbacks between government debt, consumption, investment and output that we have measured in this paper; in that direction are also the recent results obtained by Bernardini and Peersman, (2015). Following our results, the crucial debate for policymakers in the next years stands, we believe, on the correct regulation of debt contracts (in particular concerning mortgage contracts) to be operated by means of micro and macro prudential tools.

2.7 Appendix A - The VARLiNGAM Algorithm

A. Estimate the reduced form VAR model of equation (2.2) obtaining estimates $\hat{\mathbf{A}}_\mathbf{i}$ of the matrices $\mathbf{A_i}$, $\forall\, \mathbf{i} = \mathbf{1}, \ldots, \mathbf{p}$. Denote by $\hat{\mathbf{U}}$ the $K \times T$ matrix of the corresponding estimated VAR error terms, that is each column of $\hat{\mathbf{U}}$ is $\hat{\mathbf{u}}_\mathbf{t} \equiv (\hat{u}_{1t}, \ldots, \hat{u}_{Kt})'$, $\forall\, t = 1, \ldots, T$. Check whether the u_{it} (for all rows i) indeed are non-Gaussian, and proceed only if this is so.

B. Use *FastICA* or any other suitable ICA algorithm (Hyvarinen et al., 2001) to obtain a decomposition $\hat{\mathbf{U}} = \mathbf{PE}$ where $\mathbf{P}$ is $K \times K$ and $\mathbf{E}$ is $K \times T$, such that the rows of $\mathbf{E}$ are the estimated independent components of $\hat{\mathbf{U}}$. Then validate non-Gaussianity and (at least approximate) statistical independence of the components before proceeding.

C. Let $\tilde{\tilde{\mathbf{\Gamma}}}_\mathbf{0} = \mathbf{P^{-1}}$. Find $\tilde{\mathbf{\Gamma}}_\mathbf{0}$, the row-permutated version of $\tilde{\tilde{\mathbf{\Gamma}}}_\mathbf{0}$ which minimizes $\sum_i \frac{1}{|\tilde{\mathbf{\Gamma}}_{\mathbf{0,ii}}|}$ with respect to the permutation. Note that this is a linear matching problem which can be easily solved even for high K (Shimizu et al., 2006).

D. Divide each row of $\tilde{\mathbf{\Gamma}}_\mathbf{0}$ by its diagonal element, to obtain a matrix $\hat{\mathbf{\Gamma}}_\mathbf{0}$ with all ones on the diagonal.

E. Let $\tilde{\mathbf{B}} = \mathbf{I} - \hat{\mathbf{\Gamma}}_\mathbf{0}$.

F. Find the permutation matrix $\mathbf{Z}$ which makes $\mathbf{Z}\tilde{\mathbf{B}}\mathbf{Z^T}$ as close as possible to lower triangular. This can be formalized as minimizing the sum of squares of the permuted upper-triangular elements, and minimized using a heuristic procedure (Shimizu et al., 2006). Set the upper-triangular elements to zero, and permute back to obtain $\hat{\mathbf{B}}$ which now contains the acyclic contemporaneous structure. (Note that it is useful to check that $\mathbf{Z}\tilde{\mathbf{B}}\mathbf{Z^T}$ indeed is close to strictly lower-triangular).

G. $\hat{\mathbf{B}}$ now contains $K(K-1)/2$ non-zero elements, some of which may be very small (and statistically insignificant). For improved interpretation and visualization, it may be desired to prune out (set to zero) small elements at this stage, for instance using a bootstrap approach (Shimizu et al., 2006).

H. Finally, calculate estimates of $\mathbf{\Gamma_i}$, $\forall\, \mathbf{i} = \mathbf{1}, \ldots, \mathbf{p}$ for lagged effects using $\mathbf{\Gamma_i} = (\mathbf{I} - \hat{\mathbf{B}})\hat{\mathbf{A}}_\mathbf{i}$.

2.8 Appendix B - Testing the Assumptions Needed for the Causal Search Algorithm

At the base of our causal search algorithm stand two important assumptions:

- **A1:** the residuals are linear combinations of underlying components that are mutually independent;
- **A2:** the residuals are linear combinations of underlying components that are non-Gaussian (with at most one of them Gaussian).

But the first assumptions might also be relaxed and replaced by its milder version:

- **A2 bis:** the residuals are linear combinations of underlying components that are jointly non-Gaussian.

The typical assumption required for SVAR is orthogonality – i.e. absence of dependencies in the first moment – which is testable. Our causal search strategy instead requires independence – i.e. absence of dependencies in all the moments – that instead cannot be tested. Testing for absence of correlation at least, allows us to control for the first one and is a first indication for the fact that at least, the assumption is not rejected (up to the first order). We test this via a bivariate graphical inspection and via simple OLS regressions.

The second assumption is instead testable and we do that via Shapiro-Wilk test for normality on the residuals. Indeed the residuals are linear combinations of the underlying components (the structural shocks). Since a linear combination of Gaussian components would necessarily imply Gaussian residuals as well, if the residuals are non-Gaussian, they must have been generated by non-Gaussian components. Notice here that the rejection of H_0 stands for non-Gaussianity.

2.8.1 Testing the Independence Assumption

The bivariate graphical inspections presented in 2.15 allows to check for the presence of a clear pattern between the variables. The results here are presented only for the first model for sake of brevity, but for all the models, the results are very similar, indicating that the structural shocks do not present clear patterns and are therefore considerable as independent.

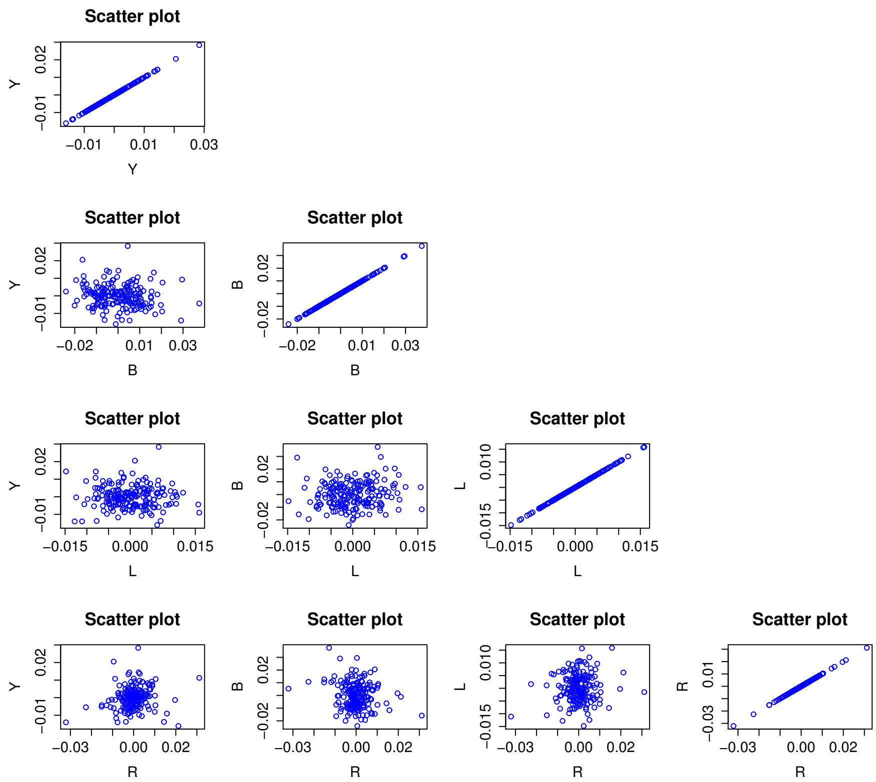

Figure 2.15: Independence test on the structural residuals of the baseline models 1.

Also a statistical analysis by means of OLS suggest that this is the case. In fact by running simple linear regressions of the type:

$$\varepsilon_i = \beta \varepsilon_j$$

where i and j stand for the residuals of two different variables, allows one again to check whether some linear relations between the structural residuals ε_i and ε_j exists. Table 2.5 presents the results for the first baseline model. For the others models the results are very similar and allow us to conclude that in this case the independence of the structural shocks is an assumption which is satisfied. Indeed only few times some parameters result slightly significant at the 5% while they never are at the 1%, indicating that their independence might be a good assumption.

Equation	*Coefficient*	*Standard Error*	*t statistic*	*p-value*
Y = f(B)	-0.08658981	0.04535515	-1.9091505	0.05774169
Y = f(L)	-0.04699905	0.08438565	-0.5569554	0.57820994
Y = f(R)	0.15871436	0.06997094	2.2682896	0.02443041
Y = f(B,L)	-0.08485211	0.04570988	-1.8563190	0.06495647
	-0.03061003	0.08431399	-0.3630480	0.71697261
Y = f(B,R)	-0.07105341	0.04569677	-1.5548891	0.12163660
	0.13977211	0.07076914	1.9750431	0.04971154
Y = f(L,R)	-0.06442700	0.08376427	-0.7691466	0.44276133
	0.16355376	0.07032789	2.3255889	0.02109609
Y = f(B,L,R)	-0.06774527	0.04612290	-1.4687989	0.14354977
	-0.04929656	0.08414325	-0.5858647	0.55866539
	0.14435692	0.07132240	2.0240053	0.04437743

Table 2.5: OLS regressions allowing to check for independence of the structural shocks

2.8.2 Testing the non-Gaussianity Assumption

In table 2.6 we check that the assumption **A1** or that at least its milder version, **A1 bis** is satisfied. From the first row, it is easy to see that at least the assumption **A1 bis** is always satisfied. After a more detailed check, we see that the strict version of it is satisfied in 3 out of 6 cases (the models with ID 1, 2 and 4) if we test this assumption at the 5%; if we test it at the 10% than, assumption **A1** is satisfied in 4 out of 6 cases (model with ID 5 adds to the list). We therefore conclude that the first of the two assumption required is always satisfied, even if sometimes only in its mildest version.

Variable	*ID 1*	*ID 2*	*ID 3*	*ID 4*	*ID 5*	*ID 6*
joint	0.000	0.000	0.000	0.000	0.000	0.000
u_Y	0.001	0.000	0.001	0.062	0.004	0.100
u_B	0.028	0.025	0.024	0.014	0.540	0.001
u_L	0.879	0.574	0.113	0.378		
u_F					0.062	
u_H						0.295
u_I			0.941		0.072	
u_C				0.030		0.107
u_R	0.000	0.000	0.000	0.000	0.000	0.000

Table 2.6: Shapiro-Wilk test for non-Gaussianity p-values. H_0 = Gaussianity

2.9 Appendix C - Robustness Analysis

In this section we estimate other 5 models in order to check whether our results might be robust to different specifications and to the inclusion of other possibly related variables.

Model 07

Variables: $(Y, B^{*}, H^{*}, F^{*}, I, C, R_{3m})$

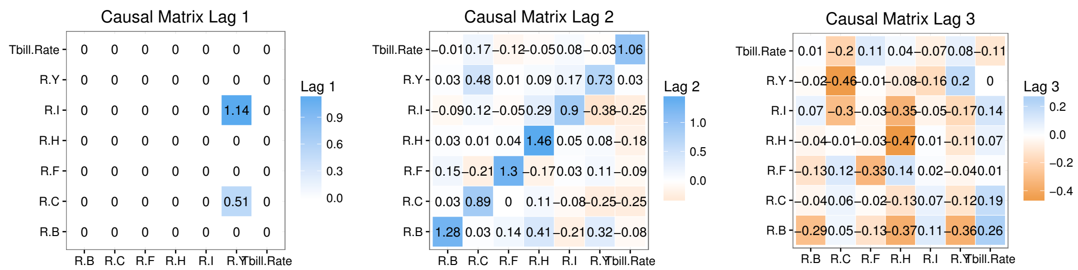

Figure 2.16: Structural VAR form of the model (up to lag-2).

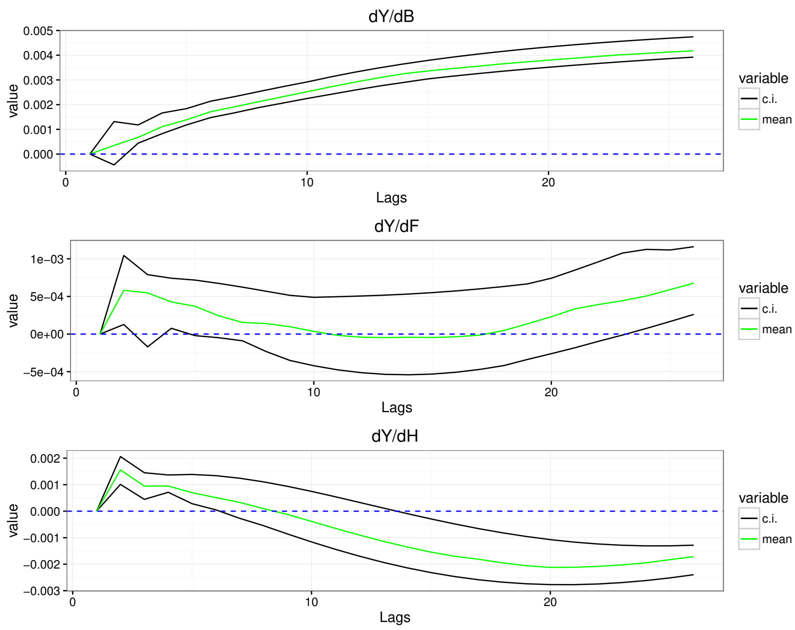

Figure 2.17: Impulse responses of the model.

Model 08

Variables: $(Y, B, H, F, I, C, R_{3m})$

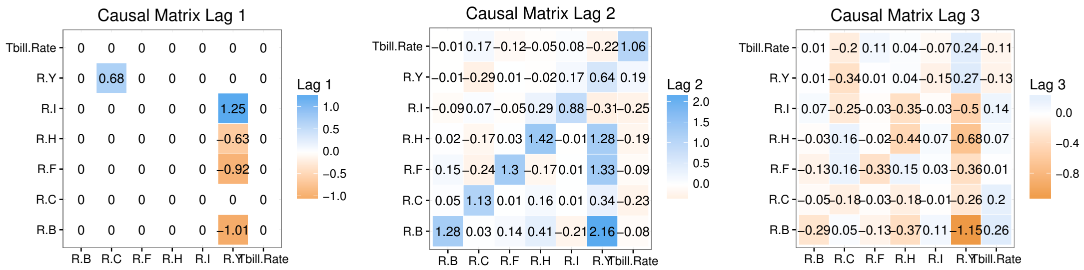

Figure 2.18: Structural VAR form of the model (up to lag-2).

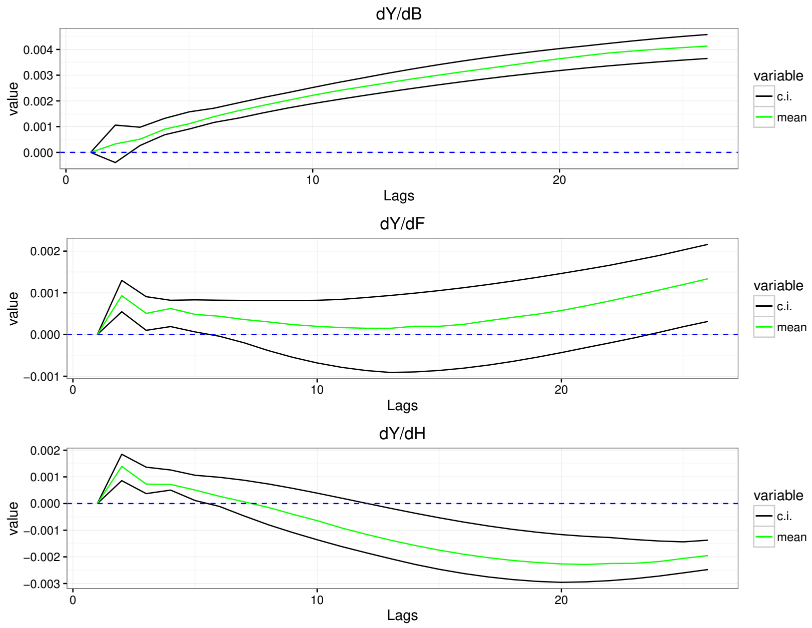

Figure 2.19: Impulse responses of the model.

Model 09

Variables: (Y, B, L, R_{10y})

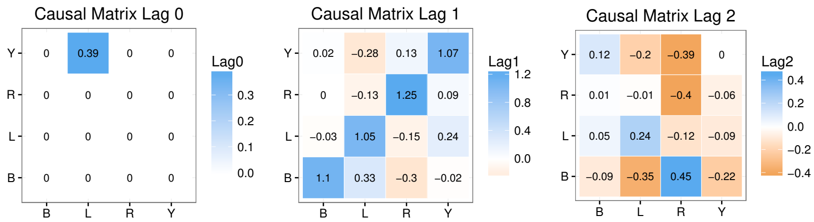

Figure 2.20: Structural VAR form of the model (up to lag-2).

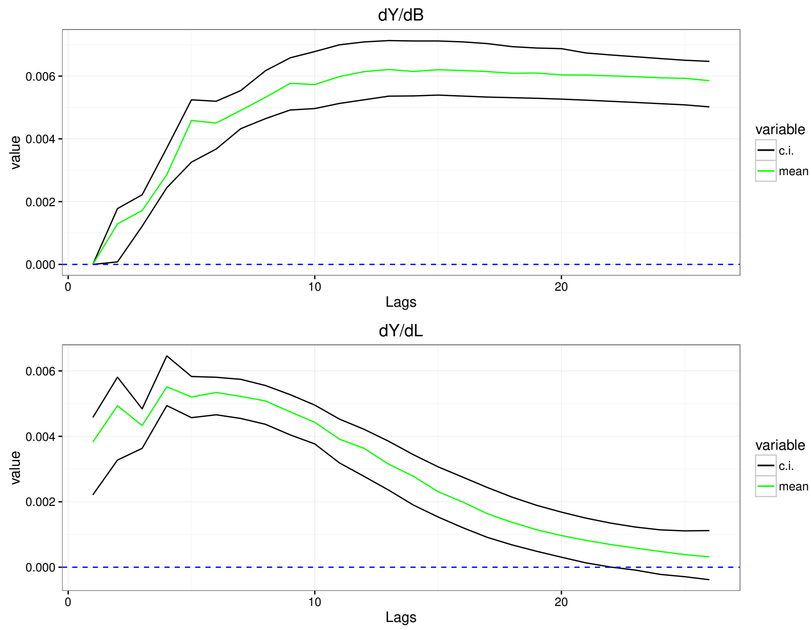

Figure 2.21: Impulse responses of the model.

Model 10

Variables: (Y, B, L, G)

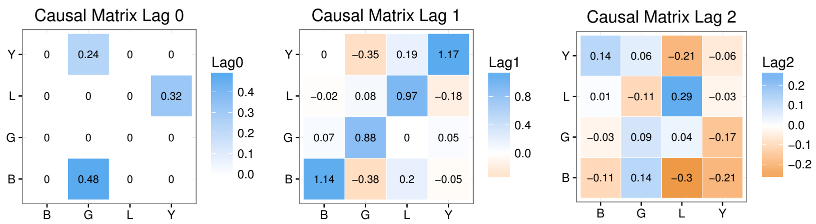

Figure 2.22: Structural VAR form of the model (up to lag-2).

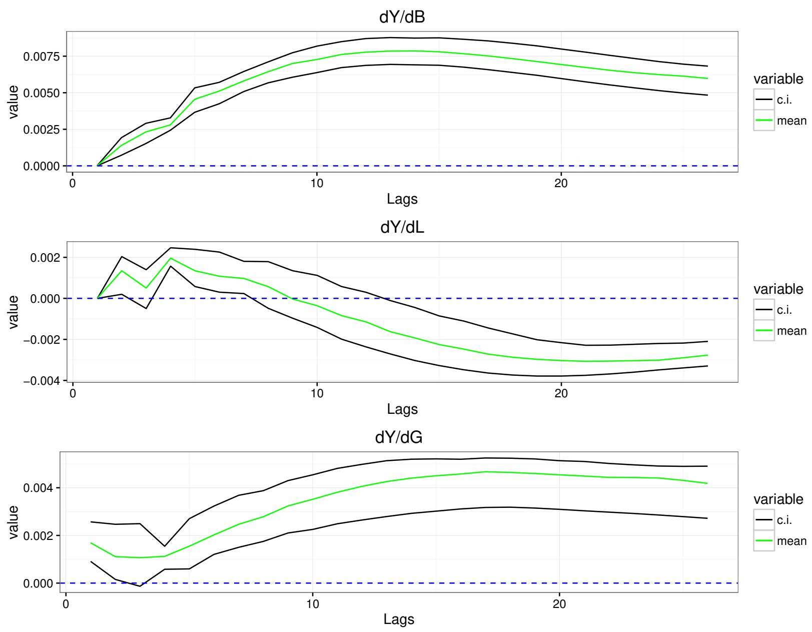

Figure 2.23: Impulse responses of the model.

Model 11

Variables: (Y, B, L, G, R_{10y})

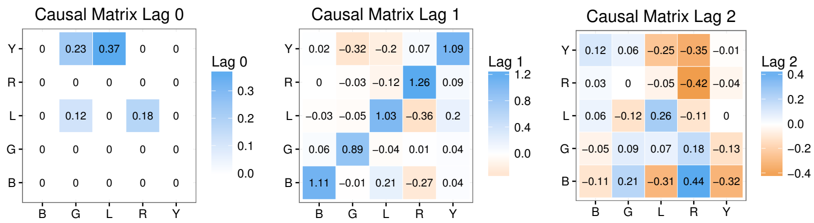

Figure 2.24: Structural VAR form of the model (up to lag-2).

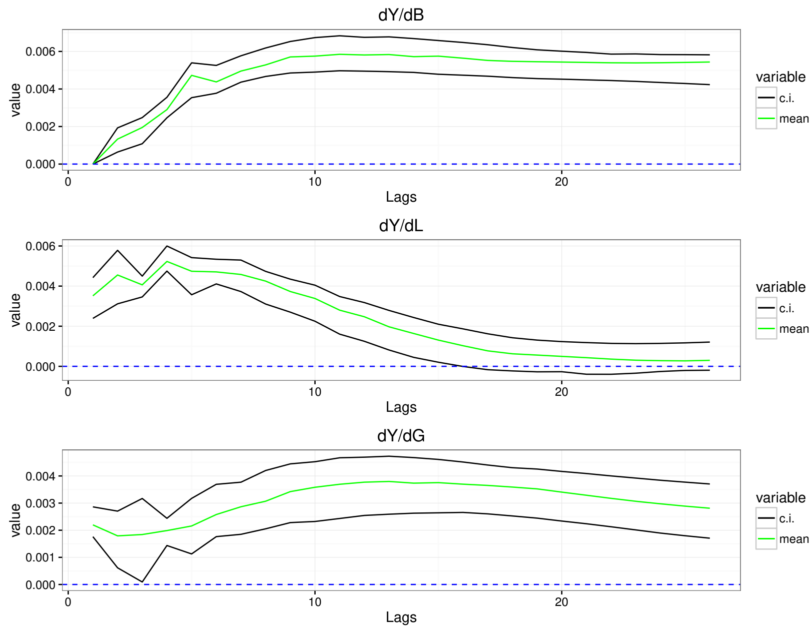

Figure 2.25: Impulse responses of the model.

CHAPTER

THREE

THE IMPACT OF HETEROGENEITY AND LOCAL INTERACTIONS ON MACROECONOMIC DYNAMICS

> *According to the cybernetic approach an economy, like biological evolution or any other algorithm, operates myopically, with no long-run goal. It constitutes a dynamical system whose behaviour over time can be studied regardless of whether or not it approaches any particular reference point. Like many other algorithms it might exhibit a strong attraction to certain reference points, but there will be circumstances under which it will not converge, or will converge slowly or non-monotonically. The central question Leijonhufvud was trying to get the profession to address was how, and under what circumstances, the algorithm of a decentralized market economy might exhibit attraction to a state of full employment equilibrium.*
>
> Howitt, (2002)

In this work, we develop an agent-based model to study the macroeconomic outcomes (e.g. full employment, coordination failures, involuntary unemployment) emerging out of the interactions occurring between heterogeneous firms and households in good and labor markets.

Since the "New Classical" revolution, most macroeconomists have been developing micro-founded macroeconomic model where a fully rational, representative household or firm maximizes an intertemporal utility or profit function under some constraints. Such a methodological commitment has allowed the profession to circumvent the problems of existence and stability of the general equilibrium (Kirman, 1989). Nevertheless, the price paid for such a shortcut has not been cheap: agents' heterogeneity and local interactions have been disregarded (see Kirman, 1992, for a sharp critique of the representative agent assumption).

At the same time, since the seminal contribution of Leijonhufvud, (1970), a research venture has been studying how coordination mechanisms in decentralized markets can possibly lead to full employment equilibrium or to persistent disequilibria (see e.g. Solow and Stiglitz, 1968; Clower and Leijonhufvud, 1975). In the latter case, mismatches between demand and supply of goods and labor are the norm, coordination failures (Cooper and John, 1988) can arise, and one can explain the emergence of involuntary unemployment without assuming a plethora of imperfections and frictions.

The natural outcome of such a program is to consider the economy as a *complex evolving system*, i.e. as an ecology populated by heterogeneous agents whose far-from-equilibrium interactions continuously change the structure of the system (Farmer and Foley, 2009; Kirman, 2010a; Rosser, 2011; Dosi, 2012a; Kirman, 2016a; Battiston et al., 2016). This is the methodological core of agent-based computational economics (ACE, Tesfatsion and Judd, 2006; LeBaron and Tesfatsion, 2008). Agent-based models (ABM) have "behavioral" microfoundations (Akerlof, 2002): in line with the micro-empirical evidence, agents (e.g. firms, workers, households) behave adaptively and employ heuristics in their decision and forecasting processes (see e.g. Tversky and Kahneman, 1986; Gigerenzer and Brighton, 2009; Camerer et al., 2011; Gigerenzer and Goldstein, 2011; Hommes, 2014).

An increasing number of agent-based models has studied decentralized interactions of heterogeneous agents in goods and/or labor markets.[1] In this work, however, we take a different path. Our aim is to develop a parsimonious model which bridges

[1]The number of macroeconomic agent-based model is increasing fast and an exhaustive list is beyond the scope of this work. For germane macro ABMs, see Russo et al., (2007), Dosi et al., (2010), Delli Gatti et al., (2010), Ashraf et al., (2012), Dosi et al., (2013), Dawid et al., (2014), Dosi et al., (2015), Riccetti et al., (2015), Assenza et al., (2015), Popoyan et al., (2015), Seppecher and Salle, (2015), and Dosi et al., (2016b,a). See also Fagiolo and Roventini, (2016) for a survey of macro agent-based models.

the agent-based framework with the DSGE one (see Fagiolo and Roventini, 2012, 2016, for a comparison of the DSGE and ACE paradigms) in order to study the role of coordination mechanisms in decentralized market economies. Indeed, our ABM is characterized by the presence of a full employment symmetric equilibrium, which can be considered as the reference point for the dynamics of the economic system. Moreover, as in the DSGE framework, the model sports a deterministic skeleton that can be hit by exogenous stochastic shocks. Such a structure allows one to directly compare the impulse-response functions (IRF) produced by both models and to assess the conditions (if any) under which the economy goes back to the full employment equilibrium after a shock.

The model considers an economy where heterogeneous firms and households trade in the goods and labor markets. Market interactions occur according to two different protocols. Similarly to DSGE models, in the *centralized matching scenario*, a fictitious auctioneer solves any possible coordination problem among the agents. On the contrary, in the *decentralized matching scenario*, agents locally interact in the markets. In such a regime, matching frictions and agents' heterogeneity may lead to imperfect allocations of goods and labor.

In both scenarios, we study the response of the economy to negative productivity shocks. Simulation results show that in the fully centralized scenario, the economy always come back to the full employment equilibrium, thus exhibiting a dynamics consistent with standard DSGE models. The presence of a "benevolent social planner" that organizes information efficiently works as a deus ex machina, thus solving any possible coordination issue among agents. On the contrary, in the fully decentralized regime, where information is dispersed and interactions are local, the economy fluctuates around an underemployment equilibrium characterized by persistent heterogeneity in firm and household populations. In addition, in this scenario the emerging coordination failures prevent real wage movements from driving the economy back to the full employment equilibrium. The latter results depends on the interplay between demand feedbacks and matching frictions in a population of heterogeneous agents. This suggests that macroeconomic models should seriously take into account agents' heterogeneity and decentralized market interactions.

The rest of the work is organized as follows. In Section 3.1, the model is introduced. Simulation results are presented in Section 3.2. Finally, Section 3.3 concludes.

3.1 The model

We consider a closed economy populated by F firms and H households. Firms produce a consumption good by using a linear technology that employs only labor. Households supply labor inelastically and consume the final good using the wage received by firms and their stock of liquid wealth. In the good and labor markets, firms and households are matched according to different protocols. The model is stock-flow consistent (SFC, see e.g. Godley and Lavoie, 2012): the transaction flow matrix is reported in Appendix 3.7.

3.1.1 Timeline of events

In any given time period (t), the following microeconomic decisions take place in sequential order:

1. Financial state variables are updated. Firms update their net-worth and households update their wealth.
2. Firms set their offered wage, the selling price and determine their expected demand.
3. Households compute their desired consumption levels.
4. The labor market opens. Employers and employees are matched using different protocols (see Section 3.1.3 below). Production takes place. Households receive their wages.
5. The goods market opens. Firms and consumers are matched using different protocols (see Section 3.1.3 below). Firms compute their profits and distribute dividends to households.
6. Households calculate their consumption expenditure and their savings.

7. Bankrupted firms exit from the economy and are replaced by new ones on a one-to-one basis. The wealth of defaulted households is reset to a constant value.

At the end of each time step, aggregate variables (e.g. GDP, investment, employment) are computed summing over the corresponding microeconomic variables.

3.1.2 Consumption, production, prices and wages

Firms fix production as well as the price and the wage they offer to the workers. At the same time, households set their desired consumption.

In line with the spirit of agent-based models and with microeconomic evidence, agents have adaptive behaviours and employ heuristics (see e.g. Tversky and Kahneman, 1986; Gigerenzer and Brighton, 2009; Camerer et al., 2011; Gigerenzer and Goldstein, 2011; Hommes, 2014), which usually boil down to linear decision rules. This also allows to keep the dimensionality of the parameter space as low as possible. Each decision rule is a linear combination of two effects: (i) a *within'* effect reflecting decisions based on the past levels of agent's state variables; (ii) a *network* effect accounting for the position of each agent with respect to its own peers. The latter effect allows to study how social interactions with neighbours (see Brock and Durlauf, 2001; Durlauf, 2004) influence the decisions of each agent.

The wage of a typical firm f is set as:

$$W_{f,t} = W_{f,t-1} + \gamma \Delta P_{f,t-1} + \alpha z^{lab}_{f,t-1} + \beta(\bar{W}_{f,t-1} - W_{f,t-1}), \qquad \gamma > 0, \alpha > 0, \beta > 0 \tag{3.1}$$

where $\Delta P_{f,t-1} = P_{f,t-1} - P_{f,t-2}$ relates price growth to wage dynamics (as in Solow and Stiglitz, 1968). The term $z^{lab}_{f,t-1} = n^d_{f,t-1} - n^s_{f,t-1}$ represents the firm excess demand for labor and implies that a gap between open and filled vacancies will lead to an increase in the wage offered by the firm, thus reflecting the attempts of the latter to become more competitive in attracting workers (see e.g. Mortensen and Pissarides, 1999; Diamond, 1982). The third term captures social interaction effects, measuring the deviations of firm wage with respect to the average wage set by its N_f neighbors in the previous period, i.e. $\bar{W}_{f,t-1} = \sum_{j=1}^{N_f} \omega_{f,j} W_{j,t-1}$. We assume that the network is complete so that $N_f = N-1$ for any firm f and that, in the computation of the average wage, each firm f randomly assigns heterogeneous weights $\omega_{f,j}$ to its neighbors.[2]

In a similar way, firms fix price in an imperfect competition framework according to the linear rule:

$$P_{f,t} = P_{f,t-1} + \gamma \Delta W_{f,t-1} + \alpha z^{good}_{f,t-1} + \beta(\bar{P}_{f,t-1} - P_{f,t-1}), . \quad \gamma > 0, \alpha > 0, \beta > 0 \tag{3.2}$$

The first term indexes price to wage growth. Notice that in the model, wage and price setting rules are linked one with the other, reflecting dynamic wage-indexation to prices and mark-up pricing in the spirit of Solow and Stiglitz, (1968). Moreover, in line with "customer market" models (Phelps and Winter, 1970; Diamond, 1971; Greenwald and Stiglitz, 2003), firms increase their price in presence of positive excess demand $z^{good}_{f,t-1} = q^d_{f,t-1} - q^s_{f,t-1}$ to exploit market power. Finally, the latter term in Eq. (3.2) captures the distance between the firm's price and the average one of its neighbors in the previous period ($\bar{P}_{f,t-1} = \sum_{j=1}^{N_f} \omega_{f,j} P_{j,t-1}$). Again, we assume that the firms network is complete, i.e. $N_f = N - 1, \forall f$.

The production of the consumption good takes place by means of a linear production process employing only labor ($n_{f,t}$):

$$q^s_{f,t} = a_{f,t} n_{f,t}, \tag{3.3}$$

where $a_{f,t}$ is the firm-specific labor productivity. Output is perishable and cannot be stored for the next period. Firms set desired production ($\hat{q}_{f,t}$) using a rule accounting for both within and network effects:

$$\hat{q}_{f,t} = \tilde{q}_f + \alpha z^{good}_{f,t-1} + \beta(\bar{q}_{f,t-1} - q_{f,t-1}). \quad \alpha > 0, \beta > 0 \tag{3.4}$$

The term $\tilde{q}_f$ captures reference production level, in line with the insights from behavioral economics about reference-dependence and the role of status quo biases in decision-making (see e.g. Kahneman et al., 1991; Koszegi and Rabin, 2009). The

[2] In order to generate the random graph we have adopted the Matlab functions built by Bounova and de Weck, (2012) and available online at `http://strategic.mit.edu/downloads.php?page=matlab_networks`.

above rule implies that deviations from the reference level of production are due to past excess demand $z^{good}_{f,t-1}$ and to the relative position of the firm vis-á-vis its neighbors $q_{f,t-1} - \bar{q}_{f,t-1}$, with $\bar{q}_{f,t-1} = \sum_{j=1}^{N_f} \omega_{f,j} q_{j,t-1}$ being the average production level set by firm f's neighbors in the previous period. .

Similarly to firms, households have a reference level for consumption, $\tilde{c}_h$. In addition, consumption is determined by the real value of wealth growth ($\Delta A_{h,t}/P_{t-1}$) to take into account the empirically relevant effect of wealth variation on consumption (see Sousa, 2009; Jawadi and Sousa, 2014). Moreover, household consumption is affected by social interaction effects, captured by the average level of past consumption across neighbors, $\bar{c}_{h,t-1} = \sum_{j=1}^{N_h} \omega_{h,j} c_{j,t-1}$. Such a effect allows one to account for external habits (see Duesenberry, 1949; Abel, 1990). To sum up, desired consumption is fixed according to:

$$\hat{c}_{h,t} = \tilde{c}_h + \alpha \frac{\Delta A_{h,t}}{P_{t-1}} + \beta(\bar{c}_{h,t-1} - c_{h,t-1}), \qquad \alpha > 0, \beta > 0 \tag{3.5}$$

3.1.3 Search and matching

In both goods and labor markets, there are two alternative matching scenarios. In the *centralized matching* scenarios, the presence of a fictitious auctioneer allows to avoid possible coordination issues among agents in the market. On the contrary, in the *decentralized matching* scenario, firms and workers interact locally in both the goods and labor market (in line with an increasing literature in agent-based models, see e.g. Ashraf et al., 2012; Riccetti et al., 2015; Assenza et al., 2015; Popoyan et al., 2015; Seppecher and Salle, 2015; Dosi et al., 2016b). Such a scenario allows us to study the relevance of heterogeneity and interactions and the possible emergence of coordination failures in a fully decentralized economy subject to shocks (more in Section 3.2 below).

The labor market

Firms demand labor to fulfill their production plans. Workers supply labor inelastically and have a zero reservation wage. Labor is measured in working hours terms.

Centralized matching regime. An "auctioneer" collects vacancies posted by firms and allocate workers to firms in proportion to their relative wage offers. Given the total number of households (H) and firms (F), the amount of labor supply allocated to each firm f is:

$$n^s_{f,t} = \frac{H}{F}\left(\frac{W_{f,t}}{\bar{W}_t}\right). \tag{3.6}$$

where $W_{f,t}$ is the firm wage and $\bar{W}_t$ is market average wage. The labor demand of each firm is

$$n^d_{f,t} = \left(\frac{\hat{q}_{f,t}}{a_{f,t-1}}\right)\left(\frac{W_{f,t}}{P_{f,t}}\right)^{-\varphi}. \tag{3.7}$$

The first term accounts for "Keynesian" demand expectations, while the second one links labor demand to the real wage.

The effective number of hours worked at the firm level is determined by the short side of the market:

$$n_{f,t} = min\left\{n^s_{f,t}, n^d_{f,t}\right\}. \tag{3.8}$$

It follows that if the demand constraint is binding, i.e. $n^d_{f,t} > n^s_{f,t}$, the firm is not able to cover all the opened vacancies, and it will produce $q_{f,t} < \hat{q}_{f,t}$. On the contrary if the supply constraint is binding, unemployment arises. In the centralized matching scenario, there is no frictional unemployment, and disequilibria at the micro-level can emerge only if total labor demand is higher or lower than total labor supply.

Decentralized matching regime. The matching between firms and workers is local. Firms post their vacancies and wage quotes. Workers decide to queue up or not for the job opened by a firm with a probability increasing in the offered wage. Labor demand is determined as in (3.7), but workers will search for open vacancies and will queue-up ($\Phi_{h,t} = 1$) or not ($\Phi_{h,t} = 0$) for a job according to the following Bernoulli trial:

$$\Phi^{LM}_{h,t} = \begin{cases} 0 & with\ \ probability\ \ p^{LM}_{f,t} \\ 1 & with\ \ probability\ \ 1 - p^{LM}_{f,t} \end{cases} \tag{3.9}$$

A worker can queue up for one job only, suppling inelastically one unit of labor. The probability of queuing $(1-p^{LM}_{f,t})$ is proportional to the wage offered by the firm relative to the market-average one:

$$1 - p^{LM}_{f,t} = 1 - \frac{1}{\rho^{LM}}\left[1 - \left(\frac{W_{f,t} - \bar{W}_t}{\bar{W}_t}\right)\right], \tag{3.10}$$

where $\bar{W}_t$ is the market average wage and $\rho^{LM} \in (1, \infty)$ is a parameter determining the degree of search frictions (and imperfect information) in the market. The higher the value of ρ^{LM}, the higher the probability that workers will queue up for any given difference between the firm's wage and average one. It follows that higher values of ρ^{LM} also imply higher intensity of competition in recruiting workers, which become more sensitive to wage differences across firms.

Finally, as in the previous scenario, the effective hours at the firm level are determined by the short side of the market, according to (3.8). However, notice that, differently from the centralized scenario, decentralized matching implies that frictional unemployment (or labor rationing) may arise even when the notional aggregate labor demand and aggregate labor supply are equal.

The goods market

The determination of supply is common in both scenarios: right after the labor market closes and workers have been allocated to the firms, the production of goods take place by means of the linear production process specified in Eq. (3.3).

Centralized matching scenario. Desired consumption (cfr, equation 3.5) is aggregated over households. Then total consumption, $\hat{C}_t = \sum_h \hat{c}_{h,t}$ is allocated to each firm f on the basis of the firm's price relative to the average one in the market. The (real) demand of the good for a single firm f is computed as follows

$$q^d_{f,t} = \frac{\hat{C}_t}{F}\left[1 - \left(\frac{P_{f,t}}{\bar{P}_t} - 1\right)\right]. \tag{3.11}$$

Notice that the above allocation is equivalent to the one that would emerge in equilibrium in Dixit-Stiglitz monopolistic competition. Moreover, the quantity of the consumption good effectively sold by a firm depends on the shortest side of the market:

$$q_{f,t} = min\left\{q^d_{f,t}, q^s_{f,t}\right\}. \tag{3.12}$$

If demand is higher than supply, then consumers are rationed in a symmetric fashion. On the contrary, if supply is higher than demand, the firm is not able to sell all its output and may experience losses.

Decentralized matching scenario. Contrary to the previous scenario, there is no centralized device attributing consumption shares to firms, and demand allocation is an emergent property of a costly search and matching process. In addition, similarly to the decentralized labor market scenario, we assume that consumers decide whether to queue-up ($\Phi^{GM}_{h,t} = 1$) or not ($\Phi^{GM}_{h,t} = 0$) for the goods sold by firms with a Bernoulli trial, which is formulated as follows

$$\Phi^{GM}_{h,t} = \begin{cases} 0 & with\ \ probability\ \ 1 - p^{GM}_{f,t} \\ 1 & with\ \ probability\ \ p^{GM}_{f,t}. \end{cases} \tag{3.13}$$

The probability of a success $p^{GM}_{f,t}$ reads:

$$p^{GM}_{f,t} = \frac{1}{\rho^{GM}} \left[1 - \left(\frac{P_{f,t} - \bar{P}_t}{\bar{P}_t} \right) \right]. \tag{3.14}$$

A household queues up only in one firm, demanding $\hat{c}_{h,t}$ units of the good. Notice that the probability of queuing up falls with the price $P_{f,t}$. Accordingly, more price-competitive firms will get longer queues and higher demand for their good. Moreover, the parameter $\rho^{GM} \in (1, \infty)$ in Eq. 3.14 is inversely related to the quality of the matching in the market. The higher is the value of the parameter, the lower the reaction of firms to differences between their price and the average price in the market. Accordingly, higher values of ρ^{GM} imply higher matching frictions and less competitive markets for goods.

Once all the households have queued up, the effective amount of product sold by a firm, $q_{f,t}$, is determined by the short side of the market as in Equation (3.12). Again if demand is higher than supply, consumers are symmetrically rationed. If the opposite happens, the firm will have some unsold non-storable output that perishes.

3.1.4 Financial conditions, exit and entry

After the matching process in the goods market is concluded, households determine their effective real consumption $c_{h,t} \leq \hat{c}_{h,t}$ and their consumption expenditure $\sum_{f=1}^{F} P_{f,t} c_{hf,t}$. They also compute savings, as the difference between income and effective nominal consumption. Households' income is represented by the earned wage $W_{h,t}$, and the fraction of firms profits paid as dividends, $D_{h,t}$. Accordingly, savings, $S_{h,t}$, are determined as:

$$S_{h,t} = W_{h,t} + D_{h,t} - \sum_{f=1}^{F} P_{f,t} c_{hf,t}. \tag{3.15}$$

We assume that the only assets available in the economy is money, which pays a zero interest rate. Households update their wealth ($A_{h,t+1}$) accordingly:

$$A_{h,t+1} = A_{h,t} + S_{h,t}. \tag{3.16}$$

Whenever the current wealth is higher than the initial one, the excess wealth will fuel a fund to bail-in bankrupted households and firms. A household is declared bankrupt whenever her wealth becomes negative. In turn, her wealth is reconstituted at the initial level employing the resources in the bail-out fund.[3]

Firms' profits $\Pi_{f,t}$ are equal to total sales revenues net of labor costs:

$$\Pi_{f,t} = q_{f,t} P_{f,t} - n_{f,t} W_{f,t}. \tag{3.17}$$

Whenever profits are positive, firms pay a fraction $1-\vartheta$ as dividends to households. As firm ownerships is symmetric, each household receives a fraction *1/H* of the dividends paid by each firm. It follows that the dividends received by household h in period t are equal to:

$$D_{h,t} = \frac{(1-\vartheta)}{H} \sum_{f=1}^{F} \Pi_{f,t}^{+}. \tag{3.18}$$

If profits are negative, firm's net worth is reduced accordingly. The law of motion of $A_{f,t+1}$ is than equal to:

$$A_{f,t+1} = \begin{cases} A_{f,t} + \vartheta \Pi_{f,t}^{+} \\ A_{f,t} + \Pi_{f,t}^{-}, \end{cases} \tag{3.19}$$

[3]Note that the presence of the bail-out fund guarantees the stock-flow consistency of the model as to the entry and exit of households and firms. Simulation results show that the resources in the fund are always sufficient to rescue bankrupted agents.

where $0 \leqslant \vartheta \leqslant 1$ is a parameter governing the fraction of retained profits ($\Pi^{+}_{f,t}$), and $\Pi^{-}_{f,t}$ denotes losses.

A firm is declared bankrupt when her net-worth is negative. In such a situation, the firm exits the market and it is replaced by a new entrant. The net-worth of the new firms is drawn from the bail-out fund and it is equal to the initial one. Households own an equal share of the firm, receiving its future dividends (if any). Finally, prices, wages and desired production of the entrant are computed as the average ones of the incumbents.

3.2 Simulation results

As anticipated in the introduction, the aim of this chapter is to investigate the conditions that allows an economy populated by heterogeneous, interacting agents to converge to the full employment equilibrium. In particular, we want to study how the matching protocols in labor and good markets affect the convergence process. The model presented in the previous section contains a deterministic skeleton that can be hit by exogenous stochastic shocks affecting structural variables (e.g. productivity). Such a structure is akin to DSGE models (e.g. Clarida et al., 1999; Woodford, 2011) and it allows a direct comparison of the impulse-response functions (IRFs) generated by both types of models. However, in our model all decisions are based on heuristic rules and, in contrast with the typical DSGE model, agents' behavior is adaptive and not grounded on hyper rational, forward looking behavior (see Fagiolo and Roventini, 2012, 2016, for a direct comparison of DSGE and agent-based models). Moreover, differently from the DSGE framework where Walrasian markets clear via price and wage movements, in our model the causality goes from quantities to prices (see equations 3.1 and 3.2).

Our agent-based model is characterized by the presence of a full employment symmetric equilibrium (derived in Appendix 3.8). More precisely, we define the full employment symmetric equilibrium as a situation characterized by

$$\begin{cases} \Delta x_t = 0, & \forall x \in \Omega \\ \tilde{u}_t = 0, \qquad \tilde{y}_t = 0, & \tilde{\pi}_t = 0 \end{cases} \tag{3.20}$$

where Ω is an array containing all the model (micro and macro) variables (x), $\tilde{y}_t$ is the output gap, and $\tilde{u}_t$ and $\tilde{\pi}_t$ are respectively the deviation of unemployment and inflation from their steady state values. This means that, consistently with the DSGE framework, in our agent-based model we have a possible emerging limit case in which not only the system is characterized by full-employment equilibrium, but also by agents' homogeneity. Such a result allow us to directly compare the results generated by our model vis-à-vis those of DSGE ones.

Let us now consider several simulation exercises[4] in order to study the stability of the full employment equilibrium under different productivity shocks for alternative matching scenarios in the labor and goods markets (cfr. Section 3.2.1). We will then assess the robustness of our results in Section 3.2.2. Table 3.1 contains the values of the parameters of our baseline simulation environment.

3.2.1 The effects of productivity shocks

We begin by initializing the variables of the model (consumption, wages, prices, production, firms' net worth, households' wealth, etc.) at values compatible with the full-employment, symmetric equilibrium of the economy (cfr. conditions (3.20) above). We then let a negative technology shock hit the economy at the firm level and we study the stability of the ensuing equilibrium and the convergence properties of the model. More precisely, we consider a negative, idiosyncratic change in the value of firm productivity at time t^*. The dynamics of the shock writes as:

$$a_{f,t} = \tilde{a}(1 - \eta_{f,t}) \qquad \textit{where}: \begin{cases} \textit{if} \quad t < t^* & \eta_{f,t} = 0 \\ \textit{if} \quad t = t^* & \eta_{f,t} \sim \mathcal{N}(\mu_\eta, \sigma_\eta) \\ \textit{if} \quad t > t^* & \eta_{f,t} = \rho_\eta \eta_{f,t-1} \end{cases} \tag{3.21}$$

[4]The pseudo-code of the model is reported in Appendix 3.9. The code is available from the authors upon request.

Model Parameters		
Symbol	*Value*	*Meaning*
MC	100	Monte Carlo realizations
T	1500	time simulations
H	200	number of households
F	20	number of firms
α	0.4	sensitivity to main economic effects
β	0.4	sensitivity to social effects
γ	0.4	sensitivity of wage/price indexations
ϑ	0.5	percentage of retained profits
φ	5	sensitivity of labor demand to real wage
ρ^{LM}	2	easiness of matching in the labor market
ρ^{GM}	2	difficulty of matching in the goods market
μ_η	-0.01	supply shock average
σ_η	0.002	supply shock variance
ρ_η	0.98	supply shock persistence

Table 3.1: Baseline parametrization of the model.

where μ_η, σ_η, and ρ_η represent, respectively, the mean, the standard deviation and the autoregressive persistence of the shock. [5]

In what follows, the effect of supply shocks will be studied in both the fully *centralized* and *decentralized* scenarios. In the first regime, matching is centralized in both the labor and goods markets, while in the second one, search and matching processes are local in both markets.

The non-linearities in agents' decision rules and their interaction patterns imply that the model does not allow for analytical, closed-form solutions. This is a general feature of agent-based models[6] and it forces us to perform extensive Monte-Carlo anal-

[5]The above formulation of the productivity shock is also in line with Cooper and Schott, (2013), who introduce firm heterogeneity in a simple RBC by means of idiosyncratic technology shocks. In what follows, the shock will hit all the firms, but the results are robust also with respect to shocks that hit only sub-samples of firms.

[6]Methodological issues concerning the exploration of the properties of agent-based models are dis-

yses to wash away across-simulation variability in order to study the dynamics of micro and macro variables. Consequently, all results below refer to across-run averages over 100 replications and we report the standard-error bands.

In all simulations we set the number of households $H = 200$ and the number of firms $F = 20$, and we run the economy for $T = 1500$. We tune the shock by setting $\mu_\eta = -0.01$, $\sigma_\eta = 0.002$, $\rho_\eta = 0.98$ and $t^* = 50$. All the simulations parameters are reported in Table 3.1.

Productivity shocks in the fully centralized scenario

In presence of a negative productivity shock, firm production falls immediately causing a period of excess demand in the goods market (cfr. Figure 3.1). As a consequence, households are rationed and are forced to increase saving. Such a situation increases prices and in turn induces firms to demand more labor, putting inflationary pressure on wages. In addition, prices will rise further as they are indexed to wages and there is still excess demand in the market for goods. However, as prices change more than wages, the real wage will fall.

The centralized allocation mechanism at work in the labor market avoids any rise of frictional unemployment. This fact, together with the higher savings from demand rationing, contributes to keep aggregate demand high,[7] and the excess demand in the two markets to persist as long as production is constrained by low productivity. However, as time goes by, productivity will monotonically return to its equilibrium level. Accordingly, production will be back to the equilibrium level, causing excess demand to vanish. The system settles down in the original equilibrium (cf. Figure 3.1). In this scenario, out-of-equilibrium dynamics are only temporary and the system is able to effectively reabsorb the shock.

Figure 3.2 shows the evolution of the variance of the distributions of some key micro variables of the model. The figure provides insights about the agents' heterogeneity that underlies the aggregate dynamics exposed above. As the plots reveal,

cussed in Fagiolo et al., (2007) and Fagiolo and Roventini, (2012, 2016).

[7] In particular, real savings from demand rationing rise more than the fall in real income due to lower real wages.

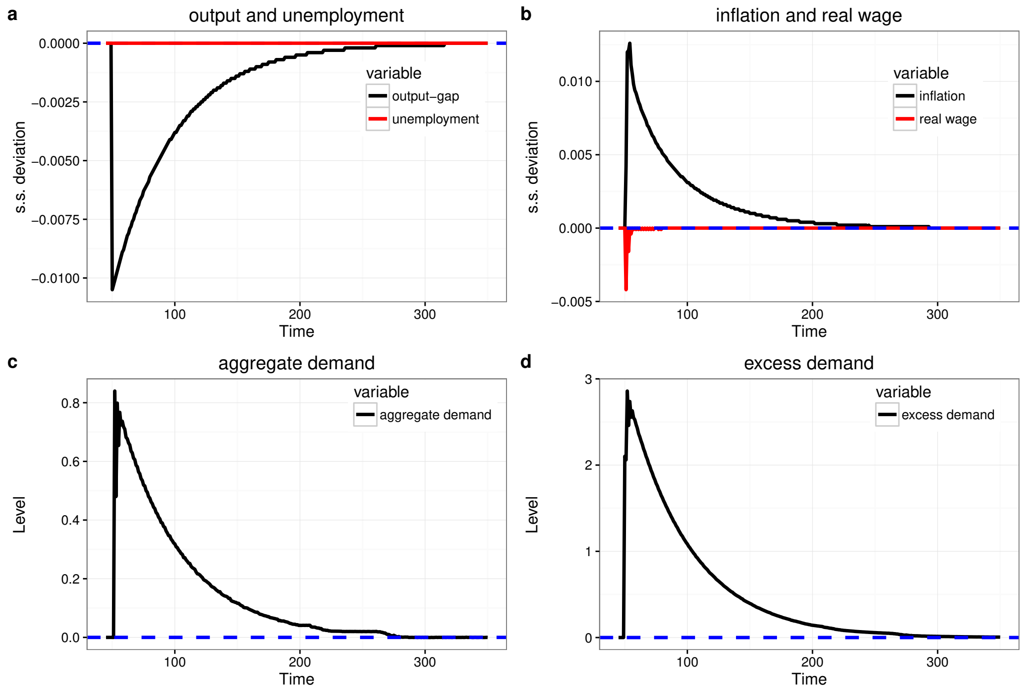

Figure 3.1: Emergent macroeconomic dynamics under supply shocks. Fully centralized scenario.

the micro-level heterogeneity introduced by the productivity shock is only temporary, very mild, and limited to few variables of the system. In particular, constant hours worked together with persistent full employment lead to homogeneity in wages. Finally, the effects of agents' heterogeneity do not persistently affect macroeconomic dynamics and eventually dies off when the effect of the shock become nil.

The foregoing results show that an economy with fully centralized matching protocols is able to restore the full-employment equilibrium without creating persistent distortions in the system and the emergence of coordination failures. This result is perfectly in line with DSGE macroeconomics. In particular, the simulation dynamics in this scenario replicates the behaviour of standard impulse response functions (IRFs) produced by Real Business Cycles and New Keynesian DSGE models (e.g. Clarida et al., 1999), as well the standard results in the empirical macro literature, showing that in presence of supply shock, prices and output move in opposite directions (see Blanchard, 1989).

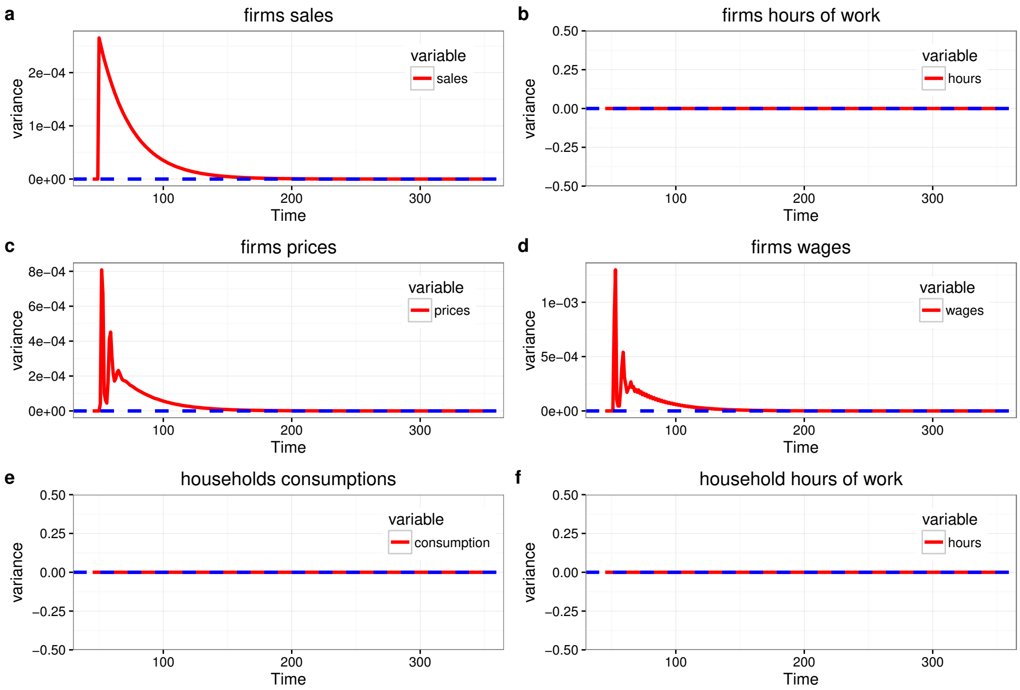

Figure 3.2: Micro-level variances under supply shocks. Fully centralized scenario.

Productivity shocks in the fully decentralized scenario

As search and matching processes are fully decentralized in both the labor and goods markets, the productivity shock creates both frictional unemployment in the labor market, and micro mismatches between demand and supply in the goods market. As a result, significant heterogeneity (see Figure 3.4) now emerges both at the firm level (in terms of prices, wage offers, output and labor demand) and at household level (in terms of hours worked and incomes).

What is more, micro heterogeneity has now consequences at the aggregate level, amplifying the effects of the initial shock. More precisely, the initial frictional unemployment stemming from decentralized matching in the labor market feeds back into lower consumption in the goods market, further contributing to depress firm output, and, in turn, labor demand, and real wages. Indeed, the fall in real wages is much stronger now than in the centralized scenario (compare the second panel in Figures 3.3 and 3.1).

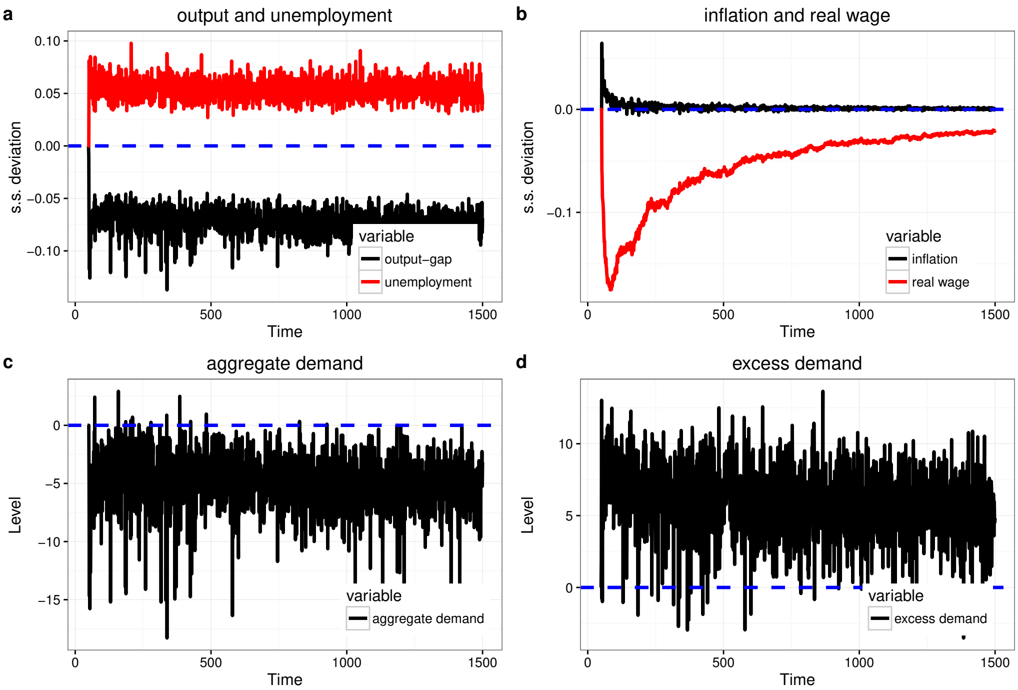

Figure 3.3: Emergent macroeconomic dynamics under supply shocks. Fully decentralized scenario.

The emerging result is a disequilibrium wherein aggregate demand is lower than in the full-employment case and fluctuates around the supply level, causing also *involuntary* unemployment to emerge (cfr. Figure 3.3; see Dosi et al., 2010, 2013, 2015, 2016b, for agent-based models where involuntary unemployment emerges because of low aggregate demand).

Furthermore, differently from the fully centralized regime, coordination failures emerge and the economy is not able to reabsorb the shock. At the aggregate level, the output-gap and unemployment keeps fluctuating around values that are, respectively, significantly lower and higher than the full-employment equilibrium (cfr. Figure 3.3). The same occurs for the levels of aggregate demand and supply, which are persistently lower than full employment ones. Finally, and again in contrast with the fully centralized scenario, micro-level variance does not fade away in the long-run (see Figure 3.4).

The only exceptions to the above general dynamics are represented by price inflation and real wage. Indeed, the fluctuations of such variable are in the long-run much

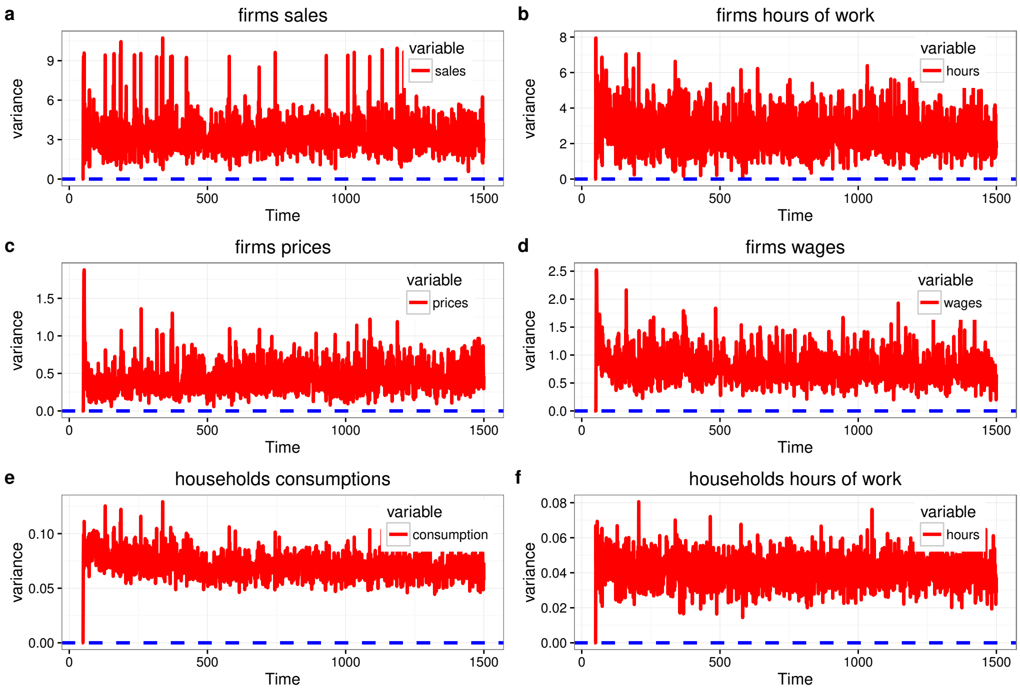

Figure 3.4: Micro-level variances under supply shocks. Fully decentralized scenario.

milder than for the other variables (basically zero for inflation) and around steady-state values.

As both the mean and the variance of all the variables in the model exhibit fluctuations around stable values in the long-run, in this scenario the economy self-organizes in a new statistical equilibrium, defined as *a state where some relevant statistics of the system are stationary* (Grazzini and Richiardi, 2015; Guerini and Moneta, 2016).

The persistent heterogeneity at the micro-level arises because frictions in the search and matching processes get now amplified by aggregate demand feedbacks in the goods market and by involuntary unemployment. As a consequence, micro-level heterogeneity now matters for the aggregate, and it is in particular responsible for the persistent deviation of aggregate variables from their full employment levels. In addition, and well in line with the original Keynes' analysis (see Clower and Leijonhufvud, 1975), price rigidity is not the source of underemployment and coordination failures. Indeed, persistent unemployment and low aggregate demand emerge notwithstanding the fact that the real wage falls and then eventually converges to values close to the old steady-state ones.

Taking stock of productivity shocks in different search and matching scenarios

Table 3.2 summarizes the results obtained so far by presenting the long-run values of the main aggregate variables following the negative supply shock under different matching scenarios. The values presented in the table are averages across 100 Monte-Carlo iterations.

	output-gap	unemployment	inflation	real-wage
Supply Shock FC	-0.00	0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Supply Shock FD	-0.06	0.05	0.00	-0.03
	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.2: Long-run values of the main aggregate variables for different matching scenarios. Values are averages over MC=100 Monte-Carlo iterations. Monte-Carlo standard errors in parentheses. FC: fully centralized scenario. FD: fully decentralized scenario.

As the table shows quite neatly, the economy is always able to return to the full-employment equilibrium in the fully centralized scenario. In contrast, the presence of an under-employment statistical equilibrium emerges as a robust property[8] across simulation runs in the fully decentralized scenario. Such a statistical equilibrium is always characterized by persistent (negative) output gap and unemployment. Moreover the real wage is lower than in full employment (see the last column of Table 3.2). However, differently from DSGE models, a fall in the real wage is not able to eradicate unemployment in the labor market.

[8]We also tested the robustness of the statistical equilibrium by performing Kolmogorov-Smirnov tests of equality in distributions of the Monte-Carlo time series generated by the model for the different macroeconomic variables (see the test for statistical equilibrium performed in Guerini and Moneta, 2016). The results of the test shows that the distributions across Monte Carlo are equivalent over time, indicating that the aggregate variables converge to a statistical equilibrium.

Our simulation results show the importance of heterogeneity and interactions for explaining persistent fluctuations in decentralized markets. Indeed, depending on the type of search and matching process, an ecology of heterogeneous agents following adaptive rules may (or not) generate a situation of persistent under-employment. Such a difference in dynamics cannot be typically observed in New Keynesian DSGE models as they are nested in the representative agent equilibrium framework.

3.2.2 Robustness analysis

In the previous section we documented how an economy endowed with a decentralized search and matching structure is not able to reabsorb the effects of an adverse supply shock and to go back to the full employment equilibrium. In this section we turn to investigate the robustness of the foregoing result to changes in some of the key parameters of the model.

We first investigate the robustness of the model with respect to the seed in the random number generator governing the impact of the shock in Equation (3.21). We find that all simulation results are robust to different sequences of random numbers.

We then study how the results of the model are affected by the persistence of productivity shocks (cfr. Equation 3.21). As expected, increasing the persistence of the shock has only effects in the fully centralized scenario, lowering the speed of convergence of the economy to the full employment equilibrium.[9]

The parameter regulating the percentage of profits firms distribute as dividends $(1-\vartheta)$ is particularly relevant to study as it provides a neat assessment of the role that aggregate demand dynamics play in the model. Indeed, higher amount of dividends (see Equation (3.18) could possibly compensate the fall in real wages experienced by workers after a negative productivity shock, increasing the resilience of the economy.

However, as Figure 3.5 shows, this is not the case. The output-gap and unemployment are basically invariant with respect to an increase in the share of dividends paid to households. Only the inflation rate and the real wage are affected for extreme high values of the parameters. A scenario where almost all profits are paid out as dividends

[9]The results related to these first two robustness exercises are available from the authors upon request.

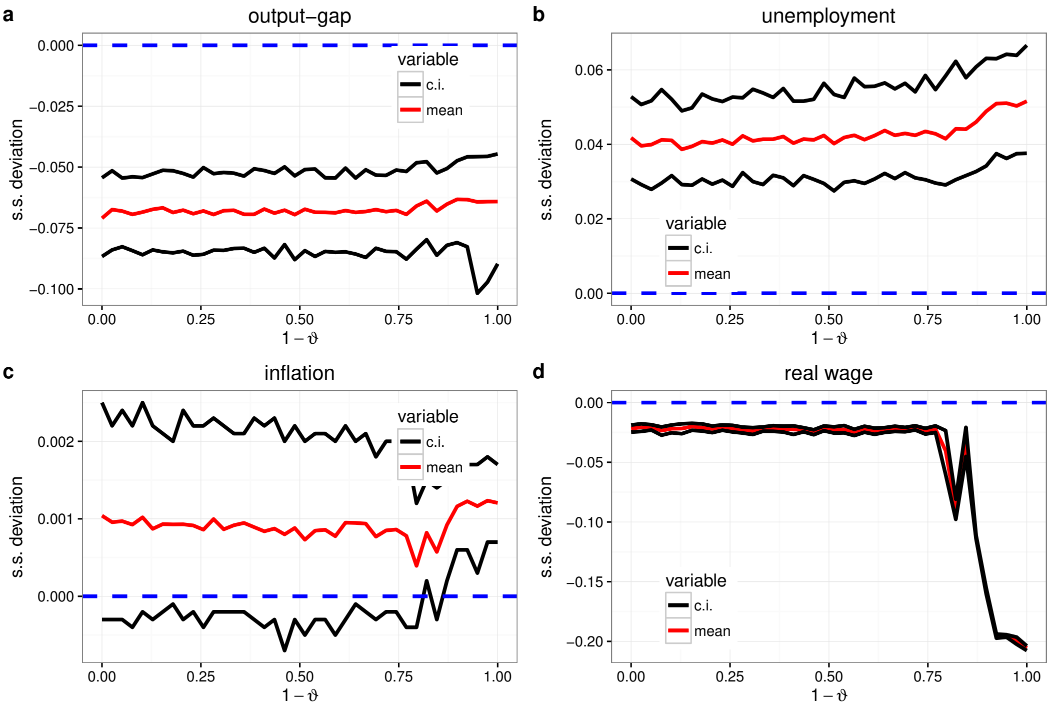

Figure 3.5: Effects of a variation on the percentage of retained profits parameters ϑ. The red line represents the mean of the last $T_{ss} = 200$ periods of the simulation, for any parameter value. The black lines represent confidence intervals which are computed as the maximum and the minimum values attained in the same period.

spur excess demand. As a consequence, firms increase prices, thus leading to the surge of average inflation observed for extremely high values of $1-\vartheta$. Finally, high inflation rate together with the depressing effect of unemployment of nominal wages explains the fall observed in the real wage.

An additional robustness analysis exercise concerns the parameters ρ^{LM} and ρ^{GM}, which capture matching frictions in the labor and goods markets. Higher values of ρ^{LM} increase the probability that workers queue up at any given firm, thus increasing the quality of matching in the labor market. Moreover, lower levels of ρ^{GM} raises the probability that households queue up at any given firm in the goods market, thereby boosting the matching quality in that market. In our sensitivity exercise we change the two parameters independently. The results are reported in the heat maps presented in Figure 3.6. We find that lower matching frictions in both markets improves the overall resilience of the economy, which show an improved ability to get closer to the full employment equilibrium after a productivity shock, like in the fully centralized scenario. (bottom left corners). Indeed, output increases, unemployment and inflation fall, and the real wage is on average smaller. Such results are not surprising: improving matching quality makes market interactions less local: workers and consumers queue up at a larger fraction of firms for any given price and wage differences. Moreover, lower matching frictions implies higher sensitivity of labor and consumption demand to cross-firms price differentials in both markets. Accordingly, price variations can quickly mop up micro-disequilibria and adjustment mechanisms mimic the ones at work in representative-agent DSGE models.

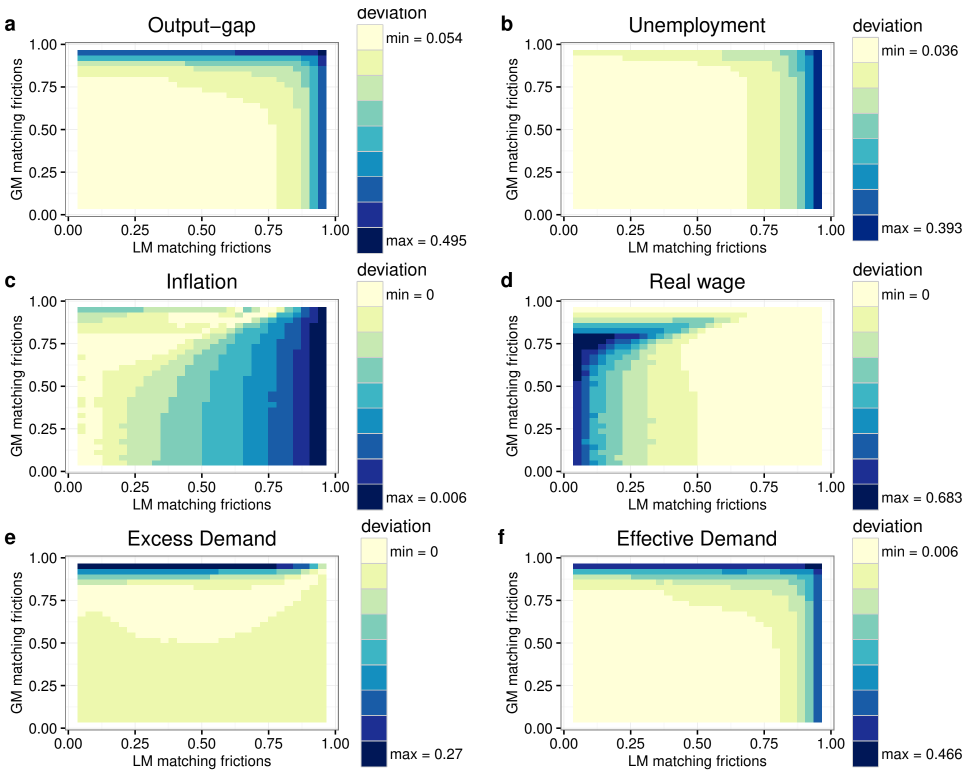

Figure 3.6: Effects of a variation in the quality of matching in the labor (horizontal dimension) and in the goods markets (vertical dimension). From left/bottom to right/top the quality of matching deteriorates.

Finally, we explore the causal mechanisms responsible for the stability (or not) of the full employment equilibrium. On one side, output and unemployment appear to be closer to the full-employment equilibrium in presence of large falls of the real wage. This correlation might suggest the presence of some Walrasian Neoclassical mechanisms at work. On the other side, the strong correlation between output, unemployment and effective demand point to Keynesian adjustment dynamics. In order to shed some light on these possible alternative explanations, we repeat the last exercise concerning matching friction parameters (ρ^{LM} and ρ^{GM}) assuming *fixed* real wage (i.e. $P_{f,t} = W_{f,t}$, cf. equation 3.2). Simulation results show that the results found with flexible real wage are mostly unaffected (see Figure 3.7), pointing then to the driving role of Keynesian adjustment mechanisms.

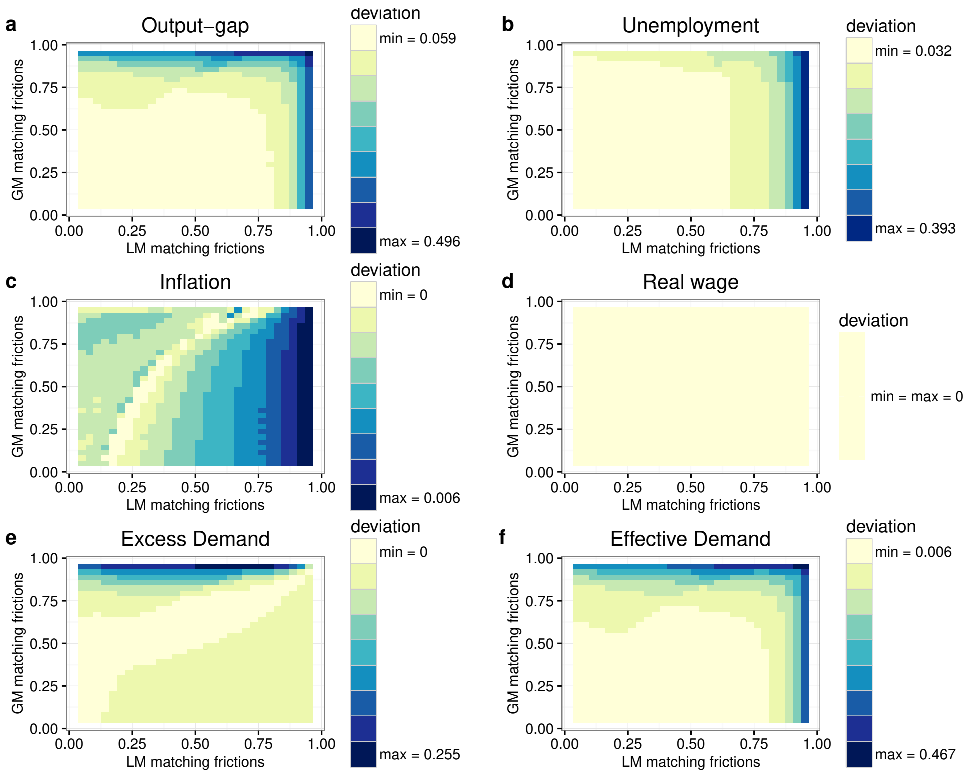

Figure 3.7: Fixed real-wages simulation. Effects of a variation in the quality of matching in the labor (horizontal dimension) and in the goods markets (vertical dimension). From left/bottom to right/top the quality of matching deteriorates.

3.3 Conclusions

In this work we develop an agent-based model (ABM) where an ecology of heterogeneous firms and households interact in labor and good markets according to centralized or local search and matching processes. The model is characterized by a full employment symmetric equilibrium and by a deterministic backbone that can be hit by exogenous, stochastic shocks. The structure of our ABM is akin to the one of DSGE models and it allows a direct comparison of the impulse-response functions observed in those frameworks. However, in DSGE models, a fully-rational representative agent take optimal decisions, whereas in our ABM, heterogeneous agents behave according to adaptive rules and explicitly interact in markets. In that, our model takes into account the insights stemming from behavioral economics (e.g. Camerer et al., 2011; Gigerenzer and Goldstein, 2011) and search theory (e.g. Mortensen and Pissarides, 1999).

We study the response of the economy to a negative productivity shock under two different institutional arrangements governing interactions in labor and goods markets. In the fully centralized scenario, a fictitious auctioneer distributes the labor force and consumption demand across firms following allocation rules similar to those emerging in the equilibrium of monopolistically competitive markets. In the fully decentralized scenario, search and matching is local. Accordingly, frictions and firms and households heterogeneity can arise due to the imperfect allocation of labor and demand across firms.

We find that in the fully centralized scenario, the economy is always able to return to the full employment equilibrium after a shock and it displays a dynamics very similar to the one generated by standard DSGE models. In contrast, when search is local the economy persistently deviates from full employment, and converges to a statistical equilibrium where output and unemployment are lower than their full employment values and firms and households display persistent heterogeneity. The interplay between coordination failures in the labor markets and positive demand feedbacks is at the core of the above result. In the fully decentralized scenario the supply shock generates heterogeneity across firms and some frictional unemployment. The latter has however a negative impact on household consumption, thus triggering Keynesian involuntary unemployment. In such a situation, the fall in the real wage contributes to foster deviations of the economy from the full employment rather than contributing to restoring it.

We also investigated the robustness of the above result to different degree of efficiencies of matching in labor and goods markets. We show that higher matching efficiency has a beneficial effect on the ability of the economy to return to full employment. Indeed, a better matching greases the wheel of the market allocation mechanisms, and the decentralized economy becomes more similar to the fully centralized one, where prices are able to put markets back to equilibrium (as in DSGE models). Such a results hold also when real wage is fixed, suggesting, again, the driving role of Keynesian adjustment mechanisms in the model.

Our results have at least two implications for the current macroeconomic theory. First they show that, under some conditions, an agent-based model embedding boundedly rational decision rules is able to generate dynamics resembling those produced by DSGE models, and in particular to display convergence to full employment equilibrium. However, the results also show that such an outcome depends on the restrictive assumptions concerning the interaction structure in labor and goods markets. When information is dispersed in the economy (as it is typically the case in reality), and interactions are local, market mechanisms can generate significant heterogeneity across economic actors and trigger positive economic feedbacks that pull the economy away from full employment.

Our model can be extended in many directions. First, we have not considered the possible stabilizing role of the interest rate. One could therefore modify the consumption rule introducing intertemporal substitution effects and then study the ability of monetary policy to put back the economy to the full employment steady state. Second, we have not considered the possible effects of demand shocks in the model and the possible differences in dynamics with respect to the ones presented here. Third, we could further explore the impact of different speed of adjustment in the goods and labor markets along the lines of Solow and Stiglitz, (1968). Finally, one could better study the role of social interaction effects in both markets, by changing the underlying structure of network interactions.

Acknowledgments

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3.4 Appendix A - Productivity shocks in the two intemediate scenarios

We here analyze the emerging dynamics in the two intermediate scenarios. Firstly we analyze a case with centralized labour market and a decentralized goods market. Then we will proceed by analyzing the opposite case, the onw with a decentralized labor market and a centralized goods market.

3.4.1 Productivity shocks in the I1 scenario

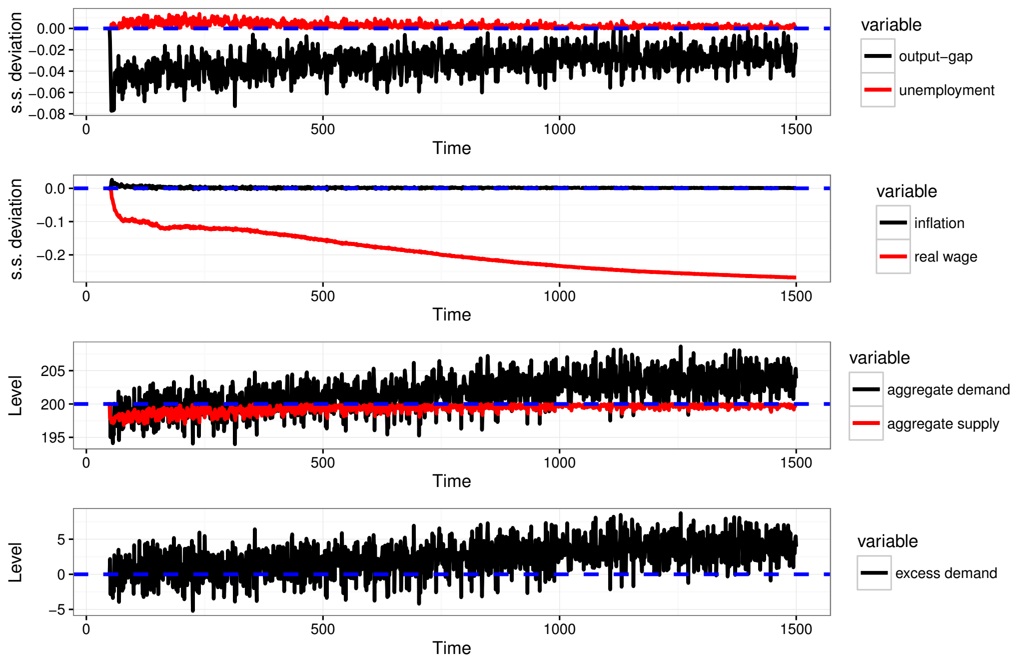

Figure 3.8: Emergent macroeconomic dynamics under supply shocks. Centralized labour market and decentralized goods market scenario.

In such a scenario, depicted in figure 3.8, after the shock hits, frictional missmatch between demand and supply on the goods market emerges. This introduces also heterogeneity (at the micro level) and due to the market selection equation, some firms experiment individual excess demand while others individual excess supply. Even if demand and wages are low, firms make profits and this creates a paradoxical situation in which the households (which are also owner of the firms) can consume and increase their demand because they experiment wealth increases and desire to consume out of distibuted profits. Such a behaviour, drives the economy toward a new statistical equilibrium with quasi full employment and near zero inflation; but still, due to market selection, even if demand is higher then supply: output does not converge to the original steady state value.

3.4.2 Productivity shocks in the I2 scenario

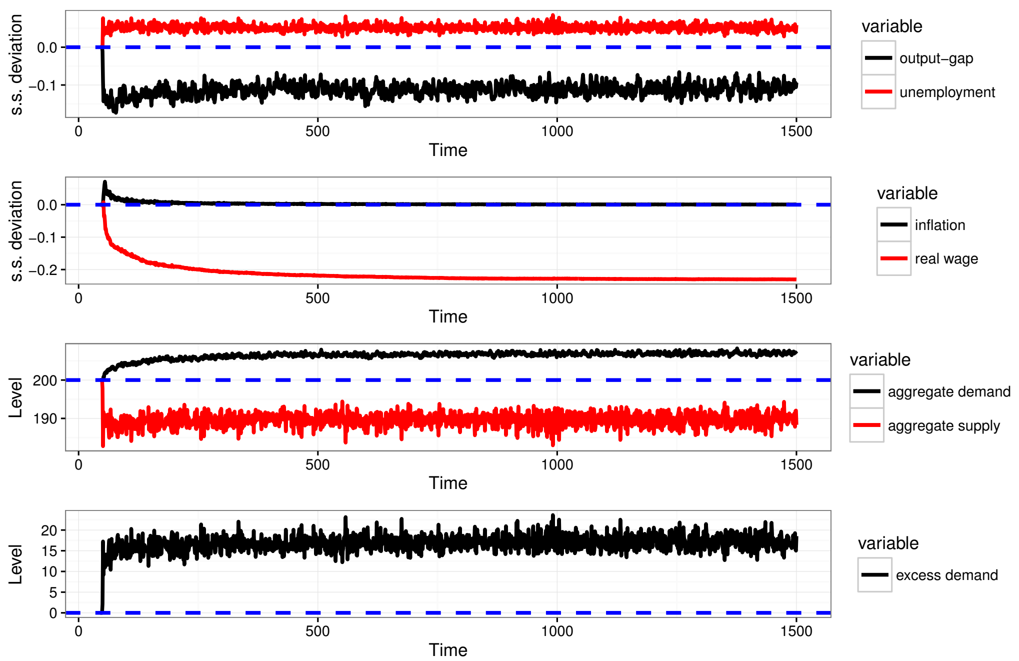

Figure 3.9: Emergent macroeconomic dynamics under supply shocks. Decentralized labour market and centralized goods market scenario.

Again, as soon as the productivity is negatively affected by the micro shocks, supply, excess demand and real wages are affected. But in this scenario, depicted in figure 3.9, the fact that the job search is market-based, causes miss-match between supply and demand of labour; this in turns creates frictional unemployment (frictional only, because the aggregate demand remains high due to the distibuted profits). This frictional unemployment keeps the production persistently lower with respect to the full-employment case (even when the persistency of the shock vanishes) a state of positive aggregate excess demand that will never be satisfied. It has to be noticed that due to the asymmetry between the two market-types, the excess demand for goods is more pronounced than in the previous case: this is so because the job market introduces strong household working time and wages heterogeneity but it kills heterogeneity in consumption (because demand is symmetrically rationed by constuction) that is not accounted by the symmetric monopolistic competition equation that applies in the goods market. It has to be noticed also the fact that that frictional unemployment and persistent excess demand for goods lead real wages to decline up to a new statistical equilibrium level, which is lower than the one present in the full-employment equilibrium. It might be puzzling that lower real wages coexists with high demand, but it is due to the fact that households are also owners of the firms and even if they are paid lower real wages, they earn higher dividends.

3.5 Appendix B - Robustness Checks

Figure 3.10 shows the results of 100 Monte Carlo simulations of the model under the fully-market based scenario and parametrized as described by table 3.1. The picture confirms that the results presented along the chapter are not a function of the specific random seed; indeed in all the 100 draws the results qualitatively hold true. It is important to notice that the robustness with respect to the RNG is here depicted only for the variable "unemployment" and only under the "fully market-based" scenario, but the same can be said referring to other scenarios and/or to other aggregate variables.

3.6 Appendix C - Comparison of long-run statistical equilibria

Stationarity tests on the aggregate variable stemming from a micro-founded model can be of great help in quantitatively detecting the existence of long-run statistical

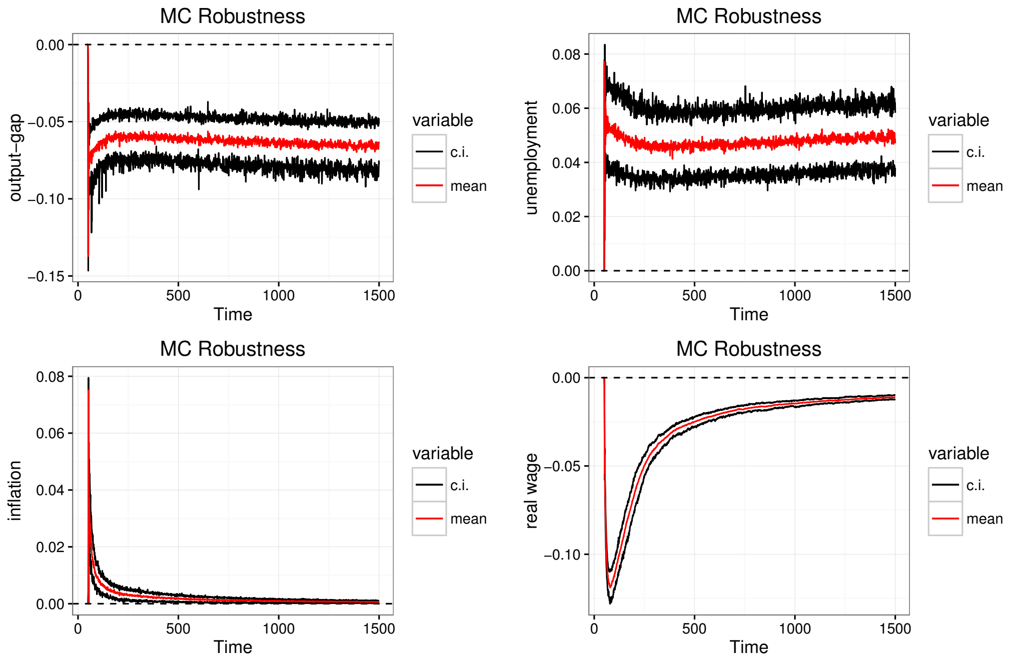

Figure 3.10: Results from $MC = 100$ Monte Carlo realizations of the baseline parametrization of the model.

equilibria.

In a context where the series have been generated by a simulation model that provides M Monte-Carlo realizations, the stationarity hypothesis (and implicitly the existence of a statistical equilibrium hypothesis) can be tested directly (Guerini and Moneta, 2016). Indeed if we consider all the M time series realization of an aggregate variable of interest k, we will collect a matrix with dimensions $M \times T$ containing all the observations $Y^m_{k,t}$. We here define as *ensambles* all the possible column vectors of such a matrix; therefore each of these vectors contains the M observations $Y^{\cdot}_{k,t}$ in which the time dimension is fixed. Then, denoting by $F_t(Y_k)$ the empirical cumulative distribution function of a single ensamble, testing for the existence of a statistical equilibrium reduces to test (via Kolmogorov, 1933 and Smirnov, 1948 test) for the following condition:

$$F_{t_i}(Y_k) = F_{t_j}(Y_k), \quad \forall i, j = 1, \ldots, T \quad i \neq j \tag{3.22}$$

Kolmogorov-Smirnov test results, which are presented in table 3.3, confirms what

scenario	output-gap	unemployment	inflation	real-wage
I1	0.96	1.00	0.81	0.81
I2	0.94	1.00	0.77	0.77
FD	0.91	1.00	0.92	0.92

Table 3.3: Test for stationarity: percentage of times that the Kolmogorov-Smirnov test cannot reject the null hypothesis of *equal distribution*. I1: intermediate case with centralized labour and decentralized goods markets. I2: intermediate case with decentralized labour and centralized goods markets.

we had already argued by graphically analysing the time series of the emergent macrodynamics from figures 3.1 to 3.4: the aggregate variables of our economic system satisfy the stationarity condition; there exist therefore a statistical equilibrium for each of the scenarios;[10] but it is a different one for each of them. Table 3.4 collects the mean of the long run statistical equilibria and summarizes them allowing an easy comparison.

The results in table 3.4 grossly confirms the indications already presented in the previous part of this section, but a puzzling aspect is worth to be considered: the third scenario, in which the labour market is decentralized but the goods market is planner-based, converges to a statistical equilibria which is worse-off the one on which a fully decentralized economy converges to. As a matter of fact, the unemployment is on average the same (5%), but the misallocation of the demand is worse (-11% against -7%). This might be explained due to the fact that in the third scenario, also the real wage is much lower with respect to the fully-decentralized one (-23% against -1%) but also to the fact that in the third scenario, demand of a single household is globally allocated to all the F firms, all of whom ration it; in the fourth scenario instead, local and decentralized demand of a single household might get partially rationed but another household might instead fully satisfy his desired demand.

From the analysis performed we therefore conclude that the assumption of the economy as populated by a representative firm and a representative households might

[10]Stationarity test on the fully-centralized case is not performed because the distribution of the long run dynamic is a Dirac Delta Function with peak in zero, since that scenario converges back to the full employment equilibrium.

scenario	output-gap	unemployment	inflation	real-wage
FC	0.00	0.00	0.00	0.00
I1	-0.03	0.00	0.00	-0.30
I2	-0.11	0.05	0.00	-0.23
FD	-0.07	0.05	0.00	-0.01

Table 3.4: Long-run equilibrium for each scenario after supply shock, in percentage deviation from the full-employment steady state.

be a good idea in an ideal world, where imperfections are absent and where a fully-informed central planner is able to fix distortions by isolating the shocks and not letting them propagate to the micro structure of the economy. But, on other side of the coin, in cases where market imperfections are important, describing the economic system as populated by a unique representative agent is unsatisfactory and does not allow to capture many important microeconomic aspects that are the drivers of the macroeconomic outcome.

3.7 Appendix D - Stock-Flow Consistency

	Households	Firms		Bailout Fund	$\sum$
		Current	Capital		
Consumption	-C	+C			0
Wages	+W	-W			0
Dividends	+ $(1-\theta)\Pi$	$-\Pi$	$+\theta\Pi$		0
Contribution to bailout	-H			+H	0
Δ Net-Worth	ΔA_h	0	ΔA_f	ΔA_{bf}	0

Table 3.5: Transaction flow matrix.

3.8 Appendix E - Equilibrium conditions

In this section we show how to compute the full employment equilibrium when agents are homogeneous. As subscript we will adopt the letters a, f and h for referring respectively to aggregate, firm-level, household-level variables. The conditions that we adopt in order to compute the equilibrium are simply based on homogeneity, market-clearing and zero profits.

Starting from the full employment definition, aggregate employment is the sum of all firms' employees and equal to the number of households:

$$N_a^* = \sum_f n_f^* = H.$$

For the homogeneity condition, all the firms must have the same number of employees. This implies that:

$$n_f^* = \frac{N_a^*}{F} \quad \forall f = 1, \ldots, F.$$

Therefore, by recalling the linear technology in equation 3.3, the production of each firm is equal to:

$$q_{f,s}^* = an_f^*.$$

Aggregate supply is equal to $q_{a,s}^* = \sum_f q_{f,s}^*$ and, in the equilibrium, it correspond to aggregate demand:

$$q_{a,s}^* = q_{a,d}^*.$$

Aggregate demand stems in turn from the sum of consumption plans of households: $q_{a,d}^* = \sum_h \hat{c}_h^*$, which, given the homogeneity of agents condition correspond to:

$$\hat{c}_h^* = c_h^* = \frac{q_{a,d}^*}{H}$$

Again due to homogeneity, the goods demand of each household to a particular firm is equal to:

$$\hat{c}_{h-f}^* = c_{h-f}^* = \frac{c_h^*}{F}$$

Quantities are uniquely defined once the full-employment condition is achieved. Employing the zero-profits condition, we cannot uniquely identify unique price and wage levels satisfying the equilibrium. However, as equilibrium employment n_f^* is necessarily different from zero, we can find a unique real-wage that satisfies it:

$$\begin{aligned}
\pi_f &= p_f q_f^* - w_f n_f^* \\
0 &= p_f a n_f^* - w_f n_f^* \\
w_f &= p_f a \\
\frac{w_f}{p_f} &= a.
\end{aligned}$$

Note that $1/a$ can be interpreted as firms' mark-up.

3.9 Appendix F - Pseudo code

```
UPDATE FIRMS NET-WORTH:
        if positive profits
                take previous net-worth
                add retained ones
        if negative profits
                take previous net-worth
                add losses

UPDATE HOUSE WEALTH:
        take previous wealth
        add returns from savings
        add quota of distributed profits

ENTRY-EXIT PROCESS:
        search bankrupt firms (net-worth < 0)
        search bankrupt house (wealth < 0)
        search "rich" house (wealth > wealth_0)
        if rich house
                take excess wealth
                put it in a "saving fund"
        compute amount in the saving fund
        if bankrupt firms
                take saving fund
```

```
                re-start net-worth (initial)
                re-start price (average)
                re-start wage (average)
                re-start expected demand (average)
        if bankrupt house
                take saving fund
                re-start wealth (initial)
                re-start price (average)
                re-start wage (average)
                re-start expected demand (average)
        compute remaining resources in the saving fund

DECISION PROCESS:
        firms set wage
        firms set price
        firms set expected demand
        house set desired counsumption

LABOR MARKET:
        if centralized
                compute labor supply to each firm
                compute labor demand by each firm
                compute effective labor of each firm
                compute excess demand of labor
                update employer status
                update employee status
                update employee wage
        if decentralized
                firms post vacancies (labor demand)
                house search for jobs and queue up (labor supply)
                sequential matching (effective labor)
                compute excess demand of labor
                update employer status
                update employee status
                update employee wage

GOODS MARKET:
        update productivity (here the shock happens)
```

```
        if centralized
                compute supply by each firm
                compute demand to each firm
                compute effective sales of each firm
                compute excess demand of goods
                update firms sales
                update house consumption
        if decentralized
                compute production by each firm (supply)
                house search for goods and queue up (demand)
                sequential matching (effective sales)
                compute excess demand of goods
                update firms sales
                update house consumption

ACCOUNTING PROCESS:
        firms compute in-period profits
        firms compute in-period distributed profits
        firms compute in-period retained profits
        house compute in-period savings
        house compute in-period returns

AGGREGATION PROCESS:
        total consumption
        total savings
        total returns
        total production
        total profits
        aggregate price level
        aggregate wage level
        empoyment
        unempoyment
        inflation
        output-gap
```

CHAPTER

FOUR

A METHOD FOR AGENT-BASED MODELS VALIDATION

Although demonstrative simulation models are useful, not least at performing "what if" *exercises of exploration of different models, policy analysis requires validated, descriptive simulation models.*

Marks, (2013)

4.1 Introduction

Economics, as any scientific discipline intended to inform policy, has inevitably addressed questions related to identification and measurement of causes and effects. This paper, by identifying and comparing causal structures, proposes a method that improves the empirical reliability of policy-oriented simulation models.

The foundation of the *Econometric Society* in 1930 paved the way for a rigorous and formal approach to the analysis of causality, which, as Heckman, (2000) points out, constituted the major contribution of econometrics.[1] In the post World War II period

[1] As Hoover, (2004) has shown, however, causal language has not always been explicit in economics and in the sciences in general. In the first half of the twentieth century, under the influence of Karl Pearson, Ernst Mach and Bertrand Russell, many research scientists endeavoured to eschew causal concepts in order to privilege functional and statistical dependencies (Illari et al., 2011). Explicit discussions of causality revived in the second half of the last century (Hoover, 2004). See also Granger, (1980)

causal claims were introduced in macroeconomics by means of aggregate, mechanic and dynamic models in which the *ex-ante* use of economic theory was pivotal. Under this approach the causal process used to be partitioned in a deterministic component and a random component. The former was meant to reflect the causal relations dictated by economic theory. The condition for it to be considered "valid" was to have the random component satisfying the standard Gauss-Markov statistical properties. Such a methodology goes under the name of *Cowles Commission* or *Simultaneous Equations Model* (SEM) approach. The most prominent proposers were Haavelmo, (1944) and Koopmans, (1950).

This approach has been strongly criticized by Lucas, (1976) and Sims, (1980) on theoretical and methodological grounds respectively: the former insisted that individuals endowed with rational expectations would have anticipated the policy interventions supported by SEMs and their behaviour would have brought results opposite to the ones predicted by SEMs; the latter instead stressed the fact that in the *Cowles Commission* approach the distinction between endogenous and exogenous variables was *ad hoc*, in order to ensure system identifiability.

Taking as starting points the Lucas, (1976) and Sims, (1980) critiques, Kydland and Prescott, (1982) paved the way for a new class of models, becoming the founding fathers of the stream of literature that goes under the name of *Real Business Cycle* (RBC) theory and which then evolved in what today is known as the *Dynamic Stochastic General Equilibrium* (DSGE) approach. These types of models are nowadays the most widely used to draw and to evaluate policy claims because they bear the advantage of simultaneously addressing two critical issues about causal structures. On the one hand, under the acceptance of the rational expectation hypothesis, the structure modeled by the RBC/DSGE approach remains invariant under policy intervention because it takes into account the forward-looking behaviour of the economic agents. On the other hand, the theoretical structure has an empirical counterpart in which the distinction between endogenous and exogenous variables is eschewed. The empirical counterpart is represented by a *Structural Vector Autoregressive* (SVAR) model.[2] But

[2]See, however, Canova and Sala, (2009) and Fukac and Pagan, (2006) for cautionary notes about the existence of the empirical counterpart of a DSGE model.

the RBC/DSGE approach is not exempt from problems: structural stability is grounded in individual behaviour, but assumes a representative agent, which neglects or even denies any form of interaction. Moreover, the identification of the empirical structure in the SVAR model is typically achieved by imposing restrictions derived from the theoretical model, which are therefore not subjected to any severe test. (See Fagiolo and Roventini, (2012, 2016) for a detailed criticism on similar issues).

An alternative approach to the problem of representing macroeconomic causal structures, in which it is possible to run reliable policy experiments, is to build a class of models that better reflect the existing economic mechanisms, including the microeconomic interactions. This is the aim of the *Agent-Based Model* (ABM) approach, also known as the *Agent-Based Computational Economics* (ACE) approach, in which the macroeconomic structure is analyzed as an emerging property from the interaction between heterogeneous and bounded rational economic actors. This modeling strategy has been applied to economic theory for only three decades, but it rapidly gained a significant success and in recent years has begun to be perceived as a new valuable paradigm, able to provide a viable alternative to the DSGE framework.[3] ABM is a useful and flexible tool for performing rich policy experiments and for evaluating their implications. Among the main advantages of the ABM strategy is the possibility of analyzing endogenously generated booms and busts and studying the reaction of the economy to different stimuli, applied not only around a fictitious locally stable steady state of the economy but also in periods of distress.

But ABMs pose a serious methodological problem because of their unclear relationship with the empirical evidence. This paper aims to address this issue. The difficulties of the ABM approach, which represent the counterpart of its flexibility, are perceived both in the model-data confrontation and in the comparison of different models investigating the same piece of evidence. The value of ABMs has been up to now evaluated according to their ex-post ability to reproduce a number of stylized facts even if other validation procedures are available (see Fagiolo et al., 2007). We argue that such an

[3]The rapid acceptance of ABM might be due both to the huge improvements in computational power and recently, to their ability to explain and reveal intriguing aspects of the world financial and economic crisis; DSGE models instead, have proven to be of little help when facing the crisis. See Trichet (2010) for clarification on this last point.

evaluation strategy is not rigorous enough. Indeed the reproduction, no matter how robust, of a set of statistical properties of the data by a model is a relatively weak form of validation, since, in general, given a set of statistical dependencies there are possibly many causal structures which may have generated them. Thus models which incorporate different causal structures, on which diverse and even opposite practical policy suggestions can be grounded, may well replicate the same empirical facts.[4]

The present work proposes a procedure to validate a simulation model which proceeds by first estimating both the causal structure incorporated in the model (using the data artificially generated by the model) and the causal structure underlying the real-world data. Secondly, it compares the two inferred causal structures. In this manner the proposed procedure offers a solution to both the issue of comparing an ABM to empirical data and the issue of comparing different simulation models. Indeed causal structures inferred from different simulation data, generated by different models can be compared in the same way. A good matching between the causal structure incorporated in the ABM and the causal structure underlying the real-world data provides a more rigorous empirical support to the policy statements drawn from the ABM, if compared with the support coming from mere replication of statistical evidence. Other validation procedures have been recently proposed based on information criteria by Lamperti, (2015) and Barde, (2015b,a); other researchers such as Grazzini and Richiardi, (2015), Lux, (2012), Recchioni et al., (2015), Gilli and Winker, (2003) have focused on estimation, or on the analysis of the emergent properties stemming from ABMs (see Grazzini, 2012); there has also been interest in parameter space exploration and parameter robustness (see Salle and Yildizoglu, 2014; Ciarli, 2012). The flourishing of all these complementary approaches devoted to the solution of such interrelated issues can be seen an indicator of their relevance and a signal of the vitality of the agent-based community.

The paper is organized as follows. Section 2 reviews the different strands of literature upon which our method is built; the validation algorithm is presented extensively in Section 3; Section 4 provides a first application of the method to the *"Schumpeter*

[4] At the root of this underdetermination problem is the fact that statistical relationships are in general symmetric, while this is not necessarily the case for causal relationships.

meeting Keynes" model proposed by Dosi et al., (2015). Section 5 concludes.

4.2 Background literature

DSGE models are confronted to the data in two ways. The first and traditional approach is through calibration, in which the parameters of the model are chosen from pre-existing microeconomic studies or in order to replicate the statistical properties of aggregate variables (Kydland and Prescott, 1996). The second approach is through the estimation of a VAR model built to represent the empirical counterpart of the DSGE model. Having estimated a VAR, one can identify a SVAR and confront its impulse response functions with the responses to policy shocks derived from the DSGE model. Alternatively, as proposed by Ireland, (2004), one can augment the DSGE model with the VAR residuals and estimate a hybrid model via maximum likelihood (for a criticism see Juselius, 2011).

Calibration and replication of statistical properties of data are practiced in the ACE community as well. To our knowledge, however, the models by Bianchi et al., (2007) and Bianchi et al., (2008) are the unique medium-scale agent-based macro-models in which parameters are estimated *ex-ante* and calibrated *ex-post* in order to replicate statistical properties of observed data.[5]

Although calibration and replication of statistical properties of data are a first step in taking the model to the data, we claim that this is not enough for the reliability of the policy implications derived from the model, since two models with alternative policy implications may well be both calibrated in order to replicate certain statistical properties of observed data. Reliability can be improved only through a validation exercise designed to provide evidence that the modeled data generating mechanism is an adequate representation of the real-data generating mechanism.

We are aware that the economic system has the characteristics of a complex system in which somehow stable macroscopic aggregate properties emerge from intricate connections at the microscopic level. But we further believe that representing as a unique

[5]There are, however, several small-scale ACE financial models which are instead calibrated or estimated such as the ones by Alfarano et al., (2005, 2006, 2007).

model every single micro-mechanism at work in a complex economy and showing it is a good match with data at different levels of aggregation is a very difficult task. A reduction in the complexity of the issue may be necessary, and hence in what follows we will analyze only the relations between macro variables.[6] Our strategy is indeed to focus only on representing causal structures among aggregate variables of the ABM and test whether they significantly differ from the causal structures that can be found in the real world from observed aggregate variables, without further considerations of the micro properties. In other words, we compare a macro-reduced version of the model generated mechanism with a macro-reduced version of the real-data generating mechanism.[7]

Our procedure will separately identify the causal structures of the two different data generating processes at their aggregate level, and then will compare the results of the estimations: if the causal structures are similar, then the model is a good characterization of the causal structure of the real-world data generating process and we will consider it as "valid". The identification method is the same for both processes: we will estimate an SVAR model using both observed and simulated aggregate data. This model, being a model with well-known properties, provides us enough power and flexibility to compare the explanatory performances of ABM with that of real-world data. But a crucial feature in the SVAR estimation is the identification procedure, which we describe in the next subsections.

4.2.1 SVAR identification: an open issue

Starting from a multiple time series dataset composed of K variables collected for T periods we can denote by $\mathbf{Y_t} = (Y_{1t}, \dots, Y_{Kt})'$ the values of these variables at a particular time t. A simple, but useful way of representing the data generating process, is to model the value of each variable Y_{kt} as a linear combination of the previous values

[6]Cfr. Haldane, (2012).

[7]Possible developments of the same method may allow to compare a micro-macro version of the modeled generated mechanism with the real-data generating mechanism, using observations at different levels of aggregation.

of all the variables as well as their contemporaneous values:

$$\mathbf{Y_t} = \mathbf{B}\mathbf{Y_t} + \mathbf{\Gamma_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{\Gamma_p}\mathbf{Y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{4.1}$$

where the diagonal elements of the matrix $\mathbf{B}$ are set equal to zero by definition and where $\boldsymbol{\varepsilon}_\mathbf{t}$ represents a vector of error terms which we will assume to be mutually statistically independent. Therefore the covariance matrix $\mathbf{\Sigma}_\varepsilon = \mathbf{E}\left[\boldsymbol{\varepsilon}_\mathbf{t}\boldsymbol{\varepsilon}'_\mathbf{t}\right]$ is diagonal. The SVAR form of this model can also be written as

$$\mathbf{\Gamma_0}\mathbf{Y_t} = \mathbf{\Gamma_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{\Gamma_p}\mathbf{Y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{4.2}$$

where $\mathbf{\Gamma_0} = \mathbf{I} - \mathbf{B}$. The problem with equations (4.1) and (4.2) is that they cannot be directly estimated without biases, being the contemporaneous variables endogenous. What is typically done in the literature is to estimate the reduced form VAR model:

$$\begin{aligned}\mathbf{Y_t} &= \mathbf{\Gamma_0^{-1}}\mathbf{\Gamma_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{\Gamma_0^{-1}}\mathbf{\Gamma_p}\mathbf{Y_{t-p}} + \mathbf{\Gamma_0^{-1}}\boldsymbol{\varepsilon}_\mathbf{t} \\ &= \mathbf{A_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{A_p}\mathbf{Y_{t-p}} + \mathbf{u_t}.\end{aligned} \tag{4.3}$$

where $\mathbf{u_t} = \mathbf{\Gamma_0^{-1}}\boldsymbol{\varepsilon}_\mathbf{t}$ is a zero-mean white noise process with a covariance matrix $\mathbf{\Sigma_u} = \mathbf{E}\left[\mathbf{u_t}\mathbf{u'_t}\right]$ that in general is not diagonal.

The problem is that, even if the parameters contained into $\mathbf{A_i},$ for $i = 1, \dots, p$ can be estimated from equation (4.3) without incurring in any particular issue, their knowledge is not sufficient for the recovery of the structural parameter contained in $\mathbf{B}$ and in $\mathbf{\Gamma_i},$ for $i = 1, \dots, p$ of equation (4.1), making impossible the inference of any causal and/or policy claim. To do such claims we need to recover the matrix $\mathbf{\Gamma_0}$ that contains the contemporaneous causal effects. But the problem is that any invertible unit-diagonal matrix might be compatible with the coefficients estimated from the VAR in equation (4.3).

The problem of finding the appropriate $\mathbf{\Gamma_0}$ (and hence also finding the matrices $\mathbf{\Gamma_1}, \dots, \mathbf{\Gamma_p}$) is called the *identification problem* and it is usually performed by imposing restrictions on the $\mathbf{\Gamma_0}$ matrix using a Cholesky factorization of the estimated covariance matrix $\mathbf{\Sigma_u}$. But this approach should only be employed when the recursive ordering implied by the identification scheme is firmly supported by theoretical consideration. A class of alternative identification procedures derives from the seminal papers by Bernanke, (1986) and Blanchard, (1989) and imposes zero restrictions that are based on economic considerations about contemporaneous interactions. An alternative identification strategy descends from Shapiro and Watson, (1988) and Blanchard and Quah, (1989) by assuming that certain economic shocks (e.g. supply shocks) have long-run effects on some variables but do not influence in the long-run the level of other variables, while other shocks (e.g. demand shocks) have only short-run effects on all the variables. Unfortunately these identification strategies are grounded on some level of theoretical apriorism which does not completely solves the critique put forward by Sims, (1980).

A relatively recent approach for solving the identification issue of a SVAR model in a more agnostic and data-driven fashion, allowing one to avoid as much as possible subjective choices and theory driven considerations, has been put forward by Swanson and Granger, (1997), Bessler and Lee, (2002), Demiralp and Hoover, (2003), Moneta, (2008) and Moneta et al., (2011) and is based on graphical causal models (see Pearl, 2000; Spirtes et al., 2000).

4.2.2 Graphical causal models and SVAR identification

A *Causal Graph* $\mathcal{G}$ is a model that consists of a set $\mathcal{V}$ of vertices (nodes) and a set $\mathcal{E}$ of edges (links) and might be written concisely as $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$. It is aimed at representing and analyzing specific features of the data-generating process underlying the set of observed variables. The vertices of such a graph correspond to random variables and the edges denote causal relationships among them. In what follows we focus on the simple case of *Directed Acyclic Graphs* (DAG) in which all the edges are directed and causal loops are not allowed.

The identification procedure based on graphical causal models consists of three steps: *(i)* estimating the reduced form VAR of equation (4.3), *(ii)* applying a search algorithm to the estimated residuals $\mathbf{u_t}$ to obtain the matrix $\mathbf{\Gamma_0}$ (cfr. equation 4.2) and *(iii)* recovering the other matrices $\mathbf{\Gamma_i}$ $(i = 1, \ldots, p)$ of the SVAR model.

The critical part of the procedure is the second step, in which an algorithm is applied in order to uncover the causal dependencies among the residuals $\mathbf{u_t}$. The literature on causal search models has developed a plethora of algorithms which differ among each other for the assumptions on which they are based and the computational properties. Assumptions typically concern the form of the causal structure (e.g. cyclic or acyclic), the presence or exclusion of latent variables (i.e. causally not sufficient or causal sufficient structures), rules of inference (more on that below), and, finally, statistical properties of the residuals (e.g. normality or linearity) which allow the application of specific tests of conditional independence.

The algorithm presented in Appendix A is the PC algorithm originally developed by Spirtes et al., (2000). In this algorithm causal loops are not allowed. Indeed it is assumed that the causal generating mechanism can be modeled by a DAG. In the SVAR framework this amounts to excluding feedbacks in the contemporaneous causal structure, while feedbacks over time are of course conceivable (e.g. X_t causes Y_{t+1} which in turn causes X_{t+2}). The PC algorithm also assumes causal sufficiency, i.e. there is no unmeasured variable which simultaneously affects two or more observed variables. Rules of inference are conditions that permit deriving causal relationships starting from tests of conditional independence. The PC, and similar algorithms of the same class, hinge on two rules of inference (see Spirtes et al., 2000):

Condition 1. (Causal Markov Condition) *Any variable in the causal graph $\mathcal{G}$ is conditionally independent of its graphical nondescendants (i.e. non-effects) — except its graphical parents — given its graphical parents (i.e. direct causes).*

Condition 2. (Faithfulness Condition) *Let $\mathcal{G}$ be a causal graph and $\mathcal{P}$ be a probability distribution associated with the vertices of $\mathcal{G}$. Then every conditional independence relation true in $\mathcal{P}$ is entailed by the Causal Markov Condition applied to $\mathcal{G}$.*

The PC algorithm, as many other of the class of *constraint-based search* algorithms, needs as input knowledge of the conditional independence relationships among the variables. There are many possibilities of testing conditional independence that in principle are all compatible with the PC algorithm. If the probability distribution underlying the data is Gaussian, zero partial correlation implies conditional independence. Then a typical procedure is to test for Gaussianity and in case this is not rejected, one can test for zero partial correlations. In many statistical packages the default option is to test zero partial correlation through the Fisher-z-transformation, as proposed by (Spirtes et al., 2000). An alternative option, suited for the SVAR framework, is to test zero partial correlations among the VAR residuals through a Wald test that exploits the asymptotic normality of the covariance matrix of the maximum-likelihood estimated VAR residuals (for details see Moneta, 2008). If Gaussianity is rejected or one is not willing to make distributional assumptions, one way to proceed is to rely on nonparametric tests of conditional independence, which, however, present the well-known problem of dimensionality (cfr. Chlass and Moneta, 2010).

The PC algorithms follow this scheme:

i. Create a complete graph on the variables $(X_1, \dots, X_k)$;

ii. Apply tests for conditional independence in order to prune unnecessary edges;

iii. Apply tests for conditional independence in order to direct remaining edges.

There are other algorithms in the literature that, following a similar scheme, allow for feedback loops or the possibility of latent variables (e.g. the CCD or FCI algorithm; cfr. Spirtes et al., 2000).

Usually, the output of the causal search is in general not a unique graph $\mathcal{G}$, but a set of *Markov equivalent* graphs which represent all the possible data generating processes consistent with the underlying probability $\mathcal{P}$. Hence the information obtained from this approach is generally not sufficient to provide full identification of the SVAR model requiring again a certain level of a priori theoretical knowledge. Moreover, if the distribution of the residuals is non-Gaussian, it is necessary to apply tests of conditional independence that are different from tests of zero partial correlation. However, Moneta et al., (2013a) have shown that if the VAR residuals are non-Gaussian, one can exploit higher-order statistics of the data and apply *Independent Component analysis* (ICA) (see Comon, 1994; Hyvarinen et al., 2001) in order to fully identify the SVAR model.

4.2.3 Independent component analysis and SVAR identification

Let us recall the fact that VAR disturbances $\mathbf{u_t}$ and structural shocks $\boldsymbol{\varepsilon}_\mathbf{t}$ are connected via

$$\mathbf{u_t} = \boldsymbol{\Gamma}_\mathbf{0}^{-1}\boldsymbol{\varepsilon}_\mathbf{t}. \tag{4.4}$$

In this framework the VAR residuals are interpreted as generated by a linear combination of non-Gaussian and independent structural shocks via the *mixing matrix* $\boldsymbol{\Gamma}_\mathbf{0}^{-1}$. Independent Component analysis applied to equation (4.4) allows the estimation of the mixing matrix $\boldsymbol{\Gamma}_\mathbf{0}^{-1}$ and the independent components $\boldsymbol{\varepsilon}_\mathbf{t}$ by finding linear combinations of $\mathbf{u_t}$ whose mutual statistical dependence is, according to some given measure, minimized. Some points should be noticed: *(i)* while the assumptions of mutual independence of the structural shocks is usually not necessary in a SVAR framework (orthogonality is usually sufficient), such an assumption is necessary to apply ICA; *(ii)* ICA does not require any specific distribution of the residuals $\mathbf{u_t}$ but only requires that they are non-Gaussian (with the possibility of at maximum one Gaussian element); *(iii)* the ICA-based approach for causal search does not require the faithfulness condition; *(iv)* in non-Gaussian settings while conditional independence implies zero partial correlation, the converse does not hold in general.

The application of ICA to the estimated VAR residuals allows identifying the rows of the matrix $\boldsymbol{\Gamma}_\mathbf{0}$, but not their order, sign and scale (for details see Hyvarinen et al., 2001). In order to obtain the correct matrix $\boldsymbol{\Gamma}_\mathbf{0}$, that is the matrix incorporating the contemporaneous causal structure and such that $\boldsymbol{\Gamma}_\mathbf{0} = \mathbf{I} - \mathbf{B}$ in equation (4.1), we further assume that the VAR residuals can be represented as a *Linear Non-Gaussian Acyclic Model* (LiNGAM) so that the contemporaneous causal structure can be represented as a DAG. On the basis of this assumption, it is possible to apply the causal search algorithm presented in Appendix B (VAR-LiNGAM), which draws on the original contributions of Shimizu et al., (2006) and Hyvarinen et al., (2001) (for an application to economics see Moneta et al., 2013a). The basic idea by which the VAR-LiNGAM algorithm solves the order indeterminacy is that if the underlying causal structure is acyclic, there must be only one row-permutation of the ICA-estimated rows of $\boldsymbol{\Gamma}_\mathbf{0}$ such that all the entries of the main diagonal are different from zero. Hence, the algorithm applies a search procedure to find such a permutation (for details see Appendix B, step C). The scaling indeterminacy is solved by normalizing the elements of the ICA-estimated matrix and rightly row-permuted $\mathbf{\Gamma_0}$, such that the main diagonal is one (and the main diagonal of $\mathbf{B}$ is zero, as dictated by equation (4.1).

In the literature there are alternative ICA-based search algorithms, which relax the assumption of acyclicity and causal sufficiency: see for example the algorithms proposed by Lacerda et al., (2008) and Hoyer et al., (2008), which respectively allow for feedback loops and for latent variables. However, since this is a first application of this type of framework to validation of simulated model, we decided to keep the analysis as simple as possible so that in future works we might relax assumptions, and understand which are the most critical for validation concerns.

4.3 The validation method

In this section we describe our validation procedure which is composed of five different steps as shown in figure (4.1). In the first step we apply some simple transformations that allow the empirical and the artificial data to be directly comparable; in the second step we analyze the emergent properties of the series produced by the simulated model; in the third step we estimate the reduced-form VAR model; in the fourth step we identify the structural form of the model by means of some causal search algorithm; in the last step we compare the two estimated causal structures according to some distance measure.

4.3.1 Dataset uniformity

Our method starts by selecting, in the model under validation inquiry, K variables of interest $(v_1, \ldots, v_K)$. We then collect a dataset that corresponds to the actual realization of these variables in the real world (we call this dataset RW-data) as well as a dataset for the realizations of M Monte Carlo simulations of the agent-based model (we call this one AB-data). We thus obtain two preliminary datasets $\mathcal{V}_{RW}$ and $\mathcal{V}_{AB}$

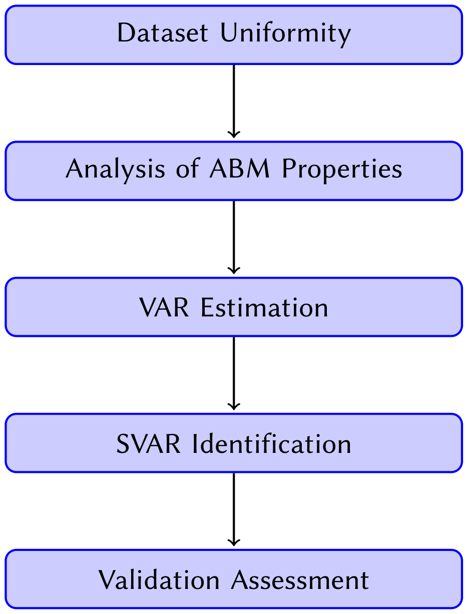

Figure 4.1: The five steps of the validation method.

which might be of different dimensions. In general we will have

$$\begin{cases} dim(\mathcal{V}_{RW}) & = 1 \times K \times T_{RW} \\ dim(\mathcal{V}_{AB}) & = M \times K \times T_{AB} \end{cases}$$

meaning that for the real world we observe only one realization of the K variables of interest for a period of length T_{RW} while for the simulated data (for which we can possibly have an infinity of observations) we will have M Monte Carlo realizations, of the same K variables, for a period of length T_{AB}; it often holds true that $T_{AB} >> T_{RW}$, so that the two datasets are not perfectly matchable.

The large availability of realizations in the simulated data is in fact an advantage and not an issue, since this allows a pairwise comparison of each run of the Monte Carlo simulation with the unique empirical realization. But the presence of different lengths in the time series might generate issues in two directions: *(i)* using the whole length of the AB-data time series creates the risk of capturing the effects present in the transient period, which does not represent the true dynamic entailed by the model, but is only due to the choice of the initial conditions, *(ii)* lag selection might be affected due to unobserved persistence in some of the modeled variables. Therefore we remove an initial subset of length $T_{AB} - T_{RW}$ from each of the M artificial datasets (as shown in figure 4.2) in order to force each pair of datasets to have the same dimensions:

$$dim(\mathcal{V}_{RW}) = dim(\mathcal{V}_{AB}(i)) = 1 \times K \times T_{RW} \quad \text{for } i = 1, \ldots, M.$$

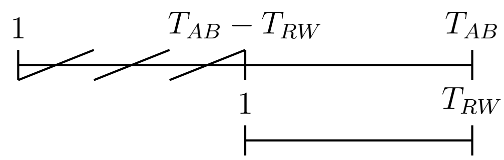

Figure 4.2: Time window selection. The first periods of the AB-data are cancelled with the objective of homogenising the series.

Moreover the order of magnitude of RW-data and AB-data are typically different; this is not perceived by the ABM community as an issue, being the concern of a large number of ABMs the replication of stylized facts (distributions, variations, statistical properties but not levels). But in our approach this might create comparability issue. We will see that in our application it is sufficient to take a logarithmic transformation in order to smooth out this scaling issue, and we speculate that in many applications any monotonic transformation might be applied.

4.3.2 Analysis of ABM properties

Some considerations about two underlying assumptions are needed. For the model to be a good proxy of the data generating process, we require that it be in a statistical equilibrium state in which the properties of the analyzed series are constant. In particular we require that the series, or a transformation of them (e.g. first differences), have distributional properties that are time-independent; secondly we require that the series are ergodic, meaning that the observed time series are a random sample of a multivariate stochastic process.

In a context where the series have been generated by a simulation model, which provides M Monte Carlo realizations, these two assumptions can be tested directly (see Grazzini, 2012). Indeed if we consider all the M time series realizations of a variable of interest k we will collect a matrix with dimensions $M \times T$ containing all the generated data $Y_{k,t}^{m}$, as represented in figure (4.3). We call here *ensembles* the column vectors of such a matrix; therefore each ensemble contains the M observations $Y_{k,t}^{(\cdot)}$ in which the time dimension is fixed. We instead define *samples* all the row vectors of the same matrix, each of which contains the T observations $Y_{k,(\cdot)}^{m}$ in which the Monte Carlo dimension is fixed. Let's denote by $F_t(Y_k)$ the empirical cumulative distribution function of an ensemble and by $F_m(Y_k)$ the empirical cumulative distribution function of a sample. Testing for statistical equilibrium and for ergodicity reduces to testing (via Kolmogorov-Smirnov test) for the following conditions:

$$F_i(Y_k) = F_j(Y_k), \quad \text{for } i, j = 1, \ldots, T \quad i \neq j \tag{4.5}$$

$$F_i(Y_k) = F_j(Y_k), \quad \text{for } i = 1, \ldots, T \quad j = 1, \ldots, M \tag{4.6}$$

Therefore we perform two tests as represented in figure (4.3): we recursively run tests of pairwise equality of distributions and we present the percentage of non-rejection of such tests. Rejecting the test would imply that the distribution under investigation are different from each other. Our two assumptions will be supported by the data if we obtain high percentages of non-rejection.

$$Y^k = \begin{pmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,T} \\ y_{2,1} & \cdots & \cdots & y_{2,T} \\ \vdots & \vdots & \vdots & \vdots \\ y_{M,1} & y_{M,2} & \cdots & y_{M,T} \end{pmatrix} \qquad Y^k = \begin{pmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,T} \\ y_{2,1} & \cdots & \cdots & y_{2,T} \\ \vdots & \vdots & \vdots & \vdots \\ y_{M,1} & y_{M,2} & \cdots & y_{M,T} \end{pmatrix}$$

Figure 4.3: The elements of comparison when testing for statistical equilibrium (left) and for ergodicity (right).

4.3.3 VAR estimation

Following the Box and Jenkins, (1970) methodology, the first task when any time series model has to be estimated, is the lag selection. In our case this means choosing the two values p_{RW} and p^i_{AB}, for $i = 1, \ldots, M$ according to some information criterion like the BIC the HQC or the AIC. Two cases might emerge from the data:

1. $p_{RW} - p^i_{AB} = 0$, which would mean that our estimations based on the artificial dataset and on the real-world dataset are perfectly comparable;

2. $p_{RW} - p^i_{AB} \neq 0$, which would mean that one of the two dataset presents at least one effect which is not present in the other; we keep this fact into account when computing the similarity measure.

Once the lag selection has been performed, our procedure estimates the VAR as explicated in equation (4.3) via OLS and also in VECM form, via maximum likelihood estimation (see Lutkepohl, 1993) using the Johansen and Juselius, (1990) procedure.

4.3.4 SVAR identification

In this step we extract the vectors of residuals $(u_1, \ldots, u_K)$ from the estimation of the VAR and analyze their statistical properties and their distributions. We test for normality applying the Shapiro-Wilk and the Jarque-Bera statistics. Then according to the outcome of the tests, we select the appropriate causal search algorithm to be adopted for the identification strategy. Two algorithms, the *PC* (to be adopted for the Gaussian case) and the *VAR-LiNGAM* (for the non-Gaussian case) are presented extensively in the appendices A and B. At the end of the identification procedure, we have estimated our structural matrices $\mathbf{\Gamma_i^{RW}}$ for $i = 0, \ldots, p_{RW}$ and $\mathbf{\Gamma_i^{AB,m}}$, for $i = 0, \ldots, p_{AB}$ and for $m = 1, \ldots, M$.

4.3.5 Validation assessment

The last step consists of the comparison of the causal effects entailed by the *SVAR*$_{RW}$ and the *SVAR*$_{AB}$ models. This will tell us how many of the real-world estimated causal effects are captured also by the agent-based model under validation inquiry.

In order to compare the causal effects we will use the similarity measure $\mathbf{\Omega}$, which we construct, as already anticipated, starting from the estimates of the SVAR.

Let's denote $\gamma_{i,jk}^{RW}$ the (j,k) element of $\mathbf{\Gamma_i^{RW}}$ for $i = 0, \ldots, p_{RW}$ and $\gamma_{i,jk}^{AB}$ the (j,k) element of $\mathbf{\Gamma_i^{AB,m}}$ for $i = 0, \ldots, p_{AB}$ and for $m = 1, \ldots, M$. We define $p_{max} = max\{p_{RW}, p_{AB}\}$ and then we set

$$\begin{cases} \mathbf{\Gamma_i^{RW}} = \mathbf{0} \quad \text{for } p_{RW} < i \leq p_{max} \quad \text{if} \quad p_{RW} < p_{max} = p_{AB} \\ \mathbf{\Gamma_i^{AB}} = \mathbf{0} \quad \text{for } p_{AB} < i \leq p_{max} \quad \text{if} \quad p_{AB} < p_{max} = p_{RW} \end{cases}$$

This allows us to penalize the value obtained by the similarity measure for the fact that the causal effects are completely mismatched after a certain lag. Then we build the indicator function:

$$\omega_{i,jk} = \begin{cases} 1 \quad \text{if} \quad sign(\gamma_{i,jk}^{RW}) = sign(\gamma_{i,jk}^{AB}) \\ 0 \quad \text{if} \quad sign(\gamma_{i,jk}^{RW}) \neq sign(\gamma_{i,jk}^{AB}) \end{cases} \tag{4.7}$$

where $i = 0, \ldots, p_{max}$ is the index for the i^{th}-order matrix containing the causal effects, while j and k are the row and column indexes of these matrices. The similarity measure is then defined as

$$\mathbf{\Omega} = \frac{\left(\sum_{i=1}^{p_{max}} \sum_{j=1}^{K} \sum_{k=1}^{K} \omega_{i,jk}\right)}{K^2 p_{max}}. \tag{4.8}$$

Our similarity measure is bounded between $[0,1]$ allowing us to have an easily interpretable index that represents the ability of the agent-based model to recover, at the aggregate macro-level, the same causal relationsips estimated in the RW-data.

4.4 Application to the "Schumpeter Meeting Keynes" model

The idea of building a simulation macroeconomic laboratory performing policy exercises dates back to Lucas, (1976) and Kydland and Prescott, (1982) but one problem of this approach has always been the external validity. Our methodology can be interpreted as a test for external validity for any simulation model and it might be of particular interest for researchers engaged in practical policy matters and policy makers who should make decisions based upon models that are shown to be reliable and valid. We already argued that a policy reliable model is one that is able not only to replicate a list of stylized facts, but also to represent the real-world causal structure as much accurately as possible. We want to test here the validity of the agent-based laboratory by Dosi et al., (2015), a model that builds upon a series of previous papers (see Dosi et al., 2010, 2013) and that has attracted great attention in the recent literature.

The Dosi et al., (2015) model aims at investigating the implications of demand and supply public policies in a model that *"bridges Keynesian theories of demand-generation and Schumpeterian theories of technology-fuelled economic growth"*. The model by itself is able to reproduce a long list of stylized facts and in particular is able to reproduce a cross correlation table close to the one usually computed with the US observed data.

4.4.1 The dataset

Since the model under validation inquiry is dedicated to the analysis of the real side of an economic system and to the analysis of fiscal and monetary policies, the $K = 6$ variables of major interest that we will consider in our system of equation are: aggregate consumption (C), gross private investments (I), unemployment rate (U), gross domestic product (Y), current price index (P) and effective federal funds rate (R). Appendix C describes the adopted model parametrization. The RW-data refer to the United States and are collected from the Federal Reserve Economic Database (FRED); we decided to cover the period from January 1959 to April 2014 with a quarterly frequency, implying a time series length $T = 222$; this is a typical selection when analyzing US business cycle. All the variables are plotted in figure 4.4.

To fulfill the dataset uniformity requirement for the AB-data, we collect the last T time observations, getting rid of possible transients and we consider $M = 100$ Monte Carlo simulations, each of them pairwise compared to the unique realization of the RW-data. Finally we take logs of the C, I, Y, P variables and we uniformize U and R by expressing them in percentage terms.[8]

We then check whether the assumptions we require for estimation of our AB model are too stringent or they are supported by the data; we perform the statistical equilibrium and ergodicity tests as described in the section 3.2 and in figure (4.3); this kind of

[8]Three Monte Carlo simulations, $m = \{55, 61, 89\}$, are excluded from the original dataset because in (at least) one period of the model, investment goes to 0, implying we cannot take the logarithm.

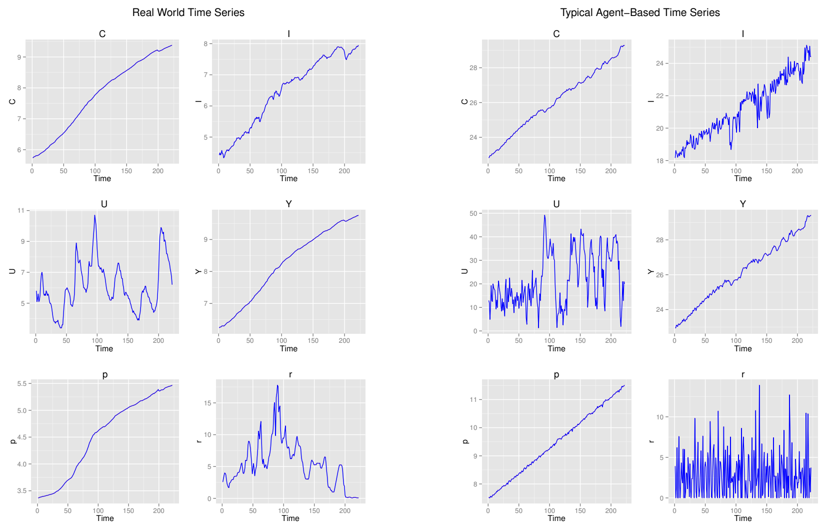

Figure 4.4: *Left columns:* time series of RW-data. *Right columns:* time series of a typical AB-data.

test is in line also with the analysis proposed by Grazzini, (2012).

Variable	*Equilibrium*	*Ergodicity*
C	0.9538	0.9479
I	0.9634	0.9564
U	0.9608	0.9396
Y	0.9532	0.9513
P	0.9560	0.9055
R	0.9609	0.9716

Table 4.1: Percentages of non-rejection of statistical equilibrium and ergodicity.

Table (4.1) presents the percentage of non-rejection of the Kolmogorov-Smirnov test of each pairwise comparison, for each stationarised series. The results come from $\frac{T\times(T-1)}{2} = 24310$ and $T \times M = 22100$ pairwise comparisons for the statistical equilibrium and for the ergodicity tests respectively. For all the series we have values higher than 90% and this allows us to conclude that the assumptions about the model having reached a statistical equilibrium and producing ergodic series are reasonable.

4.4.2 Estimation and validation results

The augmented Dickey-Fuller test does not reject the null hypotheses of unit root in all the real-world time series. For AB-data, the evidence for ubiquity of unit root is weaker since for I, U and R we can reject at the 5% level the presence of unit root (see table 4.2). This does not create any difficulty to our causal search procedure and it is only a stylized fact not replicated by the model.

(a) RW-data

Variable	*ADF p-value for levels*	*ADF p-value for 1^{st}-differences*	*Critical level*
C	0.99	0.02	0.05
I	0.89	0.01	0.05
U	0.06	0.01	0.05
Y	0.99	0.01	0.05
p	0.92	0.19	0.05
r	0.22	0.01	0.05

(b) AB-data

Variable	*ADF p-value for levels*	*ADF p-value for 1^{st}-differences*	*Critical level*
C	0.43	0.01	0.05
I	0.01	0.01	0.05
U	0.01	0.01	0.05
Y	0.19	0.01	0.05
p	0.63	0.01	0.05
r	0.01	0.01	0.05

Table 4.2: Augmented Dickey-Fuller Test.

We then estimate the model as a vector error correction model (VECM) with cointegrating relationships, without taking first difference of any variable, using the Johansen and Juselius, (1990) procedure, which is based on a maximum-likelihood estimation with normal errors, but is robust also to non-Gaussian disturbances. For sake of completeness we also check the robustness of the results by estimating the VAR in level via OLS. We select the number of lags according to the Bayes-Schwarz Information Criterion (BIC) and the number of cointegrating relationships following the

Johansen procedure. For the real-world dataset the suggestion is that of using 3 lags and 2 cointegrating relationships, while in the artificial datasets (typically) we should use 3 lags and 3 cointegrating relationships. This implies that we do not have any comparability issue for what concerns the number of lags. With respect to the cointegrating relations, it is again a stylized fact not matched by the model, which does not create any estimation issue, since we are interested in structural form of the model and cointegration is only a reduced form property.

The empirical distributions of the VAR residuals $(u_{C,t}, \ldots, u_{R,t})$ are represented in figure (4.5) both for RW-data and for a typical Monte Carlo realization of the AB-data simulation; moreover table (4.3) collects the results of the Shapiro-Wilk and the Jarque-Bera tests for normality; for all the variables, the residuals from the real-world data and all but one residual (the unemployment) from the artificial data, the tests rejects the null hypothesis of normality.

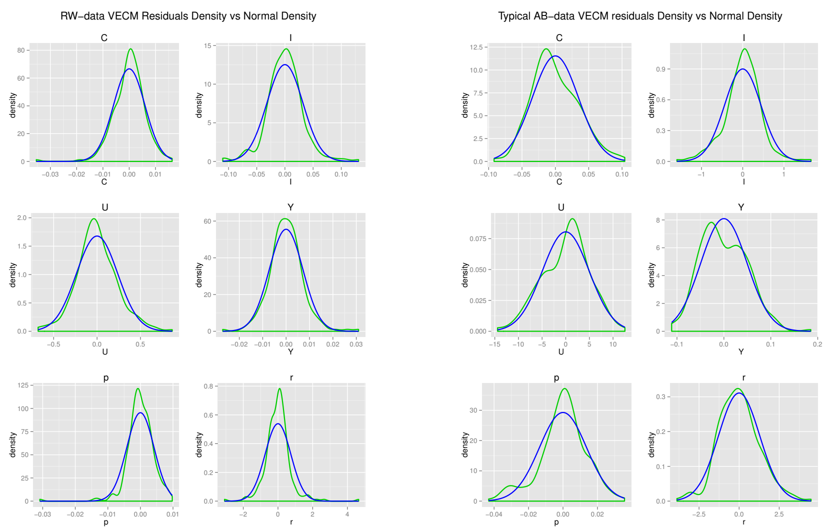

Figure 4.5: *Left columns:* RW-data VECM residuals distribution (green) and normal distribution (blue). *Right columns:* typical AB-data VECM residuals distribution (green) and normal distribution (blue).

We conclude that the residuals $\mathbf{u_t}$ are non-Gaussian and this result leads us toward the identification of the SVAR via the LiNGAM algorithm.[9]

[9]The VAR-LiNGAM algorithm is consistent even if one variable is normally distributed and therefore

(a) RW-data

Variable	*Shapiro-Wilk Test*	*Shapiro-Wilk p-value*	*Jarque-Bera Test*	*Jarque-Bera p-value*
C	0.95	0.00	271.48	0.00
I	0.96	0.00	54.96	0.00
U	0.98	0.02	12.16	0.00
Y	0.97	0.00	61.16	0.00
P	0.88	0.00	1710.58	0.00
R	0.89	0.00	698.46	0.00

(b) AB-data

Variable	*Shapiro-Wilk Test*	*Shapiro-Wilk p-value*	*Jarque-Bera Test*	*Jarque-Bera p-value*
C	0.99	0.03	6.01	0.05
I	0.97	0.00	29.64	0.00
U	0.99	0.08	3.23	0.20
Y	0.98	0.02	4.72	0.09
P	0.98	0.00	10.98	0.00
R	0.99	0.05	7.30	0.03

Table 4.3: Normality test on the VECM residuals.

After having completed the estimation, we compute the similarity measure as defined in equation (4.8). The results suggest that when we estimate the system using a OLS-VAR strategy the *Schumpeter meeting Keynes* model is able to reproduce, on a Monte Carlo average, the 78.92% of the causal relations entailed in the real-world dataset (the similarity drops to 64.9% after accounting only for bootstrapped significant parameters); on the other side, if in the first step we estimate a VECM by means of maximum likelihood, the similarity measure marks 73.85% (raising to 79.89% when considering only bootstrapped significant parameters). The results are reported also in table (4.4), containing not only the means but also standard deviations across Monte Carlo. Given that the dispersion index is quite low, we can conclude that neither very negative nor very positive outliers are present. Therefore a large fraction of simulations entail the same bulk of causal relations.

Finally, as a final result we also control how each equation of the VAR or of the VECM estimated with the simulated data replicates the real world data: that is, we decompose the similarity measure equation by equation. Doing so, we can understand

even the unemployment residuals quasi-Gaussianity does not add complications to our identification and structural estimation procedure.

Estimation Method	μ	σ
VAR-OLS (all parameters)	0.7892	0.0517
VECM-ML (all parameters)	0.7385	0.0628
VAR-OLS (significant parameters)	0.6490	0.1030
VECM-ML (significant parameters)	0.7989	0.0689

Table 4.4: Mean and standard deviation of the similarity measure.

which parts of the real world data are better described by the model and which parts are instead badly replicated. The results tell us that the variable that is described at best is inflation; the poorest replication of the explaining factors concerns GDP. We argue that this fact is due to the lack of public expenditure in our dataset, which is an important variable for explaining GDP in the Dosi et al., 2015 model. Two additional interesting characteristics emerge. Firstly, even if in the time series depicted in figure 4.4 the interest rate looked like the worst represented by the model (at least if one only look at the stylized fact), the model sis well able in replicating all the features that determines the level of interest rate (91% of the effects are well captured). Secondly, all the variables (apart from GDP with 48%) are quite well explained by the variables included in our SVAR model, with similarity values higher than 70%. We believe that this is a good property of the model since there is no part of the real economy which a behaviour completely different with respect to the behaviour observed in the real world data.

Variable	*VAR*	*VECM*
C	0.9398625	0.7736254
I	0.6331615	0.6327320
Y	0.4785223	0.4841065
U	0.8397766	0.6980241
P	0.9308419	0.9308419
R	0.9128007	0.9119416

Table 4.5: Mean of the similarity measure for each equation.

4.5 Conclusions

In this paper we have presented a new method for validating policy-oriented Agent-Based macroeconomic models able to generate artificial time series comparable with the aggregate time series computed by statistical offices, central banks and institutional organizations. The approach is based on comparing Structural Vector Autoregressive models which are estimated from both artificial and real-world data by means of causal search algorithms. In the paper we also have presented a first application of our method to the Dosi et al., (2015) model. We have calculated that by using the simulated data and according to the proposed similarity measure, the model is able to resemble between 65% and 80% of the causal relations entailed by a SVAR estimated on real-world data. We posit that this is a positive result for the *Schumpeter meeting Keynes* model but in order to reinforce this claim, we would need to compare this result with those coming from other models. In our opinion, this paper sets a new benchmark upon which members of the agent-based community might build. Convinced about the fact that the validation issue cannot be settled in an ultimate manner, other approaches for model validity can emerge and might bring evidence complementary to ours. Indeed a possible strategy, for researchers wishing to bring their agent-based models to the audience of policymakers, is that of applying a plurality of methods.

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Appendix A - PC Algorithm

A. Connect everything

Form the complete undirected graph $\mathcal{G}$ on the vertex set $(u_{1t}, \ldots, u_{Kt})$ so that each vertex is connected to any other vertex by an undirected edge.

B. Cut some edges

$n = 0$

REPEAT:

REPEAT:

select an ordered pair of variables u_{ht} and u_{it} that are adjacent in $\mathcal{G}$ such that the number of variables adjacent to u_{ht} is equal or greater than $n + 1$. Select a set $\mathcal{S}$ of n variables adjacent to u_{ht} suchthat $u_{it} \notin S$. If $u_{ht} \perp u_{it}|\mathcal{S}$ delete edge $u_{ht} - u_{it}$ from $\mathcal{G}$.

UNTIL all ordered pairs of adjacent variables u_{ht} and u_{it} such that the number of variables adjacent to u_{ht} is equal or greater than $n + 1$ and all sets $\mathcal{S}$ of n variables adjacent to u_{ht} such that $u_{it} \notin \mathcal{S}$ have been checked to see if $u_{ht} \perp u_{it}|\mathcal{S}$;

$n = n + 1$;

UNTIL for each ordered pair of adjacent variables u_{ht}, u_{it}, the number of adjacent variables to u_{ht} is less than $n + 1$.

C. Build colliders

For each triple of vertices u_{ht}, u_{it}, u_{jt} such that the pair u_{ht}, u_{it} and the pair u_{it}, u_{jt} are each adjacent in $\mathcal{G}$ but the pair u_{ht}, u_{jt} is not adjacent in $\mathcal{G}$, orient $u_{ht} - u_{it} - u_{jt}$ as $u_{ht} \rightarrow u_{it} \leftarrow u_{jt}$ if and only if u_{it} does not belong to any set of variables $\mathcal{S}$ such that $u_{ht} \perp u_{jt}|\mathcal{S}$.

D. Direct some other edges

REPEAT:

if $u_{at} \rightarrow u_{bt}$, u_{bt} and u_{ct} are adjacent, u_{at} and u_{ct} are not adjacent and u_{bt} belongs to every set $\mathcal{S}$ such that $u_{at} \perp u_{ct}|\mathcal{S}$, then orient $u_{bt} - u_{ct}$ as $u_{bt} \rightarrow u_{ct}$; if there is a directed path from u_{at} to u_{bt} and an edge between u_{at} and u_{bt}, then orient $u_{at} - u_{bt}$ as $u_{at} \rightarrow u_{bt}$;

UNTIL no more edges can be oriented.

Appendix B - VAR-LiNGAM Algorithm

A. Estimate the reduced form VAR model of equation (4.3) obtaining estimates $\hat{\mathbf{A}}_\mathbf{i}$ of the matrices $\mathbf{A_i}$, $\forall\ \mathbf{i} = \mathbf{1}, \ldots, \mathbf{p}$. Denote by $\hat{\mathbf{U}}$ the $K \times T$ matrix of the corresponding estimated VAR error terms, that is each column of $\hat{\mathbf{U}}$ is $\hat{\mathbf{u}}_\mathbf{t} \equiv (\hat{u}_{1t}, \ldots, \hat{u}_{Kt})'$, $\forall\ t = 1, \ldots, T$. Check whether the u_{it} (for all rows i) indeed are non-Gaussian, and proceed only if this is so.

B. Use *FastICA* or any other suitable ICA algorithm (Hyvarinen et al., 2001) to obtain a decomposition $\hat{\mathbf{U}} = \mathbf{PE}$ where $\mathbf{P}$ is $K \times K$ and $\mathbf{E}$ is $K \times T$, such that the rows of $\mathbf{E}$ are the estimated independent components of $\hat{\mathbf{U}}$. Then validate non-Gaussianity and (at least approximate) statistical independence of the components before proceeding.

C. Let $\tilde{\tilde{\mathbf{\Gamma}}}_\mathbf{0} = \mathbf{P^{-1}}$. Find $\tilde{\mathbf{\Gamma}}_\mathbf{0}$, the row-permutated version of $\tilde{\tilde{\mathbf{\Gamma}}}_\mathbf{0}$ which minimizes $\sum_i \frac{1}{|\tilde{\mathbf{\Gamma}}_{\mathbf{0,ii}}|}$ with respect to the permutation. Note that this is a linear matching problem which can be easily solved even for high K (Shimizu et al., 2006).

D. Divide each row of $\tilde{\mathbf{\Gamma}}_\mathbf{0}$ by its diagonal element, to obtain a matrix $\hat{\mathbf{\Gamma}}_\mathbf{0}$ with all ones on the diagonal.

E. Let $\tilde{\mathbf{B}} = \mathbf{I} - \hat{\mathbf{\Gamma}}_\mathbf{0}$.

F. Find the permutation matrix $\mathbf{Z}$ which makes $\mathbf{Z}\tilde{\mathbf{B}}\mathbf{Z^T}$ as close as possible to lower triangular. This can be formalized as minimizing the sum of squares of the permuted upper-triangular elements, and minimized using a heuristic procedure (Shimizu et al., 2006). Set the upper-triangular elements to zero, and permute back to obtain $\hat{\mathbf{B}}$ which now contains the acyclic contemporaneous structure. (Note that it is useful to check that $\mathbf{Z}\tilde{\mathbf{B}}\mathbf{Z^T}$ indeed is close to strictly lower-triangular).

G. $\hat{\mathbf{B}}$ now contains $K(K-1)/2$ non-zero elements, some of which may be very small (and statistically insignificant). For improved interpretation and visualization, it may be desired to prune out (set to zero) small elements at this stage, for instance using a bootstrap approach (Shimizu et al., 2006).

H. Finally, calculate estimates of $\mathbf{\Gamma_i}$, $\forall\ \mathbf{i} = \mathbf{1}, \ldots, \mathbf{p}$ for lagged effects using $\mathbf{\Gamma_i} = (\mathbf{I} - \hat{\mathbf{B}})\hat{\mathbf{A}}_\mathbf{i}$.

Appendix C - Parametrization of the Simulated Model

As explained in the paper, our procedure applies to the baseline parametrization of the Dosi et al., (2015) model. The unique difference across the 100 Monte Carlo replications is the random seed.

Description	*Symbol*	*Value*
Monte Carlo replications	MC	100
Time sample	T	600
Number of firms in capital-good industry	F_1	50
Number of firms in consumption-good industry	F_2	200
Number of banks	B	10
Capital-good firms' mark-up	μ_1	0.04
Consumption-good firm initial mark-up	$\bar{\mu}_0$	0.25
Uniform distribution supports	$[\varphi_1, \varphi_2]$	$[0.10, 0.90]$
Wage setting $\Delta \bar{AB}$ weight	ψ_1	1
Wage setting Δcpi weight	ψ_2	0.05
Wage setting ΔU weight	ψ_3	0.05
Banks deposits interest rate	r^d	0
Bond interest rate mark-up	μ^{bonds}	-0.33
Loan interest rate mark-up	μ^{debt}	0.3
Bank capital adequacy rate	τ^b	0.08
Shape parameter of bank client distribution	$pareto_a$	0.08
Scaling parameter for interest rate cost	k_{const}	0.1
Capital buffer adjustment parameter	β	1
RD investment propensity	ν	0.04
RD allocation to innovative search	ξ	0.5
Firm search capabilities parameters	$\zeta_{1,2}$	0.3
Beta distribution parameters (innovation)	(α_1, β_1)	$(3, 3)$
Beta distribution support (innovation)	$[\chi_1, \bar{\chi}_1]$	$[-0.15, 0.15]$
New customer sample parameter	$\bar{\omega}$	0.5
Desired inventories	l	0.1
Physical scrapping age	η	20
Payback period	b	3
Mark-up coefficient	υ	0.04
Competitiveness weights	$\omega_{1,2}$	1

CHAPTER

FIVE

CONCLUSIONS AND PATHS FORWARD

The principal enemy is orthodoxy: to use the same recipe, administer the same therapy, to resolve the most various types of problems; never to admit complexity and try to reduce it as much as possible, while ignoring that things are always more complicated in reality.

Hirschman, (1998)

A recent debate on the state of macroeconomics (see Blanchard, 2016; Wren-Lewis, 2016; Krugman, 2016; Keen, 2016) shows once again the presence of a huge (nowadays more normative than descriptive) disagreement in the profession. First, about how the research in macroeconomics should be produced in order to explain facts. Second, about how the analysis on business cycle needs to be done with the aim of providing sound policy prescriptions. Indeed after the economic profession has been unable to predict and to counteract responsively to the recent economic downturn, a number of these debates have emerged. The large number of such debates provide an additional suggestion for an interpretation that sees the economy as possessing the characteristics of a complex system and that all the insights that different economists offer – derived from the guidelines of different schools of thoughts – are only different pieces of information and different perspectives of the complex economic system. A pluralist approach (interpreted here as both a variety of ideals and a variety of tools to be adopted) is hence nowadays believed by many economists (see Blanchard, 2016; Keen, 2016; Rodrick, 2014) to be a good strategy for performing economic research; nonetheless because it allows for more open debates and for a larger open mindedness. Indeed, in this thesis it has been shown that by means of different methods and with their integration, a study of the economic system is possible and provides fruitful results. As a general conclusion it is therefore argued that a wider spectrum of tools, methods, models and ideas would surely allow to better understand – if not to better predict – the mechanisms underlying the next financial or economic crisis.

In this spirit therefore this thesis have analyzed different properties of the business cycles by means of three different approaches, all of them under a complexity perspective and all of them grasping a particular aspect of the business cycle.

Chapter 2 grasps some aspects of the business cycle from an empirical viewpoint. Indeed with this paper we investigate about the causal relations between public debt, private debt and economic performance, finding that one of the sources of the recent financial and economic crisis has been the outstandingly huge increase in mortgage credit during the 2000-2007 period. This paper also provides additional evidence of the beneficial effects that expansionary fiscal policy might have on output via the investment and the consumption channels – i.e. via the presence of crowding-in effects.

In chapter 3, we capture some features of the business cycle from a fully theoretical perspective. This paper, shows which are the effects that local interaction and heterogeneity have on the aggregate macroeconomic outcome when a supply shock hits the economy. By means of a very stylized agent-based model that contains characteristics and effects typically found in DSGE models, we reveal that the full-employment equilibrium might be locally stable or unstable according to the level of decentralization of the economy, which also is linked to the level of micro level heterogeneity that emerges. Moreover, we show that Keynesian demand shortages might emerge due to coordination failures, driving toward unpleasant (sub-efficient) aggregate outcomes with involuntary unemployment.

Finally, the last paper, discussed in chapter 4, by integrating the empirical tools adopted in the first paper and a theoretical framework similar to the one developed in the second paper, captures some causal evidence on the business cycles and on the ability of simulated models to replicate and to explain it. This paper is both methodological and practical. First because it proposes an innovative approach to the validation issue; second because it applies this new approach to one among the most prominent models of the agent-based macroeconomic community. This paper, also establishes a reference point for future research and for other policy oriented agent-based models.

Plans for Future Research

Convinced by the fact that only a variety of approaches might unravel the nodes of the complex economic system and provide a more accurate picture of it, also my future research plans build on a pluralist approach and on a complexity perspective. But the complexity perspective still is an incomplete framework, with a load of potential yet to be explored and exploited. In particular, related to the causal studies of the business cycles, to agent-based models and to policy analysis, two questions are of great momentousness.

1. **How to empirically validate agent-based models?**
   I am convinced that a definitive and certain answer to the validation issue of any simulated model cannot be provided; such an answer would indeed require full knowledge and full replication of the original data generating process of the real world. Still, a plethora of approaches might be attempted in order to verify the plausibility that a simulated model (in particular an agent-based model) is a good representation of the causal mechanisms that are at work in a complex world. As already anticipated, a new approach to the validation issue has been developed and applied in chapter 4 of this thesis: this method, goes further than the customary habit of merely replicating stylized facts, by requiring not only the replication of cross and auto correlations, but also the replication of aggregate causal relations.

   Most likely, in a very broad view, the best approach toward such a great issue is the integration and the comparison of different perspectives to be applied on several macroeconomic agent-based model. In this direction a new work I am carrying on, evaluates a model according to three features: its ability in replicating (micro-level and macro-level) stylized facts (as in Dosi et al., 2015), its ability in capturing the univariate dynamics of aggregate variables (as in Lamperti, 2015), its ability in resembling aggregate multivariate causal relations (as in Guerini and Moneta, 2016). We are applying this strategy to many ABMs in order to have a full picture of the recent literature that also allows to make a comparison, useful for policy purposes.

Another stream of research that is emerging is related to the calibration and the parameter space exploration for ABM. In this directions the works of Ciarli, (2012), Salle and Yildizoglu, (2014), Dosi et al., (2016c), and van der Hoog, (2016) are among the first ones. A possible integration of calibration and validation might be pursued, with the aim of generating a unified framework for parameter selection and for replication of some feature of the real-world that allow a model to gain credibility and additional validity.

2. **How to evaluate the effect of macro policies in ABMs?**
Agent-based macroeconomic models offer a wonderful simulation environment, allowing the experimentation of any kind of policy (g.e. fiscal, monetary or regulatory). But up to now – apart from works that investigates the relation between policy choices and inequality such as the ones by Dosi et al., (2013) and Caiani et al., (2016)– the quality of a policy has been typically evaluated according to the effects it bears on the aggregate variables (g.e. the level or the volatility of GDP or unemployment). Possible improvements for policy analysis in an agent-based framework, might involve the utilization of a measure of welfare that allows to understand which policy might be really improving also the micro agents that populate a simulated complex system economy. Indeed, while an analysis only at the macro level is appropriate for DSGE models, which are "populated" by a representative agent, it might also be that a seemingly better outcome at the macro level (g.e. higher GDP or lower unemployment) is not fully compatible with a better quality of living for all the agents.

   In this direction it might be possible to use statistical properties of the micro level distributions to verify, when a policy is activated, whether there is some form of stochastic dominance of one scenario with respect to the others. This would allow to better understand how a policy will impact on the single economic entities (firms, consumers, workers) and will allow to bring more convincing arguments to policy makers when they have to decide about which policy is needed and how should be implemented. Moreover, in terms of academic debate, such an analysis would also allow a more direct comparison of ABM with DSGE and will set the base for a new and open debate between the communities of the two modeling strategies.

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