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Abstract

This thesis consists in three essays that study the linkages between real and financial factors from different perspectives. Chapter 1, co-authored with Ester Faia and Valeria Patella, introduces a full set of *ambiguity attitudes*, which endogenously induces agents' optimism in booms and pessimism in recessions, in a model where borrowers face occasionally binding collateral constraints. We use GMM techniques with latent value functions to estimate the ambiguity attitudes process, showing that agents update their belief over the credit cycle in a way coherent with our preferences specification. By simulating a crisis scenario, we show that optimism in booms is responsible for strong leverage build-up before the crises while pessimism in recessions implies sharper de-leveraging and asset price bursts. Analytically and numerically, using global non-linear methods, we show that our ambiguity attitudes coupled with the collateral constraints help to explain relevant asset price and leverage cycle facts around the unfolding of financial crises. Chapter 2, co-authored with Carmelo Salleo, studies the strategic interactions between monetary and macroprudential authorities through the lens of an open-economy monetary model featuring trade and financial flows between two symmetric countries. Characterizing a set of *Within-Country Cooperative and Nash Equilibria* for different degrees of trade and financial integration, the analysis identifies large costs associated to the strategic interaction between the domestic authorities. Moreover, the gains from cooperation are strongly affected by the degree of cross-country integration and by the channel through which the integration is realized: larger trade flows reduce the gains, while higher financial globalization makes cooperation more valuable. Then, moving to a *Between-Countries Cooperative and Nash Equilibria* analysis, we confirm that cooperation is beneficial from both the country-specific and the global perspective. Chapter 3, co-authored with Javier Ojea Ferreiro and Elena Rancoita proposes an innovative methodology for the design of adverse scenarios for macroprudential policies calibration and impact assessment. Our methodology allows building tailored scenarios characterized by two main features. First, there is a stable and transparent mapping of the cyclical systemic risk level into the path of the scenario's target variables, which are those variables that determine the overall scenario's severity. Second, the path of the other complementary variables is calibrates with a multivariate copula model estimated with macro and financial data (*MacroFin Copula*). Simulating the model for Euro Area countries, we show that our methodology is able to calibrate adverse scenarios that properly replicate the global financial crises dynamics in terms of severity and co-movement between the key macroeconomic and financial variables.[1]

[1] The views expressed in the chapters composing this thesis are those of the authors and do not necessarily reflect those of the ECB or the Bank of England.

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Chapter 1

Ambiguous Leverage Cycles

1.1 Introduction

Most financial crises originate in debt markets and the leverage cycles have important effects on the real economy. Opacity and collateral constraints are the two most notable features of debt markets and both can be a source of instability (See Holmstrom (2015)). First, collateral constraints expose debt markets to the fluctuations in collateral values and the anticipatory effects associated to their endogenous changes trigger large reversal in debt and asset positions. Second, agents trading in debt markets hold doubts about the fundamental value of the collateral. Indeed, contrary to equity markets in which buyers of the asset wish to exert monitoring and control on the investment activity, participants in debt markets usually trade under the ignorance of the fundamental value of collateral. For this reason in debt markets a collateral guarantee is part of the contractible set-up. This indeed serves the purpose of overcoming the pervasive asymmetric information. However even if the information asymmetry underlying the specific debt relation is solved through the contracts, doubts remain about the fundamental value of the asset, implying that optimism or pessimism of subjective beliefs affect the agents' saving and investment problem, hence the dynamic of asset prices and leverage.

In this context ambiguity attitudes and endogenous beliefs formation are crucial in determining the dynamic of asset values and debt, also since the latter is tied to the first through the collateral constraint. The surge in asset prices and leverage observed prior to most financial crises and, their collapse observed following it, have often been linked to a combination of institutional factors, captured by collateral constraints, and endogenous beliefs formation. Optimism in booms, generated by assigning higher subjective beliefs to gains than to losses, can explain the surge in asset demand, prices and, through the collateral channel, in debt. Pessimism in recessions produces the opposite chain of events.[1] Despite the joint relevance of those elements in explaining the unfolding of financial crises, as well as the dynamic of asset prices and leverage over the business cycle, they are absent from the literature.

We fill this gap by assessing the role of ambiguity attitudes in a small open economy model

[1]See Barberis (2013) for discussion on the role of over-confidence and under-confidence in particular for asset prices and leverage also at around the 2007-2008 financial crisis.

where borrowers, investing in risky assets, are subject to occasionally binding collateral constraints that tie the scarcity or availability of debt to asset valuations. The latter is then affected by ambiguity attitudes, which render beliefs formation endogenous. Indeed the borrower, endowed with a sequence of subjective beliefs upon which he holds different amount of confidence, optimally chooses the degree of entropy, namely the distance between subjective and objective probability distributions, subject to bounds on it. The confidence in subjective beliefs are captured by an ambiguity parameter. Given the optimal entropy or likelihood ratio (LR hereafter), which affects also the value of risky assets through the stochastic discount factor (SDF here-after), the borrower solves optimal portfolio and leverage decisions.

Importantly, we depart from the standard ambiguity aversion framework[2] and introduce a preferences structure which combine ambiguity aversion and ambiguity seeking. We call this specification *Kinked Multiplier Preferences*, which can be derived as a dynamic and state-contingent extension of the *biseparable preferences* axiomatized in Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2004). They convexify the decision maker problem of finding the optimal beliefs by combining (depending on the weights) both aversion and seeking behaviour. Extended ambiguity attitudes have also strong support in experimental studies.[3] Specifically we model the decision marker problem using *multiplier preferences* à la Hansen and Sargent (2001) and we then mix them to combine the entropy minimization problem (ambiguity aversion) and maximization problem (ambiguity seeking). Consistently with Ghirardato, Maccheroni and Marinacci (2004) the weight (or the indicator function) in the optimal decision problem depends upon expected utilities and through them to the current state of the economy (state-contingency nature). In order to validate our preferences empirically, we determine the mapping between the ambiguity attitudes and the expected utility through structural estimation of the model. Specifically, we develop a novel estimation method by adapting the non-linear method of moments to our model-based combined Euler equation, in debt and risky asset.[4] We find that ambiguity aversion prevails when the value function is below its expected value (a case which we often label the loss domain), while ambiguity seeking characterizes the gain domain. Those attitudes endogenously result in optimism (or right-skewed beliefs) in booms and pessimism (or left-skewed beliefs) in recessions.[5] This structure of the beliefs coupled with the anticipatory effects, which are typically associated with occasionally binding collateral constraints,[6] have important implications for asset price, debt capacity and

[2]See pioneering work by Hansen and Sargent (2001, 2007*b*) and Maccheroni, Marinacci and Rustichini (2006).

[3]Ambiguity seeking is strongly supported in experimental evidence. See Dimmock et al. (2015), Dimmock et al. (2016), Baillon et al. (2017) and Trautmann and van de Kuilen (2015) among others.

[4]For this we use the procedure developed in Chen, Favilukis and Ludvigson (2013), where one step involves the estimation of a latent unobservable variable given by the continuation value ratio.

[5]Our macro estimates are well in line with experimental evidence.Abdellaoui et al. (2011) provide foundations for S-shaped preferences with changing ambiguity attitudes and show through experimental evidence that pessimism (left-skewed beliefs) prevails in face of losses, while optimism prevails in face of gains. Further experimental evidence by Boiney (1993) Kraus and Litzenberger (1976) has associated ambiguity seeking (aversion) with right (left) skewed beliefs. On another front, survey evidence by Rozsypal and Schlafmann (2017), shows that low-income households hold pessimistic beliefs about the future, while the opposite is true for high-income households.

[6]Mendoza (2010) shows that the occasionally binding nature of the collateral constraints gives a role to anticipatory effects. As agents expect the constraint to bind in the future, they off-loads risky assets and debt in

leverage dynamic. Consider a boom. Borrowers endogenously tend to act optimistically and increase their demand of risky assets. This boosts asset prices and through anticipatory effects also the demand of debt, which in turn endogenously relaxes the constraint. This is also consistent with the fact that in booms the evaluation of optimistic agents drives the debt capacity. The opposite is true in the loss domain.

With the above model, we obtain a series of analytical and numerical results related to asset prices and debt dynamic. Analytically we discuss implications for asset prices and the Sharpe ratio. For the first, we show that the conditional LR heightens asset price growth in booms and depresses it in recessions. Second, the kink in the stochastic discount factor induced by the shift from optimism to pessimism helps to move the model-based Sharpe ratio closer to the Hansen and Hansen and Jagannathan (1991) bounds. Then, we calibrate the model's parameters by minimizing the distance between some targeted model-based moments and their empirical counterparts using data for the US economy over the sample 1980-2016, namely the sample of both a rapid growth in leverage and then a sudden collapse in debt positions. Under the optimized calibration, the model can match asset price volatilities and equity premia (both the long run and the dynamic pattern), returns, Sharpe ratios, volatilities of debt and its pro-cyclicality.[7] The comparison with the model featuring solely the collateral constraint shows that our model performs better in the data matching. To explain asset price facts borrowers' ambiguity attitudes over the tails are crucial. Moreover, contrary to ambiguity aversion, which typically induces persistence, but little volatility, our preferences which combine the two in a kinked fashion induce the right amount of persistence and volatility needed to match asset price facts and debt dynamic.

Next, we solve our model numerically by employing global non-linear methods with occasionally binding constraints.[8] The policy functions and a simulated crisis event study (Bianchi and Mendoza (2018)), which allow us to discuss the economic intuition behind our model, show that optimism increases the build-up of leverage in booms, while pessimism steepens the recessionary consequence of the crisis. In both cases the comparison is done relatively to a model featuring solely collateral constraints, but no deviations between subjective and objective beliefs. More specifically, we show that while the rational expectations model is not able to generate any build-up of leverage before the crises (the aggregate credit remains close to the ergodic mean before that the deleveraging starts), the model with ambiguity attitudes is able to reproduce a growth of leverage during the upswing of the cycle similar to the one characterizing the US leverage cycle before the global financial crises. Ambiguity attitudes play a crucial role in this result. Indeed, during booms optimism boosts asset demand and the asset price growth, hence, by relaxing the constraint, it facilitates the build-up of leverage. In recessions, instead,

anticipation.

[7] It is well documented by Jorda, Schularick and Taylor (2016) at aggregate level and using historical data. But it is also well document for consumer debt, see for instance Fieldhouse, Livshits and MacGee (2016) among others.

[8] We employ and endogenous grid approach (Carroll (2006)) accommodating for different regimes (portions of the state space) with binding or non-binding constraints (Jeanne and Korinek (2010)). Functions are approximated using piecewise linear interpolation and the exogenous state processes are discretized with Tauchen and Hussey (1991) method

pessimism materializes and drive the transmission channel in the opposite direction.

The rest of the chapter is structured as follows. Section 1.2 compares the paper to the literature. Section 1.3 describes the model and the ambiguity attitudes specification. Section 1.4 presents the estimation procedure and results. Section 1.5 investigates analytical results. Section 1.6 discusses quantitative findings. Section 1.7 concludes.

1.2 Comparison with Past Literature

Following the 2007 financial crisis which was triggered by panics in various debt markets (for structured products, for short-term bank funding and in repo markets, see Gorton and Metrick (2012)) there has been a growing interest in understanding the determinants and the dynamics of the leverage cycle and the role of the underlying externalities (pecuniary and demand) for the real economy. Most recent literature tends to assess the dynamic of debt over the business cycle through models with occasionally binding constraints. Papers on this topic include Geanakoplos (2010), Lorenzoni (2008), Mendoza (2010), which among many others examine both positive and normative issues related to the leverage cycle. Papers focusing on the positive aspects show that anticipatory effects produced by occasionally binding constraints are crucial in generating sharp reversals in debt markets and in establishing the link between the tightening of the constraint and the unfolding of financial crisis. None of the past papers however assesses the joint role of financial frictions, in the form of collateral constraints, and belief formation, while both play a crucial role in determining the asset price and leverage cycle in normal times and in explaining endogenously the unfolding of crises even in face of small shocks. One exception is Boz and Mendoza (2014) which introduces learning on asset valuation in a model with occasionally binding collateral constraints. Contrary to them our beliefs are endogenously formed based on ambiguity attitudes toward model mis-specification. Moreover none of the past papers conducts a quantitative analysis aimed at assisting the quantitative relevance of those elements in jointly matching asset price and debt facts and cyclical moments.

Since we choose to model endogenous beliefs formation through ambiguity attitudes our model is also connected to the literature on ambiguity aversion (see Hansen and Sargent (2001, 2007*b*) and Maccheroni, Marinacci and Rustichini (2006)). In this context some papers assess the role of ambiguity aversion for asset prices or for portfolio allocation. For instance Barillas, Hansen and Sargent (2007) show that ambiguity aversion is akin to risk-sensitive preferences a' la Tallarini (2000) and as such it helps the model's Sharpe ratio to get closer to the Hansen and Jagannathan (1991).[9] Most of the papers focusing on ambiguity aversion are able to explain well price patterns persistence, but less so price volatility. To improve on the latter some papers introduce time-varying ambiguity aversion to study asset prices properties, such as Epstein and Schneider (2008), Drechsler (2013), Leippold, Trojani and Vanini (2008), Bianchi, Ilut and Schneider (2017) and Bhandari, Borovička and Ho (2017). Ilut and Schneider (2014), in particular, explain crises with a loss of confidence obtained by adding a shock to the ambiguity

[9] On a different line of research Benigno and Nisticó (2012) show how ambiguity averse preferences can be used to explain the home bias in international portfolio allocations due to the need to hedge against long run risk.

averse framework. All of the above papers focus only on ambiguity aversion that, finally, is a theory of pessimistic attitudes and thus precautionary behaviour. For this reason ambiguity aversion is not suitable for characterizing agents' behaviours during the upswings of the leverage cycle in which waves of optimism can driven the aggregate borrowing decisions. Moreover, ambiguity seeking as well as the state contingent nature of the ambiguity attitudes is also well documented in experimental studies (see Dimmock et al. (2015), Baillon et al. (2017), Trautmann and van de Kuilen (2015), and Roca, Hogarth and Maule (2006) among others).

We depart from the ambiguity aversion literature in two important ways. First, we model ambiguity attitudes that encompass both ambiguity aversion and ambiguity seeking behavior. Our preferences are indeed a dynamic generalization of the biseparable preferences axiomatized in a static context by Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2004). Both papers show that ambiguity attitudes can be formalized within a general decision model by constructing a biseparable preference, which can combine both ambiguity aversion and ambiguity seeking. Effectively preferences are mixed with respect to the problem of finding the optimal beliefs, so that under a weight of one the decision maker solves a minimization problem (ambiguity aversion) and viceversa. The weights in their formalization depend upon expected utility mapping. In our work we construct a value function, which embeds a multiplier on the entropy, that can be mixed, thereby combining ambiguity aversion (with a positive multiplier on entropy) and ambiguity seeking (negative multiplier). Consistently with Ghirardato, Maccheroni and Marinacci (2004), the indicator function, which non-linearly shifts the preferences from pessimistic to its dual, depends upon the deviations of the current value function from a reference level, represented by its mean. Importantly, our state-contingent multiplier preferences are estimated. This validates empirically the preferences and, to the best of our knowledge, this is the first attempt to test ambiguity attitudes with time series analysis. Moreover, the preferences estimation also allows us to pin down the exact form of the state contingency in the multiplier (negative in the gain domain and positive in the loss domain). Equipped with these preferences, we find that agents update their belief over the leverage cycle, and more precisely, they endogenously become pessimistic in the loss domain (when the value function is below its mean) and optimistic in the gain domain (the opposite case). This has important consequences in our case. Indeed, by embedding those preferences into a leverage cycle and risky investment problem we can show that optimism induces price acceleration and excessive leverage, while pessimism induces the opposite. Moreover, the combination of ambiguity attitudes with an occasionally binding collateral constraint delivers the right amount of persistence and volatility needed to explain jointly asset price and debt dynamic.

At last, our paper relates to the extensive literature on the estimation of the SDF with behavioural elements. More closely, we build upon the latent factor estimation method of Chen, Favilukis and Ludvigson (2013). We depart from them along the following dimensions. First, we adapt their estimation procedure to preferences with state-contingent ambiguity attitudes and in presence of an occasionally binding collateral constraint. Secondly, our latent factor is derived analytically, while in their case it is estimated semi-nonparametrically.

1.3 A Model of Ambiguous Leverage Cycle

Our baseline model economy is an otherwise standard framework with borrowers facing occasionally binding collateral constraints. The novel ingredients stems from the interaction between ambiguity attitudes and debt capacity. Debt supply is fully elastic with an exogenous debt rate as normally employed in most recent literature on the leverage cycle.[10] Collateral in this economy is provided by the value of the risky asset funded through debt. To this framework we add ambiguity attitudes, which includes both ambiguity aversion and seeking. The underlying logic is similar to the one pioneered and proposed by the game-theoretic approach à la Hansen and Sargent (2007*a*) in which agents are assumed to have fears of model misspecification and thus explore the fragility of their decision rule with respect to various perturbations of the objective probability distribution. However, in our specification, agents endowed with ambiguity seeking attitude look for deviations from the objective model because they think that utility gains can be generated by these deviations. Moreover, we show that ambiguity aversion results endogenously in left-skewed or pessimistic beliefs, relatively to rational expectation, namely relatively to the case in which objective and subjective beliefs coincide. On the other side ambiguity seeking results in right-skewed or optimistic beliefs. Importantly the changing nature of the ambiguity attitudes contributes to the occasionally binding nature of the collateral constraint. As agents become optimist their demand for risky assets contributes to boost collateral values and to expand debt capacity. The opposite is true with pessimism.

1.3.1 Beliefs Formation and Preferences

The source of uncertainty in the model is a shock to aggregate income y_t, which is our exogenous state and follows a finite-space stationary Markov process. We define the state space as S_t, the realization of the state at time t as s_t and its history as $s^t = \{s_0, s_1, \ldots, s_t\}$ with associated probability $\pi(s^t)$. The initial condition of the shock is known and defined with s_{-1}.

Borrowers are endowed with the approximating model $\pi(s^t)$ over the history s^t but they also consider alternative probability measures, indicated by $\tilde{\pi}(s^t)$, which deviate from $\pi(s^t)$.[11] Following the relevant literature, we introduce the measurable function $M(s^t) = \tilde{\pi}(s^t)/\pi(s^t)$, which we define as the likelihood ratio. We can also define the conditional likelihood ratio as, $m(s_{t+1}|s^t) = \tilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. For ease of notation since now onward we use the following notation convention: $M_t = M(s^t), M_{t+1} = M(s^{t+1})$ and $m_{t+1} = m(s_{t+1}|s^t)$, where the sub-index refers to the next period state. The above definition of M_t allows us to represent the subjective expectation of a random variable x_t in terms of the approximating probability models, $\tilde{\mathbb{E}}_t\{x_t\} = \mathbb{E}_t\{M_t x_t\}$, where $\mathbb{E}_t$ is the subjective expectation operator conditional to information at time t for the probability $\pi(s^t)$, while $\tilde{\mathbb{E}}_t$ is the expectation operator conditional to information at time t for the probability $\tilde{\pi}(s^t)$. The function M_t follows a martingale process

[10]This model economy corresponds to a limiting case in which lenders are risk-neutral. Alternatively the model can be interpreted as a small open economy with debt supplied from the rest of the world.

[11]The alternative probability measure $\tilde{\pi}$ is absolutely continuous with respect π. This means that events that receive positive probability under the alternative model, also receive positive probability under the approximating model

and as such it satisfies the following condition $\mathbb{E}\{M_{t+1}\} = M_t$. We can decompose M_t as follows

$$m_{t+1} \equiv \frac{M_{t+1}}{M_t} \qquad \text{for} \quad M_t > 0 \tag{1.1}$$

and $m_{t+1} = 1$ for $M_t = 0$. These incremental deviations satisfy condition $\mathbb{E}_t\{m_{t+1}\} = 1$. Moreover, the discrepancy between the approximating and the subjective models is measured by the relative entropy, $\varepsilon(m_{t+1}) = \mathbb{E}_t\{m_{t+1} \log m_{t+1}\}$ that is a positive-valued, convex function of $\pi(s^t)$ and is uniquely minimized when $m_{t+1} = 1$, which is the condition characterizing the case with no ambiguity attitudes.

Given this framework, we introduce a full set of ambiguity attitudes, including ambiguity aversion and loving. Agents are ambiguity avers if they fear that deviations from the approximated model can imply some utility losses. As a consequence, they form subjective expectations according to a worst-case scenario evaluation in order to define a lower bound for these potential losses. This attitude is coherent with a period of strong economic uncertainty in which agents are not able to make precise economic forecasts. Contrary, agents are ambiguity seekers when they look for potential utility gains deviating from the approximated model. This is a situation that characterizes period of markets' exuberance as in the case of stock markets bubble. In this case agents form objective expectations according to a best-case scenario evaluation. In other words, borrowers can have different degrees of trust in their own subjective beliefs, so that act as ambiguity averse when they fear deviations from the approximated model and they act as ambiguity seeking when they hold high confidence in their beliefs.

The coexistence of these two attitudes is introduced in the model with a new preferences structure that we call *kinked multiplier preferences*. These preferences can be derived as a dynamic extension of the biseparable preferences axiomatized in Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2004). In appendix A.1 we provide technical details about the mapping between the two preferences formalizations. Both papers show that ambiguity attitudes can be formalized within a general decision model by constructing a biseparable preference, which can combine both ambiguity aversion and ambiguity seeking. Preferences are mixed with respect to the problem of finding the optimal belief. Consider the instantaneous utility function, $u(c_t)$, and the problem of finding the optimal belief. As a consequence, we can represent our kinked multiplier preferences as follows:

$$V_t(c_t) = \mathbb{I}_{V_t \leq \mathbb{E}_{t-1}\{V_t\}} \underbrace{\left\{ \min_{\substack{\{m_{t+1}, M_t\}_{t=0}^{\infty} \\ \mathbb{E}\{m_{t+1}\}=1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t M_t u(c_t) + \beta\theta^+ \varepsilon(m_{t+1}) \right] \right\}}_{\substack{\textit{ambiguity aversion side:} \\ \text{worst-case scenario evaluation}}} +$$

$$\mathbb{I}_{V_t > \mathbb{E}_{t-1}\{V_t\}} \underbrace{\left\{ \max_{\substack{\{m_{t+1}, M_t\}_{t=0}^{\infty} \\ \mathbb{E}\{m_{t+1}\}=1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t M_t u(c_t) + \beta\theta^- \varepsilon(m_{t+1}) \right] \right\}}_{\substack{\textit{ambiguity seeking side:} \\ \text{best-case scenario evaluation}}} \tag{1.2}$$

In the equation above $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$. The kink in preferences is related to the current state of the economy, defined on the basis of the difference between the agent's value function V_t and its historical mean $\mathbb{E}\{V_t\}$. When $V_t < \mathbb{E}_{t-1}\{V_t\}$ the economy is in a bad state and agents behave according to an ambiguity aversion attitude. The opposite is true during good states defined by the condition $V_t > \mathbb{E}_{t-1}\{V_t\}$. Finally, when the condition holds with equality ($V_t = \mathbb{E}_{t-1}\{V_t\}$), this preference structure collapses to the rational expectation representation, namely $V_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. In the above expression, $\theta \in \mathbb{R}$ is a process capturing the degree of doubts about the prevailing model, and it is a state-contingent binary variable which takes positive (negative) values for states of the world for which the value function is below (above) its average. Mathematically the value function under θ^- is essentially the dual representation of the value function under θ^+. The state-contingency in preferences implies that ambiguity aversion (thus pessimism) prevails in bad states, while ambiguity seeking (thus optimism) prevails in good states. This property will be object of empirical validation in Section 1.4.

Three theoretical notes are worth at this point. First, as noted in Ghirardato, Maccheroni and Marinacci (2004) most decision theory models of ambiguity employ those biseparable preferences, but add additional assumptions. For instance ambiguity aversion arises under the assumption of ambiguity hedging, namely the fact that between two indifferent alternatives the ambiguity averse decision marker prefers a convex combination of the two to each one in isolation. Under ambiguity seeking this assumption should be reversed. Second, the dependence of the indicator function upon the expected utility effectively creates a dependence with respect to the state of the economy. Indeed it is only after a sequence of negative shocks to wealth that the value function passes its mean and viceversa.[12] Therefore, formally we should condition the indicator function and the θ_t upon the state of the economy. With a slight abuse of notation and for convenience we maintain our notation of a time dependent θ_t as in the context of our model we deal with random shocks in a time series context. At last, note that the general formalization of the decision problem is not explicit about the exact dependence of the indicator function upon the gain or the loss domain. This is effectively an empirical question. Indeed as explained above, it has been addressed in the context of experimental studies. For this reason later below we estimate our model and we assign to θ_t a state-contingent process which is consistent with the data and the evidence that we find. This effectively also serves as an indirect validation of the preferences.

Some additional considerations are worth on the interpretation of our preferences and on their implication for the asset price and the leverage cycle. First, as we show below, when solving the decision maker problem of finding the optimal beliefs, our kinked multiplier preferences deliver pessimism (or left-skewed) beliefs in loss domain and optimism (right-skewed) beliefs in gain domain. Framed in the context of the Hansen and Sargent (2007*a*) game with nature, the optimal belief problem has the following interpretation. Under the loss domain nature acts malevolently and increases uncertainty. In this way nature also tests the decision maker,

[12] In this respect the preferences are also akin to the news dependent preferences a' la Kőszegi and Rabin (2007). See also recently Pagel (2014). The main difference is that news dependence affects risk aversion, while in our case it affects attitudes toward uncertainty. Second, once again we consider aversion but also its dual.

who fearing uncertainty acts pessimistically, hence assigns more weights to adverse states. In a consumption-saving problem this naturally induces more precautionary saving, while in our framework, where financial crises endogenously materialize, pessimistic beliefs are responsible for stronger deleveraging (and fire sales) during the downturn. This effect is well in line with post-crises dynamic. Under the gain domain nature acts benevolently and reduces uncertainty. This induces the decision maker to take more risk and assign more weight to the upper tail.[13] This leads to the emergence of risk-taking and excessive leverage. In both cases nature shifts decision makers' behaviour toward the tails. Hence, our preferences are well in line with the prevalent interpretation of model ambiguity. As we show extensively below however considering also ambiguity seeking helps greatly in explaining asset price facts as well as in the context of our leverage model also debt dynamic.

1.3.2 Budget and Collateral Constraint

The rest of the model follows a standard leverage cycle model with risky assets that serve as collateral (see e.g. Mendoza (2010)). The representative agent holds an infinitely lived asset x_t, which pays a stochastic dividend d_t every period and is available in fixed unit supply. The asset can be traded across borrowers at the price q_t. In order to reduce the dimension of the state space, we assume that the dividend is a fraction α of the income realization. Therefore, we indicate with $(1-\alpha)y_t$ the labor income and with $d_t = \alpha y_t$ the financial income. Agents can borrow using one-period non-state-contingent bonds that pay an exogenous real interest rate R. The budget constraint of the representative agents can be expressed as following:

$$c_t + q_t x_t + \frac{b_t}{R} = (1-\alpha)y_t + x_{t-1}[q_t + d_t] + b_{t-1} \tag{1.3}$$

where c_t indicates consumption and b_t the bond holdings. The agents' ability to borrow is restricted to a fraction ϕ of the value of asset holding:

$$-\frac{b_t}{R} \leq \phi q_t x_t \tag{1.4}$$

The collateral constraint depends on the current period price of the asset in order to reproduce fire-sales driven amplification dynamics, which for this simple model would not be produced with a different formulation of the constraint. In appendix A.2 provide a micro-founded derivation of this constraint, based on a limited enforcement problem.

1.3.3 Recursive Formulation

Following Hansen and Sargent (2007*b*), we rely on the recursive formulation of the problem, which allows us to re-write everything only in terms of m_{t+1}. The recursive formulation shall of course be adapted to capture the changing nature of the ambiguity attitudes.

[13] Given this interpretation, such beliefs formation process is also akin to the one considered in Brunnermeier and Parker (2005) in which a small optimistic bias in beliefs typically leads to first-order gains in anticipatory utility.

We now partition the state space S_t in the two blocks, given by the endogenous and the exogenous states, $S_t = \{B_t, y_t\}$, where B_t is the aggregate bond holdings and y_t the income realization. Note that the aggregate asset holdings is not a state variable because it is in fixed supply. Moreover, the problem is also characterized by the two individual state variables (b_t, x_t). For the recursive formulation we employ a prime and sub-index to indicate variables at time $t+1$ and no index for variables at time t. The borrowers' recursive optimization problem reads as follows. Conditional on $\theta_t > 0$, the recursive two-stage optimization reads as follows:

$$\begin{aligned} V(b,x,S) = \max_{c,x',b'} \min_{m'} \Big\{ & u(c) + \beta \mathbb{E}_S\{m' V(b',x',S) + \theta m' \log m'\} \\ & + \lambda \left[y + q(S)(x+\alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] \\ & + \mu \left[\phi q(S)x' + \frac{b'}{R} \right] + \beta\theta\psi \left[1 - \mathbb{E}_S m'\right] \Big\} \end{aligned} \tag{1.5}$$

Conditional on $\theta_t < 0$, the recursive two-stage optimization reads as follows:

$$\begin{aligned} V(b,x,S) = \max_{c,x',b'} \max_{m'} \Big\{ & u(c) + \beta \mathbb{E}_S \left[m' V(b',x',S) - \theta m' \log m'\right] \\ & + \lambda \left[y + q(S)(x+\alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] \\ & + \mu \left[\phi q(S)x' + \frac{b'}{R} \right] + \beta\theta\psi \left[1 - \mathbb{E}_S m'\right] \Big\} \end{aligned} \tag{1.6}$$

where the aggregate states follow the law of motion $S' = \Gamma(S)$. In the above problem λ and μ are the multipliers associated to the budget and collateral constraints respectively, while the term $\beta\theta\psi$ is the multiplier attached to the constraint $\mathbb{E}_S[m'] = 1$.

The above optimization problems are solved sequentially. First an inner optimization and then an outer optimization problem are derived sequentially. In the first stage agents choose the optimal incremental probability distortion for given saving and portfolio choices. In the second stage, for given optimal likelihood ratio, they solve the consumption/saving problem and choose the optimal amount of leverage. Intuitively, the problem is modelled as a game of strategic interactions between the maximizing agents, who face Knightian uncertainty,[14] and a malevolent/benevolent agent that draws the distribution (see Hansen and Sargent (2007*b*) who proposed this reading).

The Inner Problem

Through the inner optimization problem the borrowers choose the optimal entropy or conditional likelihood ratio, namely the optimal deviation between his own subjective beliefs and the objective probability distribution. The first order condition with respect to m', which is

[14] Knight (1921) advanced the distinction between risk, namely the known probability of tail events, and uncertainty, namely the case in which such probabilities are not known. Ambiguity usually refers to cases of uncertainty where the state space is well defined, but objective probabilities are not available.

functionally equivalent under the two cases, is given by:

$$V(b', x', S') + \theta(\log m' + 1) - \theta\psi = 0 \tag{1.7}$$

Rearranging terms, we obtain:

$$\begin{aligned} 1 + \log m' &= -\frac{V(b', x', S')}{\theta} + \psi \\ m' &= \exp\left\{\frac{-V(b'^2, x', S')}{\theta}\right\}\exp\{\psi - 1\} \end{aligned} \tag{1.8}$$

Finally, imposing the constraint over probability deviation m', and defining $\sigma = -\frac{1}{\theta}$ we derive the optimality condition for the conditional likelihood ratio:

$$m' = \frac{\exp\{\sigma V(b', x', S')\}}{\mathbb{E}\left[\exp\{\sigma V(b', x', S')\}\right]} \tag{1.9}$$

Equation (1.9) also defines the state-contingent incremental probability deviation from the rational expectation case. The magnitude and the direction of this deviation depends on the agents' value function and the value for the inverse of σ. We will return on the role of the optimal conditional likelihood ratio later on.

The Outer Problem

For given optimal LR $m^{'}$ the borrower solves an outer optimization problem in consumption, risky assets and debt. Upon substituting the optimal LR into the value function, the maximization problem reduces to find the optimal allocations of consumption, bond holding and asset holdings. The resulting recursive problem is:

$$\begin{aligned} V(b, x, S) = \max_{c, x', b'} \Big\{ & u(c) + \frac{\beta}{\sigma}\log\left[\mathbb{E}_S \exp\left\{\sigma V(b', x', S')\right\}\right] \\ & + \lambda\left[y + q(S)((x + d) + b - q(S)x' - c - \frac{b'}{R}\right] \\ & + \mu\left[\phi q(S)x + \frac{b'}{R}\right] \Big\} \end{aligned} \tag{1.10}$$

We will now derive and list all the competitive equilibrium conditions. Since now we return to the notation with t and $t+1$ indices as this is needed for our analytical derivations in section 1.5. The borrowers' first order condition with respect to bond holding and risky assets reads as follows:

$$u_c(c_t) = \beta R \mathbb{E}_t\left\{m_{t+1}u_c(c_{t+1})\right\} + \mu_t \tag{1.11}$$

$$q_t = \beta\frac{\mathbb{E}_t\left\{m_{t+1}u_c(c_{t+1})[q_{t+1} + \alpha y_{t+1}]\right\}}{u_c(c_t) - \phi\mu_t} \tag{1.12}$$

where u_c indicates the marginal utility of consumption. Equation (1.11) is the Euler equation for bonds and displays the typical feature of models with occasionally binding collateral constraint. In particular, when the constraint binds there is a wedge between the current and the expected future consumption marginal utility, given by the shadow value of relaxing the collateral constraint. Equation (1.12) is the asset price condition.

Note that ambiguity attitudes, hence beliefs, affect asset prices since m_{t+1} enters the optimality conditions for risky assets, equation (1.12), and they affect the tightness of the debt limit as m_{t+1} enters the Euler equation (1.11). In other words the optimal m_{t+1} affects the stochastic discount factor and through this it affects the pricing of all assets in the economy. The model characterization is completed with the complementarity slackness condition associated to the collateral constraint:

$$\mu_t \left[\frac{b_{t+1}}{R} + \phi q_t \right] = 0 \tag{1.13}$$

and with the goods and stock markets clearing conditions:

$$c_t + \frac{b_{t+1}}{R} = y_t + b_t \tag{1.14}$$

$$x_t = 1 \tag{1.15}$$

Definition 1.3.1 (Recursive Competitive Equilibrium)**.** *A Recursive Competitive Equilibrium is given by a value function V_t, allocations $(c_t; b_{t+1})$, probability distortions m_{t+1} and prices q_t such that:*

- *given prices and allocations the probability distortions solve the inner problem;*
- *given prices and probability distortions, allocations and the value function solve the outer problem;*
- *the allocations are feasible, satisfying (1.14) and (1.15);*
- *the aggregate states' law of motion is consistent with agents' optimization;*

1.3.4 Pessimism and Optimism

To determine under which states the ambiguity process θ_t, turns positive or negative we will estimate our model implied Euler equations through GMM in the next section. In the meantime it is useful to discuss how the ambiguity averse or ambiguity seeking attitudes generate endogenous waves of optimism and pessimism. For simplicity of exposition we report the optimal condition for variable m_{t+1}:

$$m_{t+1} = \frac{\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}}{\mathbb{E}_t\{\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\}} \tag{1.16}$$

The conditional deviation m_{t+1} affects how agents assign different subjective probabilities (with respect to the objective ones) to future events, which can be characterized by high and low utility. In particular, if $m_{t+1} > 1$ agents assign an higher subjective probability, while if $m_{t+1} < 1$

the opposite holds. Given this, the sign of the parameter σ_t affects how these conditions are linked to positive or negative future state realizations.[15] The following lemma summarizes this consideration and defines optimism and pessimism in the agents' attitude.

Lemma 1.3.2. *When $\theta_t < 0$, $m_{t+1} > 1$ is assigned to future good states (high utility events), while $m_{t+1} < 1$ to bad future states. Hence, beliefs endogenously emerge as right-skewed and agents act with optimism. When $\theta_t > 0$ the opposite is true and agents are pessimistic.*

Proof. First we define good states as those in which the current state value function is above its expected value. When $\theta_t < 0$; then $\sigma_t > 0$ in good states $\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\} > \mathbb{E}_t\{\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\}$ namely the risk-adjusted value function for the good states is larger than the average one. Based on equation (1.16), this implies that $m_{t+1} > 1$. The opposite is true in bad states. When $\theta_t > 0$ then $\sigma_t < 0$ this implies that in good states $\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\} < \mathbb{E}_t\{\exp\{\sigma_t V(b_{t+1}, x_{t+1}, S_{t+1})\}\}$, namely the risk-adjusted value function for the good states is lower than the average one and $m_{t+1} < 1$. The opposite is true in bad states.

Beliefs Formation: A binomial state space example

To gain some intuition we discuss a particular case with only two income states, which we define as high, with a sup-index h, and low, with a sup-index l. We also consider only two periods which we label as $t = 0, 1$. By assumption the high state is high enough that the collateral constraint is slack, while the opposite is true for the low state. This facilitates the computation of the expectation operators. The states have a binomial probability structure such that state h realizes with probability π, while the state l with its complement $1 - \pi$. Equipped with these assumptions we can characterize the dynamic between time 0 and time 1. In this case the likelihood ratio can be specified as follows:

$$m_1 = \frac{\exp\{\sigma_0 V_1\}}{\pi \exp\left\{\sigma_0 V_1^h\right\} + (1-\pi)\exp\left\{\sigma_0 V_1^l\right\}} \tag{1.17}$$

where $V_1^h > \mathbb{E}_0\{V_1\}$ and $V_1^l < \mathbb{E}_0\{V_1\}$. Note that depending on the time zero realization of the state we have two different values of the inverse of the penalty parameter, σ_0. To fix ideas imagine that the income realization at time zero is the low state, l. Given our Lemma 1.3.2 we have that $\sigma_o^l < 0$. The latter implies that $\exp\{\sigma_0^l V_1^h\} < \mathbb{E}_0\left\{\exp\left\{\sigma_0^l V_1\right\}\right\}$ and $\exp\{\sigma_0^l V_1^l\} > \mathbb{E}_0\left\{\exp\left\{\sigma_0^l V_1\right\}\right\}$. Therefore, the marginal likelihood ratio are $m_1^h < 1$ and $m_1^l > 1$. As a consequence, we can define the following subjective probabilities as:

$$\omega^h = \pi m_1^h < \pi \quad \omega^l = (1-\pi)m_1^l > (1-\pi) \tag{1.18}$$

As we can see, agents assign a higher (lower) subjective probability - with respect to the objective probability - to the future negative (positive) events, typical of a pessimistic attitude.

[15] Concerning the size of the distortion, we can say that a large absolute value of θ increases the probability distortion in all future states, meaning that m' is close to unity. At the contrary, a small absolute value of θ, implies that the decisions are far from the rational expectation setting.

The opposite is true when $\sigma_o^l < 0$. In this case $\exp\{\sigma_0^h V_1^h\} > \mathbb{E}_0 \left\{\exp\left\{\sigma_0^h V_1\right\}\right\}$ and $\exp\{\sigma_0^h V_1^l\} < \mathbb{E}_0 \left\{\exp\{\sigma_0^h V_1\}\right\}$ producing $m_1^h > 1$ and $m_1^l < 1$. Therefore, agents assign higher (lower) subjective probability to the future positive (negative) events, showing an optimism attitude:

$$\omega^h = \pi m_1^h > \pi \quad \omega^l = (1-\pi) m_1^l < (1-\pi) \tag{1.19}$$

The interesting feature of this state-contingent behaviour concerns its connections with asset prices, the value of collateral and leverage. Further below we explain this in more details through analytical derivations and quantitative analysis. Intuitively, optimism explains why asset price booms and leverage build-ups are steeper in booms and relatively to the model with no beliefs formation. To fix ideas consider the case with a negative θ_0 and that the borrower experiences a good state today and expects a good state tomorrow. Asset price would grow even in the case with no ambiguity attitudes, however under our kinked multiplier preferences, borrowers form today subjective beliefs that induce an LR of $m_1^h > 1$. As this makes the borrowers' SDF right-skewed distributed, it induces higher demand for both. This is why we label this case as optimism. Consider now the opposite case, namely θ_0 lower than zero. According to Lemma 1.3.2 now the optimal LR is left skewed, namely lower than one if associated to future good states and larger than one to bad states. In other words the borrower becomes pessimistic. In this case, if a bad state is expected asset prices will fall according to equation (1.12) and they would do so more sharply than under when $m_1 = 1$ across all states of nature.

1.4 Estimation of the Model-implied SDF

To provide empirical ground to how ambiguity attitudes are formed depending on the current state of the economy we estimate the model-implied Euler equations. This delivers a process for θ_t, whose state-contingent nature empirically supports our preference specification.

We devise a novel estimation method apt to a model with collateral constraints and kinked multiplier preferences. It is based on adapting the minimum distance estimation conditional on latent variables to our modelling environment. In a nutshell we derive a moment condition by using the combined non-linear expression for the Euler equations (1.11) and (1.12). As we show in Appendix A.3, the latter depends on the value function. We therefore follow the approach in Chen, Favilukis and Ludvigson (2013), who write the Euler moment condition as function of the estimated value function. A crucial difference between our method and theirs is that their value function has an unknown functional form, which is estimated semi-nonparametrically, while ours can be derived analytically. Specifically, following Hansen, Heaton and Li (2008), we derive its functional form, which is then estimated using maximum likelihood.

More specifically, the estimation procedure (whose detailed derivations are contained in Appendix A.3) can be described as follows. First, one shall re-write the value function in terms of an ambiguity factor. For this, we adapt the steps used in the recursive preference literature to the case of our kinked multiplier preferences (see Appendix A.3.1). Next, the implied SDF is derived (see Appendix A.3.2) and the value function is estimated (see Appendix A.3.3).

Substitution of the derived SDF into the combined Euler equations for debt and risky assets, (1.11) and (1.12), delivers the final moment condition (see Appendix A.3.4). At last, as it is common for GMM estimation, we condition on a set of instruments, $\mathbf{z_t}$. The resulting moment condition reads as follows:

$$\mathbb{E}_t \left\{ \left[\underbrace{\beta \left(\frac{c_{t+1}}{c_t} \right)^{-1} \left(\frac{\frac{\exp(V_{t+1})}{c_{t+1}} \frac{c_{t+1}}{c_t}}{\sqrt[\beta]{\frac{\exp(V_t)}{c_t}}} \right)^{\sigma_t}}_{\Lambda_{t,t+1}} \left(R^s_{t+1} - \phi R_{t+1} \right) + \phi - 1 \right] \mathbf{z_t} \right\} = 0 \tag{1.20}$$

where $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$ is the cum-dividend return on risky asset and R_{t+1} is the risk-free interest rate, which is time-varying in the data. Note that the expression for the SDF can be decomposed into two factors, $\Lambda^1_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-1}$ and $\Lambda^2_{t,t+1} = \left(\frac{\frac{\exp(V_{t+1})}{c_{t+1}} \frac{c_{t+1}}{c_t}}{\sqrt[\beta]{\frac{\exp(V_t)}{c_t}}} \right)^{\sigma_t}$, where the second captures the role of ambiguity attitudes. Equation (1.20) is estimated fully non-linearly with GMM methods.[16] Note that tight restrictions are placed on asset returns and consumption data since our moment condition embodies both financial and ambiguity attitudes. For the estimation we fix the loan to value ratio at $\phi = 0.5$ and, given that $\theta_t = -\frac{1}{\sigma_t}$, we estimate the preference parameters, β and θ_t.

Table 1.1: Estimation Results

Sample	Estimated parameters[1]				
	β	θ	$\theta(\tilde{v}_t > E\tilde{v}_t)$	$\theta(\tilde{v}_t \leq E\tilde{v}_t)$	$J-test$
1980-2016	0.982		-1.701	2.434	4.385
	(.022)		(.053)	(.075)	(.495)
1985:Q1-2007:Q2	0.891	-1.959			3.811
	(.058)	(.238)			(.702)
2007:Q3-2016:Q4	0.879	7.4404			2.026
	(.015)	(.022)			(.917)

[1] *In parenthesis: the HAC standard errors for the parameter estimates and the p-values for the J-test*

Regarding the data, we use real per capita expenditures on non-durables and services as a measure of aggregate consumption. For R we use the three-month T-bill rate, while R^s is proxied through the Standard & Poor 500 equity return.[17] The choice of the instruments follows the

[16] Optimal GMM parameters minimize a quadratic loss function over the weighted distance between population and sample moments, by a two-step GMM.

[17] Data sources are NIPA Tables `https://www.bea.gov/iTable/index_nipa.cfm`, CRSP Indices database `http://www.crsp.com/products/research-products/crsp-historical-indexes`, and the Shiller database `http://www.econ.yale.edu/~shiller/data.htm`, respectively

literature on time-series estimation of the Euler equations.[18] They are grouped into internal variables, namely consumption growth and interest rates two quarters lagged, and external variables, namely the value and size spreads, the long-short yield spread and the dividend-price ratio (see also Yogo (2006)). A constant is additionally included in order to restrict model errors to have zero mean. Finally, the model's over-identifying restrictions are tested through the J-test (test of over-identifying restrictions, Hansen (1982)).[19]

Table 1.1 presents the results. The estimated values of θ_t are conditioned to the logarithm of the continuation value ratio, defined as $\tilde{v}_t = V_t - \log(c_t)$. Consistently with our previous definition, good states are those for which the latent value function is higher than its mean and vice-versa for bad states. Column 3 shows results conditioned upon the relation $\tilde{v}_t > \mathbb{E}\{\tilde{v}_t\}$, while column 4 reports the results for the complementary condition. We find that a negative value (-1.701) prevails over good states, namely those for which $\tilde{v}_t > \mathbb{E}\{\tilde{v}_t\}$, and that a positive value (2.434) prevails in bad states, namely those for which $\tilde{v}_t \leq \mathbb{E}\{\tilde{v}_t\}$. This gives clear indication on the state-contingent nature of the ambiguity attitudes, being averse to entropy deviations in bad states and opportunistic toward them in good states. According to Lemma 1.3.2 above we know that $\theta_t < 0$, which prevails in good states, implies that agents act optimistically. Similarly a $\theta_t > 0$, which prevails in bad states, speaks in favour of pessimism.

Table 1.2: Estimated Moments of the Pricing Kernel

Moments (1980-2016)	$\Lambda_{t,t+1}$	$\Lambda^1_{t,t+1}$	$\Lambda^2_{t,t+1}$
Mean SDF	0.860	0.977	0.8803
Standard deviation SDF	40.1	0.53	40.9
$Corr(SDF, \Delta c_t)$	-0.138	-0.999	-0.12
$Corr(SDF, R^s_{t+1})$	-0.121	-0.332	-0.115

To further test our result above we run unconditional estimation over two different historical periods. We choose the first to be Great Moderation sample (1985:Q1-2007:Q2), which captures the boom phase preceding the 2007-2008 financial crisis. The sub-sample representing the recessionary states is the period following the crisis, namely the (2007:Q3-2016:Q4). Estimations, reported in the last two rows, confirm the same state-contingent nature uncovered in the conditional estimates. Finally note that for each sample reported the J-test fails to reject model in equation (1.20) at conventional significance levels.

Next, given the estimated preference parameters we investigate the cyclical properties of the pricing kernel, namely the estimated SDF, and through them, those of the risk premia. To this purpose we use the decomposition of the SDF in $\Lambda^1_{t,t+1}$ and $\Lambda^2_{t,t+1}$ in order to isolate the contribution arising from the ambiguity attitudes. The empirical moments of the SDF are listed

[18] See Stock, Wright and Yogo (2002) for a survey on the relevance of instruments choice in a GMM setting

[19] This is a specification test of the model itself and it verifies whether the moment conditions are enough close to zero at some level of statistical confidence, if the model is true and the population moment restrictions satisfied.

in table 1.2. They interestingly show that the high volatility in the SDF is totally driven by the ambiguity attitudes component, which, for the same reason, contribute less to the SDF clear countercyclical properties.

1.5 Analytical Results

In this section we derive analytical expressions for asset price, premia and Sharpe ratio and show their dependence on the optimal LR and the shadow price of debt, μ_t. The analytical derivations will allow us to gain first economic intuition on the combined role of occasionally binding constraints and ambiguity attitudes for asset prices and leverage.

1.5.1 The Impact of Ambiguity on Asset Prices

Proposition 1.5.1 (Asset Price Recursion)**.** *The recursive formula for the asset price over the infinite horizon in our model reads as follows:*

$$q_t = \lim_{T\to\infty} \mathbb{E}_t \left\{ \sum_{i=1}^{T} d_{t+i} \prod_{j=1}^{i} K_{t+j-1,t+j} \right\} \tag{1.21}$$

where $K_{t,t+1} = \frac{\Lambda_{t,t+1}}{1-\phi\mu_t^{'}}$ *with* $\Lambda_{t,t+1} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1}$ *and* $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$.

Proof is described in Appendix A.4.1. The asset price clearly depends upon the optimal LR, m_{t+1}, and the shadow price of debt, μ_t. Consider first good states. In this case endogenous beliefs are right skewed toward the upper tails according to Lemma 1, hence both $\Lambda_{t,t+1}$ and $K_{t,t+1}$ are higher than when $m_{t+1} = 1$ for all positive states. In good states the asset price grows, due to increase asset demand, but it does so more under optimist beliefs. Similarly in bad states endogenous beliefs are left-skewed toward the lower tails, hence both $\Lambda_{t,t+1}$ and $K_{t,t+1}$ are higher than in the case with no ambiguity for all negative states. Asset price falls, but they do more so with pessimism. This is the sense in which ambiguity attitudes contribute to the heightened dynamic of the asset price boom and bust cycles. The asset price also depends upon the shadow price of debt, which proxies the margin or the down-payment requested to borrowers. When the constraint is binding margins are positive and increasing, in line with empirical observations (see Geanakoplos (2010)). The higher margins paid by borrowers or the higher collateral value of the asset is reflected in higher asset prices. This also contributes to heightened asset price dynamics.

Proposition 1.5.2 (Equity Premium)**.** *The return for the risky asset reads as follows:*

$$\mathbb{E}_t\{R^s_{t+1}\} = \frac{R(1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu_t^{'})}{1-\mu_t^{'}} \tag{1.22}$$

while the premium of its return over debt return reads as follows:

$$\Psi_t = \frac{1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu_t^{'}}{1-\mu_t^{'}}. \tag{1.23}$$

where $\Lambda_{t,t+1} = \beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}$ *and* $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$.

See Appendix A.4.2 for the proof. The above proposition also shows unequivocally the dependence of the premia over the beliefs as captured by m_{t+1} and the shadow price of debt. While the exact dynamic of the equity premium depends on the solution of the full-model and upon its general equilibrium effects, we can draw some general conclusions on the dependence of the equity premium upon the beliefs and the shadow price of debt.

First, a negative covariance between the SDF and the risky asset returns implies that borrowers are less hedged. This results in a higher return required to hold the risky asset. The opposite is true for positive covariances. While we cannot say with certainty the sign of the $cov(\Lambda_{t,t+1}, R^s_{t+1})$,[20] we can conjecture that optimism and pessimism increase the covariance between consumption and asset returns. One way to see this is by looking at the upper bound for this covariance. By the Cauchy-Schwarz inequality that $cov(\Lambda_{t,t+1}, R^s_{t+1}) \leq \sqrt{Var(\Lambda_{t,t+1})Var(R^s_{t+1})}$. Therefore anything that either increases the variance of $\Lambda_{t,t+1}$ or R^s_{t+1} will increase their covariance, whether in the positive or the negative domain. Endogenous beliefs formation by inducing fluctuations in m_{t+1} tend to increase the variance of the stochastic discount factor which is given by $Var(\Lambda_{t,t+1}) = Var\left(\beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}\right)$. Hence the variance of m_{t+1} adds up to the variance of the stochastic discount factor.

Second, the premium also depends upon the shadow price of debt. Taking as given again the covariance between the SDF and the risky return, one can compute the following derivative: $\frac{\partial\Psi_t}{\partial\mu_t^{'}} = \frac{(1-\phi)-cov(\Lambda_{t,t+1},R^s_{t+1})}{(1-\mu_t^{'})^2}$. If the $cov(\Lambda_{t,t+1}, R^s_{t+1})$ is negative the derivative is certainly negative.[21] In other words when there are low hedging opportunities a tightening of the constraint implies that borrowers require higher premia to hold the risky asset. The asset already conveys poor insurance opportunities, a tightening of the constraint by reducing the asset collateral value, reduces its demand. Hence borrowers are willing to hold only at higher premia. Endogenous beliefs also play an indirect role in this dependence. Indeed as explained above fluctuations in beliefs generally raise the absolute value of the covariance. Hence, consider again the case of a negative covariance. In this case fluctuations in beliefs impair even more the hedging abilities of the risky assets and this in turn increases the premium that borrowers ask in face of a tightening of the borrowing limit.

Proposition 1.5.3 (Sharpe Ratio)**.** *The Sharpe ratio in our model reads as follows:*

$$SR = \frac{\mathbb{E}_t\{z_{t+1}\}}{\sigma_z} = \left[\frac{\sigma^2_{\Lambda^*_t}}{\bar{\Lambda}^{*2}} - 2\mu_t^{'}\frac{(\phi-1)\mathbb{E}_t\{z_{t+1}\}}{\sigma_z^2} - \frac{\mu_t^{'2}}{\bar{\Lambda}^{*2}}\frac{(\phi-1)^2}{\sigma_z^2}\right]^{\frac{1}{2}} \tag{1.24}$$

where $z_{t+1} = R^s_{t+1} - R$ *is the asset excess return* $\bar{\Lambda}$ *is the long run value for the SDF,* $\sigma^2_{\Lambda^*_t}$ *is the volatility of the SDF and* σ^2_z *is the volatility of the excess return.*

[20] This indeed depends on whether $\mathbb{E}_t(\Lambda_{t,t+1}, R^s_{t+1}) > \mathbb{E}_t(\Lambda_{t,t+1})\mathbb{E}_t(R^s_{t+1})$ or $\mathbb{E}_t(\Lambda_{t,t+1}, R^s_{t+1}) < \mathbb{E}_t(\Lambda_{t,t+1})\mathbb{E}_t(R^s_{t+1})$.

[21] If the $cov(\Lambda_{t,t+1}, R^s_{t+1}) > 0$, then whether $\frac{\partial\Psi_t}{\partial\mu_t^{'}}$ is positive or negative depends upon whether the $cov(\Lambda_{t,t+1}, R^s_{t+1}) > (1-\phi)$ or not.

Proof is given in Appendix A.4.3. The presence of endogenous beliefs raises the Sharpe ratio and brings it close to the empirical values as we show in Table 1.4. Matching the empirical values of the Sharpe ratios is typically hard for models with asset pricing and/or financial frictions. The reason being that typically an increase in the excess returns of the risky assets is accompanied by an increase in its volatility. Analytically it is easy to see why the Sharpe ratio raises in our model. First fluctuations in m_{t+1} raise fluctuations in the stochastic discount factor, Λ_t^*, hence in its variance. This in turn raises the Sharpe ratio. Second, fluctuations in θ_t enhance fluctuation in beliefs, m_{t+1}. Third, the kinked nature of the value function steepens fluctuations in m_{t+1} and the SDF also since marginal utilities tend to infinity around the kink. In turn any increase in the variance of m_{t+1} raises the variance of Λ_t^* and the Sharpe ratio. Intuitively in presence of *uncertainty* or ambiguity agents require a premium which goes beyond the one needed to cover risk[22] as measured by the volatility of the excess return. If agents knew the objective probability distribution, they would need to be compensated only for bearing tail risk. As the tail itself is uncertain, borrowers require a higher premia.

1.6 Quantitative Results

In this section we solve the model numerically employing a global solution method, namely policy function iterations with occasionally binding constraints. We provide details on the solution method in Appendix A.5. We group our results in three. First, we search for the optimal model calibration. To do so we choose some target moments in the data and we search for the set of parameters that minimizes the distance between the targets and the model-implied moments. This gives further empirical validation of our model. Second, under the optimal calibration we verify if the model can match several volatilities and correlations for asset prices, returns, equity premia and leverage. We show that in fact the model does it well. At last, under this optimal calibration we examine policy functions and we conduct a crisis event exercise. Our main result is that with ambiguity the model produces steeper asset prices and leverage increases in booms, which are then followed by sharper de-leverage and crises in recessions.[23]

1.6.1 Calibration Strategy

This section describes the calibration strategy. We divide the set of structural parameters in three groups, as Table 1.3 shows. The first group includes parameters which are calibrated using external information. Those are the risk free rate, the loan-to-value ratio, the fraction of financial wealth over total wealth and the autocorrelation parameter of the income shock. The second group refers to θ, the ambiguity attitudes parameter, which is calibrated according to the the GMM estimation outcomes. The last group, instead, includes the remaining parameters which are calibrated using a matching moments routine.

[22] Here we refer to the distinction between uncertainty and *risk* introduce by Knight (1921).

[23] The quantitative results of this section are robust to the specification of the collateral constraint. Indeed, in section A.7 we show that the the main message does not change introducing an intermediation shock assuming a binomial process for the loan-to-value ratio ϕ.

In order to calibrate the second group of parameters, we choose to match six empirical moments (the matching is shown in Table 1.4, where also other moments are displayed), namely the volatility of debt σ^b, the autocorrelation of debt ρ^b, the correlation between debt and consumption $Corr(\Delta b^t, \Delta c^t)$, the expected return on risky asset $\mathbb{E}_t(R_t^s)$, the volatility of return on risky asset σ^{R^s}, the correlation between return on risky asset and consumption growth $Corr(R_t^s, \Delta c_t)$. To compute the empirical equivalent we focus on the data sample 1980:Q1-2016:Q4, which captures a period of both of large debt growth and subsequent de-leverage. More details on the data sources are in Appendix A.1. We do not include the equity premium among our targets because the risk free rate is exogenous in the model, but we show later on that our model can match it well.

Table 1.3: Values for the calibrated parameters

Parameter	Meaning	Strategy	Value
R	*Risk-free rate*	3month T-bill rate	1.0114
ϕ	*Loan-to-value ratio*	Crises Probability	0.15
α	*Share of dividend*	Fraction of financial wealth	0.10
ρ_y	*Income Persistence*	Curatola and Faia (2016)	0.634
$\theta^+(V_t \leq \mathbb{E}_{t-1}\{V_t\})$	*Pessimism*	GMM estimation	2.434
$\theta^-(V_t > \mathbb{E}_{t-1}\{V_t\})$	*Optimism*	Matching Moments	-1.701
γ	*Risk aversion*	Matching Moments[1]	2.075
β	*Discount factor*	Matching Moments	0.930
σ_y	*Income Volatility*	Matching Moments	0.0415

[1] *The matching moment routine is based on the following grid:* $\sigma^y \in [0.02, 0.07]$, $\beta = [0.92, 0.98]$, *and* $\gamma = [1, 2.2]$. *For each parameter we check that the optimal values do not hit the bounds.*

1.6.2 Empirical Moments Matching

In this section we evaluate the model's ability to match the empirical moments under the optimal calibration determined above. We also compare the theoretical moments of our model with ambiguity attitudes (labelled AA since now on) with those of the equivalent model but with rational expectation (labelled RE since now on). Table 1.4 summarizes the main results. The overall message is that our model fits well the empirical moments. First, it is better capable of matching empirical debt and risky asset return volatilities, relatively to the RE model. This is so despite both models exhibit amplification induced by the occasionally binding collateral constraint. This shows that endogenous beliefs are also needed to explain asset and debt markets dynamics. The equity premium as well as its cyclical properties are also well captured and again the presence of ambiguity attitudes seem to improve even above the benchmark model featuring solely the collateral constraint. As explained in Cochrane (2005), the ability to match contemporaneously the long run equity premia and asset returns and their cyclical properties is related to the agents' attitude toward events on the tails. In our model borrowers are endogenously optimistic, hence risk-takers, on the upper tail, while they are pessimistic, hence risk-sensitive, on the lower tail. This additional effect, stemming from the endogenous waves of optimism, improves the ability of the model to match the equity premium and its cyclical properties. At last, both model match the pro-cyclicality of leverage which is well documented in the data. Leverage indeed increases in booms due to a combination of exuberance and lax debt constraints and declines in recessions due to a combination of pessimism and increasing margins, namely borrowers' down-payments. Here neither our model nor the RE model seem to match the empirical value with precision, as the first underestimates, while the second overestimates. Finally, the model with ambiguity attitudes predict a probability of financial crises[24] equal to 3.16, close to what Bianchi and Mendoza (2018) finds empirically for the developed countries (4 crises every 100 years).

Table 1.4: Empirical and model-based moments

Moments	Mnemonics	Empirical	Model AA[1]	Model RE
Debt volatility	σ^b	12.52	12.37	7.24
Debt persistence	ρ^b	0.846	0.539	0.331
Debt cyclicality	$Corr(\Delta b_t, \Delta c_t)$	0.668	0.378	0.821
Equity return	$E_t(R_t^s)$	9.38	8.19	7.38
Equity return volatility	$\sigma^{R_t^s}$	16.21	17.46	12.40
Equity return cyclicality	$Corr(\Delta R_t^s, \Delta c_t)$	0.474	0.989	0.989

[1] *Column 3 and 4 compare theoretical moments under ambiguity versus rational expectation. For both model specifications a different moment matching exercises are run, then the two models could differs in the parameter values.*

1.6.3 Model Simulations

Given the above, we proceed analysing the dynamics of our in comparison to the a rational expectation benchmark. More specifically, we compare two identical models with the same parameters specification (see Table 1.3 fore reference) which differs only for the preferences specification. In the first model, identified with the label *AA (Ambiguity Attitudes)* agents are endowed with kinked multipliers preferences, while in the *RE (Rational Expectations)* model agents are fully rational. In the following section two types of analysis are performed. First, we run a simulated crisis event study in order to study the model dynamics around the materialization of a financial crisis. Then, we look at the policy functions of the two models in order to identify which factors drive the crises event study outcomes.

[24] In the model a financial crises is defined as the situation in which the collateral constraint is binding and and the there are massive capital outflows (current account is two standard deviations above its mean). We will come back on this point in the next section.

Simulated Crises Event Study

The crisis event study displayed in figure 1.1 proves the model's ability to endogenously generate financial crises and studies relevant macro dynamics around it. More in detail, the event analysis is realized using model-simulated data and defining as crises the events in which the collateral constraint binds and there is massive capital outflows. Then, we construct a seven-periods event window centred on the crisis, which materialize at time 0, and analyse the pre- and post-crises patterns. The first four panels of figure 1.1a display the path of the main macroeconomic variables (leverage, asset price, equity premium and consumption) for the AA model. The pre-crises period is characterized by a strong leverage build-up, a significant increase in the asset price and a decline in the equity return. Instead, during the crisis, the combination of Fisherian debt deflation effects and agents' beliefs distortions generates large declines in leverage, asset prices and consumption, as well as, a strong increase in the equity premium. Finally, the last two panels of figure 1.1a show the income income shock (on the left) and the evolution of the agents' beliefs (on the right). The latter panel plots the values of θ and clearly shows that agents update their beliefs over the credit cycle. Indeed, coherently with our empirical evidence, agents are optimistic in boom ($\theta < 0$) but switch to be pessimistic during the crises ($\theta > 0$).

In order to identify how the switching in agents belies affect the above results, figure 1.1b replicates the exercise but comparing the dynamics of the AA and the RE model. Two insights are worth noticing. First, before the crisis, optimism implies a stronger leverage build-up, which is almost absent in the RE expectation model. Indeed, in the RE model the leverage remains close to the ergodic mean before the crises materialization. From this result we can infer that a model with an occasionally binding collateral constraints and rational agents is not able to produce the strong leverage build-up that we have observed before the global financial crises.[25] Consequently, the interactions among financial frictions and beliefs distortions are fundamental in order to explain the leverage dynamics before the crises. Indeed, the magnitude of this leverage build-up build is close to the percentage deviation of the aggregate credit from its long-term trend registered in the last two US credit boom (see figure A.1 in Appendix A.6).[26]. Second, when the crisis materializes, the pessimistic attitudes are responsible for a stronger deleveraging and asset price decline.

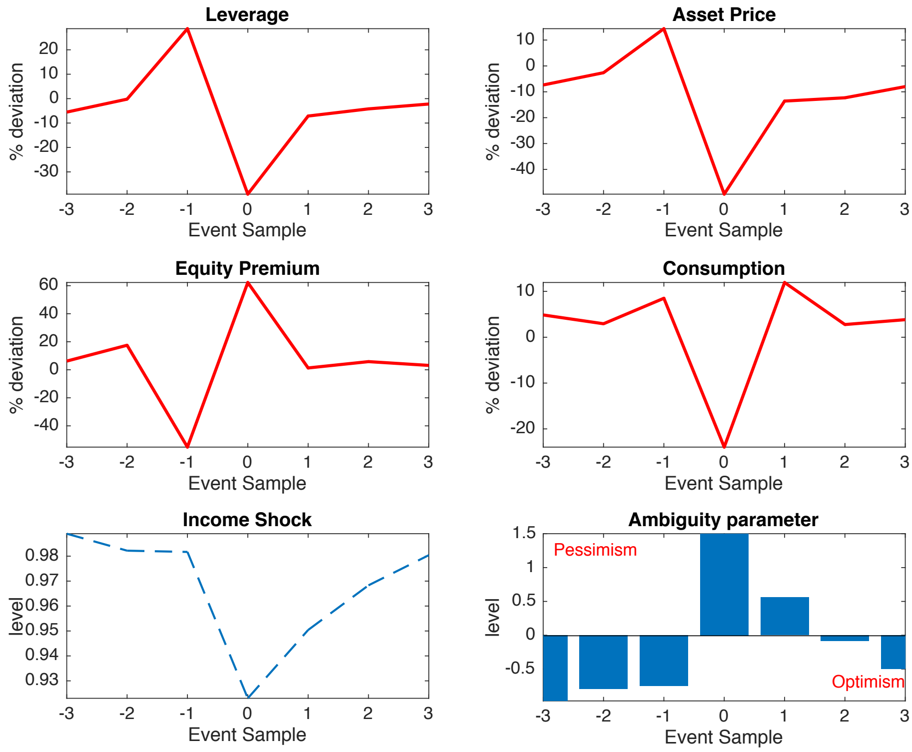
In a nutshell, we have shown how that ambiguity attitudes produce larger leverage cycle fluctuations around the financial crisis materialization. The state contingent nature of our beliefs formation has a crucial role in explaining this result. Indeed, in booms optimistic agents overweight future gains and then demand a larger amount of risky assets, boosting the price. As a consequence, the agents' borrowing capacity raises and a strong phase of leverage build-up starts. In recessions the opposite happens. Pessimism amplifies the assets fire sales spiral triggered by the collateral constraint because agents assign a too high weight to future negative

[25] This shortcoming associated to the agents' rationality of this class of models is known and not specifically related to our model (see the crises event studies in Bianchi (2011) and Bianchi and Mendoza (2018) among many others.

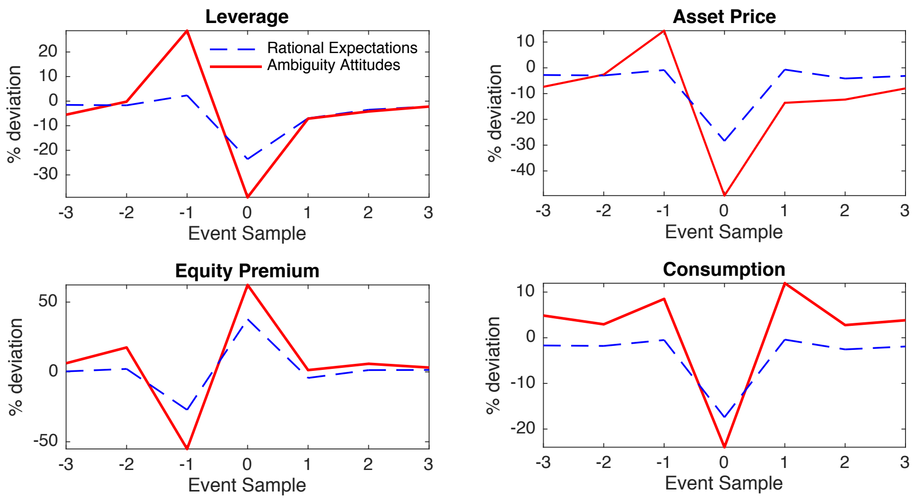
[26] More specifically, figure A.1 shows that at the peak-of-the-cycle US leverage was 8.4% in 1988 and 32.3% in 2007 higher then the long-term trend. Our simulations in 1.1 predict a deviation from the ergodic mean of about 25 %

Figure 1.1: Simulated Crises Event Study

(a) Ambiguity Attitudes



(b) Ambiguity Attitudes vs Rational Expectations



Note: The simulated crises event is performed following Bianchi and Mendoza (2018). First we run the model unconditional simulation long path (100.000 periods). Second, over the simulated path a crises is defined as the situation in which the collateral constraint is binding and there is a massive capital outflows (the current account is at least two standard deviation greater then its ergodic mean). Then we study the average behaviour of the main model variables around (three period before and three after) the identified crises events. The path of the endogenous macroeconomic variables (leverage, asset price, equity premium and consumption) is expressed in terms of deviation from the respective ergodic mean.

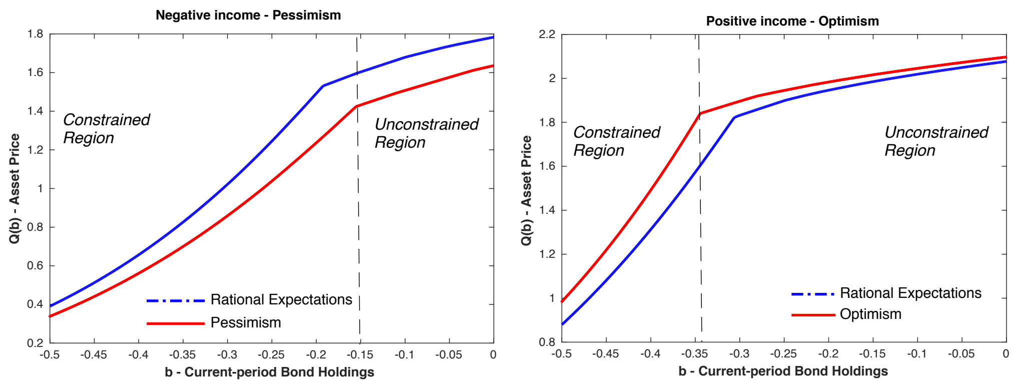
realizations in which the collateral constraint remain binding. This generates a sharper decline of the collateral values forcing borrowers to a more severe deleverage.

Policy Functions Analysis

In order to identifies the role of the different model ingredients in driving the above results, we perform a further experiment looking at the model's policy functions. Figure 1.2 shows the decision rules for debt and asset prices, comparing the model with ambiguity attitudes (red lines) and the one with rational expectations (blue-dotted lines). In order to appreciate the non-linearity coming from the kinked multiplier preferences, we show the policy functions associated to a positive income realization (+5% from the ergodic mean; left panels) and those associated to a negative realization (−5% from the ergodic mean; right panels). With these extreme income realization we can compare the decision rules of both optimistic (associated to positive realizations) and pessimistic (associated to negative realizations) attitudes.

Figure 1.2: Policy Functions Analysis

(a) Asset Price



(b) Bond holdings

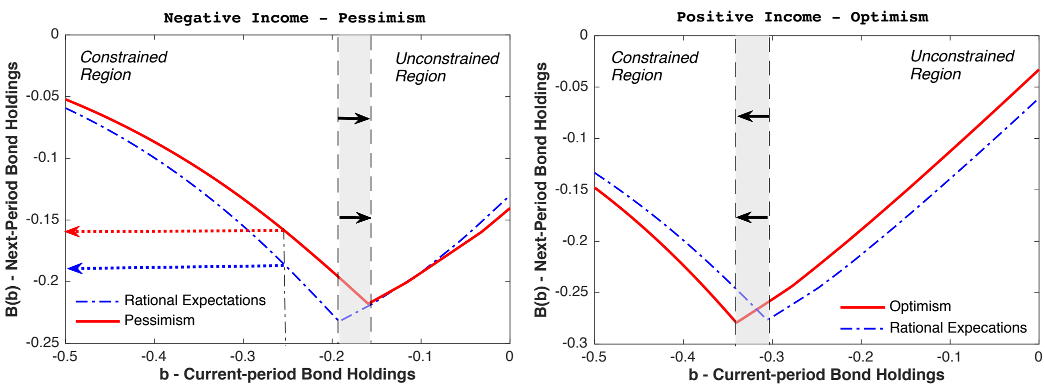


Figure 1.2a plots the asset price decision rule as a function of the current-period debt holdings (the endogenous state variable of the model). The non-linearities characterizing our model are clearly captured by this plot. First, the policy functions show a kink corresponding to the level of current-period bond holdings that makes the collateral constraint marginally binding. On the

left of the vertical line (for high level of current-period debt), the collateral constraint is binding and financial amplification occurs within this region. Indeed, the asset prices respond very strongly to changes in the debt position. Contrarily, on the left of the vertical line (for low level of current-period debt) the constraints are loose and financial amplification effects are absent. Second, we can identify the non-linearity of the beliefs distortions with respect to the income realizations. Indeed, for positive realizations and hence optimistic beliefs, the policy function of the AA model are always above of the corresponding function of the RE specification, while the opposite happens for extreme negative realization, hence pessimistic attitude. This means that, given the same level of the exogenous and endogenous state variables, agents endowed with optimistic beliefs demand a higher level of risky assets with respect rational agents, boosting the price. Contrarily, pessimistic beliefs imply a weaker demand of risky assets and then a lower price.

Figure 1.2b shows that the non-linearities detected in the asset price decision rules are transmitted into the debt markets. The collateral constraint generates *V-shaped* bond holdings decision rules, which are a typical feature of models with high deleveraging and financial crises (see Bianchi (2011) and Bianchi and Mendoza (2018) among many others). To the right of the kink, the policy functions are upward-sloping, corresponding to the unconstrained values of debt, while to the left they are downward-sloping identifying the constrained region where next-period bond holdings decrease in current bond holding. Comparing the AA and RE models allow us to study the role of beliefs distortions. Two results are worth noticing. First, agents' attitude affects asymmetrically the size of the constrained region: it becomes smaller in good states and larger in bad states. This implies that optimistic agents have a larger current and perceived (future) debt capacity in good states, driving a stronger leverage build-up in boom. Second, focusing on the binding region of left panel (negative realizations), we can notice that starting from the same level of current-period debt, the next-period debt is lower in the AA model than in the RE one. This justifies the sharper deleveraging during the crises when agents are endowed with left-skewed belief.

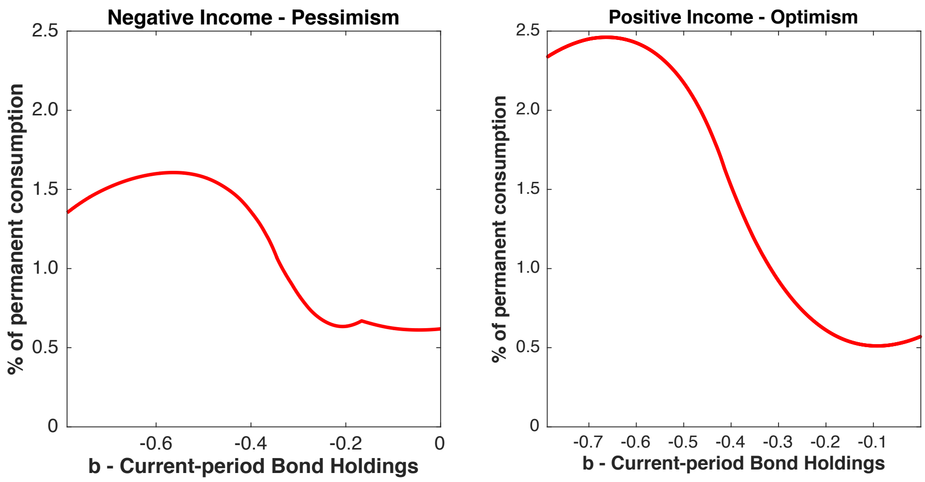
Beliefs distortions and welfare losses

Even if this paper does not study the macroeconomic policies that can reduce the impact of the externalities characterizing our model, in this section we study the welfare implications of the beliefs distortions. The welfare losses associated to ambiguity attitudes are calculated as compensating consumption variations for each initial state that equalize expected utility across the AA and the RE models. Formally, for a given initial state at date 0, the welfare losses of the beliefs distortions is computed as the value of γ that satisfies the following condition:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{AA}(1+\gamma)\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{RE}) \tag{1.25}$$

The welfare losses are illustrated in Figure 1.3, where we plot the level of γ as a functions of the current-period bond holdings for negative (on the left) and positive (on the right) income realizations. As we can notice, beliefs distortions produce significant welfare losses independently for both positive and negative realizations. Moreover, we can see that the magnitude depends on the current-period debt holdings. The highest levels of the welfare losses are associated with high level of current-period debt, because agents for those level of debt are less able to adjust their choices. This means that these welfare losses strongly interacts with the occasionally binding constraint and the associated pecuniary externalities. Moreover, the losses are significantly higher in case of positive income realizations meaning that right-skewed beliefs are much more costly in terms of social welfare.

Figure 1.3: Welfare losses



The above results suggest some tentative policy implications. From the previous literature (see Bianchi and Mendoza (2018) among others) we know that the pecuniary externalities associated to the endogenous nature of the collateral constraint justifies the intervention of a social planner, who imposing a state-contingent Pigouvian tax can alleviate the incentives to excess leveraging before the crises and then reduce the probability and severity of the financial downturns. In our model, the pecuniary externalities continue to operate, but the simple exercises discussed above shows that they interact with an additional source of inefficiency generated by the beliefs distortions. This insight, open the room for a discussion on how the social planner can deal with this new driver of welfare losses. We leave this issue for future studies.

1.7 Conclusions

Financial crisis are most often triggered by endogenous instability in debt markets. The latter are typically characterized by collateral constraints and opacity in asset values. Under lack of transparency the beliefs formation process acquires an important role since eventually it affects the value of collateral and with it the debt capacity. The narrative of most crises depict sharp increases in debt and asset prices prior to them and sharp reversal afterwards.

We therefore introduce in a model in which borrowers fund risky assets through debt and are subject to occasionally binding collateral constraints, beliefs formation, driven by ambiguity attitudes that endogenously induce optimism in booms and pessimism in recessions. In booms optimistic borrowers demand more risky assets, which results in higher asset price growth (compared to the case with only collateral constraints), and lever up more. In recessions pessimistic borrowers de-leverage sharply and off load risky assets. This beliefs formation process coupled with the occasionally binding nature of the collateral constraint is a crucial element in explaining the combined amplified dynamic of asset prices and leverage as well as the whole span of their long run and short run statistics. Importantly we assess the empirical validation of our model both through GMM estimation of the Euler equation and through data-model moment matching.

Chapter 2

Monetary and Macroprudential Strategic Interactions in Closed and Open Economy

2.1 Introduction

In the aftermath of the global financial crisis, policy makers and regulators have been focusing on instruments promoting financial stability. Countercyclical macroprudential policy rules have been adopted in many countries to prevent and mitigate systemic risks associated with excessive credit growth and asset prices bubbles, aiming to build a safer and more resilient financial system[1]. The general consensus in favor of a countercyclical prudential regulation relates to its effectiveness in targeting the main sources of financial instability due to the amplification effect of leverage. Moreover, the discussion of macroprudential policy cannot abstract from its potential interactions with monetary policy. Regarding the latter point, many also stress the importance of clear and distinct mandates for monetary and macroprudential policies[2]. However, some forms of interactions between the objectives and instruments of macroprudential and monetary authorities cannot be avoided. Therefore, a set of questions naturally arise. How do monetary and macroprudential policies interact and through which transmission channel? Which are the costs of strategic interactions compared to a fully cooperative institutional design?

The answer probably depends on the degree of real and financial integration of economies, which has been increasing over the last decades. After the 1980s financial markets deregulation, the size of capital flows between countries has increased substantially together with the degree of financial synchronization (see e.g. Passari and Rey (2015)). This fact contributes to explain the quick and strong transmission of the recent crises from the U.S. financial markets to the Europe and other parts of the world. As a consequence, the optimal design of a framework for monetary

[1]Up to date several countries have activated the CCyB (Countercyclical Capital Buffer): Bulgaria (0.5%), Czech Republic (1.5%), Denmark (1.0%), France (0.25%), Hong Kong (1.875%), Iceland (1.75%), Ireland (1.0%), Lithuania (1.0 %), Norway (2.0%), Slovakia (1.5%), Sweden (2.5%) and United Kingdom (1.0%)

[2]More in detail, the policy and academic debate around the optimal authorities' objectives allocation is vast. From one side, Smets (2014), Curdia and Woodford (2010, 2016) and Woodford (2012) among others, trust that the monetary policy should target also financial stability conditions, while Wadhawani (2010), Svensson (2012, 2014) and Yellen (2014) argue in favour of a system in which the monetary policy focuses only on the price stability and a macroprudential authority reacts to the financial imbalances.

and macroprudential policies cannot abstract from considering the role of cross-country flows. The objectives of both authorities are strongly affected by the global business and financial cycles' development, supporting an international dimension of strategic interactions. Therefore, the questions above are complemented by the following. How does the degree of financial and commercial openness affect the interaction between the two policy domains? What is the optimal policy setup at global level, between countries and within each policy domain?

This paper provides some tentative answers to the above questions through the lens of a flexible monetary model with both business and financial synchronization between two symmetric countries. In order to model the international flows, we borrow extensively from the consolidated literature known as the *"New Open Economy Macroeconomics"* (see Lane (2001) for a survey). The unifying feature of this literature is the introduction of nominal rigidities and market imperfections into an open economy, dynamic general equilibrium model, making the framework particularly suitable for optimal monetary policy analysis (see for example Clarida, Gali and Gertler (2002), Benigno and Benigno (2003), Pappa (2004) Gali and Monacelli (2005), De Paoli (2009) and Corsetti and Leduc (2011)). We set up a simple two-country model that accounts for fluctuations in terms of trade as the main transmission channel for inflation and monetary policy among countries. We enrich this framework with a country-specific credit cycles and international financial flows. The link between financial factors and the real economy, in each country, is modelled following Curdia and Woodford (2010, 2016) and Woodford (2012). In their approach, the yield difference between bonds issued by risky private borrowers and risk-free bonds, impact inefficiently output and inflation fluctuations. The intuition which would be delivered by a micro-founded model would be that higher credit spreads, worsening financial conditions for borrowers and producing an inefficient expenditure allocation, reduces aggregate demand and overall welfare. Finally, the model is set up in order to nest different international regimes, from complete autarky to a fully integrated world.

Within this environment we characterize two types of policy games. First of all, we introduce the concept of *"Within-Country Nash and Cooperative Equilibria"*, aimed at studying how international trade and financial flows affect the strategic interactions between monetary and macroprudential authorities in the domestic country. As far as we know, there are no other contributions providing a study of the effects of international openness on the optimal design of monetary and macroprudential policies from the domestic economy standpoint. Secondly, we evaluate the policy spillovers between integrated countries defining the concept of *"Between-Countries Nash and Cooperative Equilibria"*. In this second policy game, we evaluate the incentives to cooperation of two macroprudential authorities operating in different countries.[3] Again, as far as we know, this is the attempt to distinguish the within-country from the between-countries policy interactions. In both cases, the analysis is performed assuming that

[3] We focus on the strategic interactions between macroprudential authorities leaving aside the monetary authorities' interactions for a couple of reasons. First, while the literature studying the international cooperation between monetary authorities is vast, the macroprudential authorities' counterpart is limited. Second, the previous literature on the international cooperation among monetary authorities find that for two symmetric countries the gains from cooperation are negligible in the monetary policy one(for more details refers to Obstfeld and Rogoff (2000), Benigno and Benigno (2003), Pappa (2004), Corsetti and Pesenti (2005) among others).

each policy is implemented through a feedback rule in the spirit of Taylor (1993), in which the rule's elasticity links the policy instrument to the respective target. Regarding monetary policy, an approach based on a Taylor rule is largely standard. Moreover, it is well established that this type of rule can be derived as optimal policy when the policymakers' objective is a quadratic loss function containing the target variables (Woodford (2011)). For the macroprudential policy point of view, the issue is less trivial and, as suggested by Mendoza (2016), the optimal macroprudential model should be studied in a non-linear model through a Ramsey optimal problem. However, two considerations support our modelling choice. First, the complexity of global methods makes this kind of analysis feasible only for very stylized models. Second, there is a growing consensus around this type of policy rules also for macroprudential policy, given the fact that they represent well the countercyclical nature that the Basel III regulatory framework requires for macroprudential policies (among many contributions, please refer to Angeloni and Faia (2013), Angelini, Neri and Panetta (2014), Davis and Presno (2017) and Carrillo et al. (2017)).

Within this rich theoretical framework, we perform three main exercises. First of all, we focus on the fully autarkic domestic economy leaving the within-country strategic interactions between authorities being affected by domestic factors only. In this step, we compare the Nash and cooperative equilibria in terms of policy rules' elasticities and authorities' loss functions, identifying quantitatively the gains from policy cooperation. Secondly, we gradually open the domestic economy to international trade and financial flows and then compute a set of *"Within-Country Nash and Cooperative Equilibria"* for different degrees of trade/financial openness. In this second exercise, terms of trade fluctuations and financial flows crucially affect the optimal policy design and the authorities' incentives to cooperate. Lastly, we study the strategic interactions between authorities in the same policy domain but located in different countries. In the non-cooperative setup, the authorities only partially internalize the cross-country spillovers associated to their country-specific decisions and then the resulting equilibria might be inefficient from both the country-specific and the global perspective.

The interest for the within-country strategic interactions lies in the current institutional design in some countries, in which there is a separate body for each policy (e.g. in Sweden). And even when macroprudential policies are also in the hands of central banks, the decision-making process could involve different bodies and the alignment of the policies' objectives is not always guaranteed (for example the Bank of England has two different committees, albeit with the same chair and a degree of overlapping membership). These features motivate strategic actions and incentives to deviate from a common final objective in order to maximize individual payoffs. Regarding the latter point, our analysis puts emphasis on the distribution of the gains from cooperation among policies, with the aim to understand whenever each authority is willing to reduce its autonomy and commit itself to a full cooperation regime by using a criterion of Pareto optimality. From our model point of view, the presence of strategic interactions depends on the fact that each authority's target is affected, at least indirectly, by the other authority's instrument. Indeed, inflation is partially affected by the fluctuation of the credit spreads targeted by the macroprudential authority and, at the same time, monetary policy influences aggregate credit conditions by controlling the interest rate. Also the interest for the international dimension of the strategic interactions is vivid from the policy makers' point of view. Indeed, in the current world economy, which is composed by increasingly integrated countries, the definition of a stronger and more efficient international policy setup cannot abstract from understanding how international factors can potentially affect the interactions among authorities within each country and between the countries. Regarding the last point, when there is a strong misalignment in the economic fundamentals across economies, non-cooperative set-up could intensify the costs of lack of cooperation and put the global economy on an inefficient equilibrium path.

Our analysis yields the following set of results. First of all, we find large gains from cooperation between domestic authorities in the fully autarkic benchmark. In the Nash equilibrium, both authorities inefficiently overreact to fluctuations in the target variables, without being able to achieve a higher stabilization outcome. Indeed, the overall loss function (the weighted sum of the two quadratic loss functions) is much higher in the Nash equilibrium with respect to the cooperative one, in line with all the literature. Looking at the distribution of the gains from cooperation, the macroprudential authority massively benefits from the shift to the cooperative policy regime. This *"helping hand"* of the monetary authority in taming the financial cycle can be identified not only when the two authorities cooperate but also if the monetary policy acts according to a dual mandate (the output gap is part of the quadratic loss function). In the latter case, the monetary authority gives an indirect weight to credit spread fluctuations and, as a consequence, the macroprudential loss function is significantly lower in both the cooperative and Nash equilibria. This stronger alignment of the authorities' objectives also reduces the inefficiency of the non-cooperative setup. Finally, we identify the region in which the shift form a non-cooperative to a cooperative setting is desirable for both authorities on a basis of Pareto improvement criterion. We find that the feasible region depends on the weights assigned to the monetary authority component of the overall loss function.

Secondly, the open-economy extension of the within-country monetary-macroprudential strategic interactions confirms the presence of large costs associated with the non-cooperative design of the policy interactions. As expected, the total payoff is higher in the cooperative equilibrium with respect to the one associated to the Nash equilibrium for any value of trade or/and financial integration. However, the sensitivity of the gains from cooperation to an increase in the integration level depends on which kind of integration - financial or trade - is introduced. More precisely, when we close the channel of financial flows and let trade integration to vary, we find that the two authorities' payoffs are more indirectly aligned and then the size of strategic interactions losses shrinks. Indeed, the difference between the total payoff of the cooperative and Nash equilibria is a decreasing function of the degree of trade openness. However, the overreaction of the policies, highlighted in the autarky benchmark, becomes stronger when the degree of trade openness increases. Indeed, the trade openness implies that the domestic economy is exposed to another source of volatility coming from abroad and the two authorities, in particularly in the non-cooperative set-up, response increasing the rules' elasticities. The situation changes drastically when we add capital flows. In our model, indeed, the financial integration produces amplifications in the domestic credit cycle and the cooperation become more beneficial because it implies that also the monetary authority reacts to credit spread fluctuations, even when monetary policy is primarily concerned with inflation. Moreover, a higher degree of financial integration implies a stronger macroprudential policy reaction to domestic credit fluctuations.

Finally, the analysis of the macroprudential authorities' strategic interactions through the lens of the *"Between-Countries Nash and Cooperative Equilibria"* produces the following two main results. First, the cooperation is still beneficial from both the country-specific and global perspective, even if the size of the gains from cooperation is lower with respect to those of the within-county monetary-macroprudential cooperation. Second, the Nash equilibria, differently from the case of within-country case, are characterized by an under-reaction of the authorities with respect to the cooperative equilibria. This feature can be explained looking at the international policy spillovers spreading across countries that are not fully internalized by the authorities in the non-cooperative setup. Indeed, when a country-specific authority reacts to the country-specific targets, positive stabilization effects are partially or fully (depending on the degree of integration) transmitted to the other countries. As a consequence, the authority operating in the other country anticipates these effects and chooses an inefficiently low responsiveness to the aggregate domestic conditions. Finally the gains from cooperations depend positively on the degree of financial integration but decreases when the trade integration rises. This result, shared with the within-country highlights the disciplinary role of terms of trade fluctuations on the authorities' strategic interactions. Contrary, an higher degree of financial integration exacerbates the incentive to strategic behaviours and thus increases the gains from policies' coordination.

This paper proceeds as follows. Section 2.2 reviews the related literature. Section 2.3 presents a flexible NK model with trade and financial flows. Section 2.4 discusses the strategic interactions within the domestic economy while section 2.5 completes the analysis with the between-countries analysis. Section 2.6 concludes.

2.2 Comparison with the past literature

There is a growing literature that integrates credit factors and financial crises in standard New Keynesian models. In two seminal contributions, Curdia and Woodford (2010, 2016) introduce credit frictions, in the form of credit spreads between savers and borrowers, in an otherwise standard monetary model. These credit frictions determine an inefficient wealth distribution between borrowers and savers, depressing aggregate demand. Woodford (2012) derives a simplified version of the model, where the magnitude of credit frictions is described by a two-stage regime switching process. This formulation allows the model to reproduce endogenous transitions from a normal time to crisis events. The transition probabilities depend on the aggregate level of credit. Ajello et al. (2016) study the optimal interest-rate policy in a two-period New Keynesian model augmented with a two-state crises shock characterized by endogenous time-varying switching probability. Finally, Gerdrup et al. (2017) develop a SOE Markov-Switching model where the economy can endogenously move from a normal state to a crises state and vice-versa. All of these models are exploited in order to characterize how the monetary policy should optimally react to credit growth (*"leaning against the wind"* policy), abstracting from any kind of consideration related to the role of a macroprudential policy. The only exception is Svensson (2012) who introduces a macroprudential authority with a specific objective in the Woodford (2012) framework. Following this contribution we setup an open-economy model with real effects of the credit growth and study the strategic interactions of monetary and prudential policies, assuming separated authorities' objectives.

This paper is also related to the literature using DSGE models with financial frictions to examine monetary and macroprudential policy interactions, such as Angeloni and Faia (2013), Angelini, Neri and Panetta (2014), Bodenstein, Guerrieri and LaBriola (2014), Collard et al. (2017) and Davis and Presno (2017). These papers adopt different formulations of financial frictions and macroprudential policy instruments in order to characterize different aspects of the strategic interactions between the two authorities. For example, Angeloni and Faia (2013) study the optimal design (in terms of welfare-maximizing rules) of capital ratios and monetary policy in a model with nominal rigidities and bank runs. The authors find that capital ratios should be countercyclical and that monetary policy should respond to leverage and asset prices. Davis and Presno (2017) analyses non-cooperative games between a monetary authority and a macroprudential regulator in a New Keynesian model with financial sector and financial frictions. They find that gains from policy cooperation are relevant when policies are discretionary and when markup shocks hit the economy, while they become negligible for games with commitment or for shocks to intermediaries' net worth or productivity. Collard et al. (2017) study the jointly optimal plans for monetary and macroprudential policies in a model with limited liabilities and deposit insurance. All the contributions mentioned are based on a closed-economy model, abstracting from any international factors. Contrarily, our analysis characterizes the macroprudential-monetary interactions both in a closed economy framework and in a two-country world.

Closer to our analysis is the paper of Carrillo et al. (2017), which studies strategic interactions between monetary and financial authorities in a New Keynesian model augmented with the financial accelerator mechanism à la Bernanke, Gertler and Gilchrist (1999). We share with this paper the approach to characterize the Nash equilibria deriving explicitly the authorities' reaction functions. They find that the Nash equilibrium is significantly inferior to the cooperative one and that it produces a *"tight money-tight credit regime"*, namely the two authorities chose an inefficient high level of responsiveness to the respective targets. We extend the analysis questioning how the strategic interactions between the two authorities and thus the gains from coordination depend on the degree of trade and/or financial integration between countries.

Finally, this paper relates to a large literature studying optimal macroeconomic policies in an international setting. In the past, large effort has been devoted to the analysis of the optimal conduct of monetary policy in open economy DSGE models (Corsetti and Leduc (2011) thoroughly surveys this literature). However, there is an emerging and growing literature that studies the interaction between monetary and macroprudential policies in open economies. For example, Aoki, Benigno and Kiyotaki (2016) and Unsal (2013) build a small open economy model with financial intermediaries in order to study the transmission mechanism of external financial shocks on the macroeconomy. In this framework, they explore the welfare effect of different combinations between monetary and macroprudential policies, finding significant welfare gains following the introduction of a macroprudential policy[4]. Davis and Presno (2017) analyze the link between capital controls and monetary policy autonomy in a small open economy with floating currency. They find that capital controls allow optimal monetary policy to be focused less on the foreign interest rate and more on domestic variables. Our contribution to this literature is threefold. First of all, we explicitly characterize the strategic interactions between monetary and macroprudential policies in an open economy setting, quantifying the gains from cooperation. Secondly, we analyze how the degree of trade and financial openness modifies the optimal conduct of monetary and macroprudential policies. Finally, we extend the analysis to the case in which two symmetric countries interact, in order to evaluate the role of the feedback effects related to the conduct of different policies.

2.3 A New Keynesian model with international linkages

This section presents our flexible monetary open-economy model with trade and financial flows across two symmetric countries. The real and monetary side of the model replicates the main features of Benigno (2004), Pappa (2004), Gali and Monacelli (2005), Lubik and Schorfheide (2006), and Groll and Monacelli (2016), among various NK open economy models. One of the main distinctive elements of this class of models concerns the definition of terms of trade fluctuations as the main transmission channel of inflation and monetary policy across countries. This structure of commercial linkages is enriched introducing a role for credit growth, in the spirit of Curdia and Woodford (2010, 2016) and Woodford (2012). The model is completed with a macroprudential authority aiming at reduce the costs associated to the excess of credit growth.

A crucial feature of our model structure concerns the high flexibility in representing different international regimes. In order to highlight this point we express all the model parameters affecting the countries' integration as function of the degree of trade or financial openness. Starting from the real side, we define the degree of commercial openness for the domestic and foreign economies as follows:

$$\hat{\alpha}_Y = \alpha_Y(1-n), \qquad \hat{\alpha}_Y^* = \alpha_Y^*$$

where $\alpha_Y(\alpha_Y^*)$ stands for the degree of domestic (foreign) country's home bias, while n is the relative size of the home country. As can be easily noted from the above relations, the lower the degree of home bias (the higher the value of α_Y) and the smaller the country's relative size (the lower the value for n), the larger the degree of commercial openness $\hat{\alpha}_Y$. The opposite is true for the foreign country. In the baseline calibration we assume that $\alpha_Y = \alpha_Y^*$ that implies $\hat{\alpha}_Y = \hat{\alpha}_Y^*$. As a consequence, in the rest of the paper we define $\tilde{\alpha}_Y = \hat{\alpha}_Y = \hat{\alpha}_Y^*$ the degree of

[4]Moreover, Medina and Roldos (2014), in a similar framework, show that the introduction of ad hoc macroprudential policies produces welfare gains with respect to a *"lean against the wind"* monetary policy.

trade integration between the two countries. This parameter varies in the $[0,1]$ interval, where a value equal to zero means trade autarky, while a value close to one indicates a higher degree of integration. For the financial openness we define similar relations: $\hat{\alpha}_L = \alpha_L(1-n)$ and $\hat{\alpha}^*_L = \alpha^*_L$. Also in this case, we assume that $\alpha_L = \alpha^*_L$ and hence $\hat{\alpha}_L = \hat{\alpha}^*_L$. Finally, we express the degree of financial integration among countries with $\tilde{\alpha}_L$.

All the model variables are expressed as deviation from the steady state values. The demand side is characterized by an IS curve for each country:

$$\begin{aligned} y_t &= \mathbb{E}[y_{t+1}] - \sigma_\rho(\rho - \mathbb{E}[\pi_{t+1}]) + \sigma_s(\tilde{\alpha}_Y)\mathbb{E}[\Delta s_{t+1}] + \sigma_\Omega \mathbb{E}[\Delta\Omega_{t+1}] + \tilde{g}_t \\ y^*_t &= \mathbb{E}[y^*_{t+1}] - \sigma_\rho(\rho - \mathbb{E}[\pi^*_{t+1}]) - \sigma_s(\tilde{\alpha}_Y)\mathbb{E}[\Delta s_{t+1}] + \sigma_\Omega \mathbb{E}[\Delta\Omega^*_{t+1}] + \tilde{g}^*_t \end{aligned} \tag{2.1}$$

The first two terms on the right hand side of equation (2.1) correspond to the closed-economy components of the aggregate demand, where y_t stands for the output gap, ρ_t is the effective interest rate and π_t is the inflation rate. As we are going to explain later, the effective rate does not correspond to the monetary policy rate but accounts also for the effect of the macroprudential policy. The third term stands for the commercial transmission channel through the terms of trade indicated with s_t. The degree of commercial openness controls the impact of terms of trade fluctuations through $\sigma_s \geq 0$[5]. The following properties hold for this parameter: $\frac{\partial \sigma_s}{\partial \tilde{\alpha}_Y} > 0$ and $\sigma_s = 0$ for $\tilde{\alpha}_Y = 0$.

The forth term represents the magnitude of credit frictions, with Ω_t accounting for the distortions produced by credit spread fluctuations. Under the proposed calibration ($\sigma_\Omega > 0$), a higher value of Ω reduces the current level of the output gap. The theoretical justification of this effect can be described as follows: *"a higher value of Ω_t will lower the marginal utility of income associated with a given level of aggregate expenditure, as a consequence of the less efficient composition of expenditure"* (Woodford (2012)). We are going to discuss later the determinants of this credit friction.

Finally, $\tilde{g}_t$ and $\tilde{g}^*_t$ in equation (2.1) stand for the combined (domestic and foreign) demand shocks affected by the degree of trade openness:

$$\begin{aligned} \tilde{g}_t &= g_t + \gamma(\tilde{\alpha}_Y) g^*_t \\ \tilde{g}^*_t &= g^*_t + \gamma(\tilde{\alpha}_Y) g_t \end{aligned} \tag{2.2}$$

where γ satisfies the following conditions: $\frac{\partial \gamma}{\partial \tilde{\alpha}_Y} > 0$ and $\gamma = 0$ for $\tilde{\alpha}_Y = 0$. Then, when the economy is open to international markets ($\gamma > 0$), it is exposed to both domestic and foreign demand shocks. As a consequence, the output in each country is an increasing function of the other-country demand in order to account for the spillovers effect driven by the demand preferences for imported goods.

Following Fahr and Fell (2017), the effective interest rate in the IS curve is affected by

[5] As we are going to discuss later, a rise in the current terms of trade must be interpret as a real domestic depreciation that makes domestically produced goods less expensive relative to the foreign and shifts demand away from the foreign output and toward domestic one.

monetary and macroprudential policies:

$$\rho_t = R_t + \xi F_t \tag{2.3}$$

$$\rho_t^* = R_t^* + \xi F_t^* \tag{2.4}$$

where R_t is the monetary policy rate and F_t stands for the macroprudential policy instrument. This specification assumes that the policy interest rate is augmented by a spread directly affected by the macroprudential policy. There is a large empirical evidence of the short and long-term effects of macroprudential policy on lending rate (for a survey regarding capital-based measure, please refer to BCBS (2016)) supporting our modeling choice.

The supply side of the economy is characterized by a NKPC augmented by open-economy and credit components.

$$\begin{aligned} \pi_t &= \beta \mathbb{E}[\pi_{t+1}] - \kappa_y y_t + \kappa_s(\tilde{\alpha}_Y) s_t + \kappa_\Omega \Omega_t + u_t \\ \pi_t^* &= \beta \mathbb{E}[\pi_{t+1}^*] - \kappa_y y_t^* - \kappa_s(\tilde{\alpha}_Y) s_t + \kappa_\Omega \Omega_t^* + u_t^* \end{aligned} \tag{2.5}$$

The driving factors of inflation are: output gap y_t, terms of trade s_t and the credit spread Ω_t. We describe the role of Ω_t again following the words of Woodford (2012): *"larger credit frictions also reduce the average marginal utility of income, for a given level of real activity, they also increase the real marginal cost and hence the inflationary pressure resulting from a given level of real activity"*. We assume that $\kappa_\Omega \leq \kappa_\pi$ in order to maintain the standard assumption that considers the output gap as the main driver of inflation. The last term u_t stands for a cost push shock. The parameter controlling the sensitivity of inflation to terms of trade fluctuations κ_s, satisfies the following conditions: $\frac{\partial \kappa_s}{\partial \tilde{\alpha}_Y} > 0$ and $\kappa_s = 0$ for $\tilde{\alpha}_Y = 0$.

Fluctuations in the terms of trade are linked to the cross-country relative output gap:

$$s_t = \zeta(\tilde{\alpha}_Y)(y_t - y_t^*) \tag{2.6}$$

Equation (2.6) indicates that a rise in domestic output above foreign output requires, in equilibrium, a depreciation of the domestic terms of trade (s_t must increases). The associated parameter is a decreasing function of the degree of trade integration $\frac{\partial \zeta}{\partial \tilde{\alpha}_Y} < 0$.

The level of aggregate credit in the domestic economy depends on its lagged value, the domestic and foreign output gaps and the macroprudential instruments:

$$\begin{aligned} L_t &= \rho_L L_{t-1} + \phi_y y_t + \varphi_y(\tilde{\alpha}_L) y_t^* + \phi_F F_t + l_t \\ L_t^* &= \rho_L L_{t-1}^* + \phi_y y_t^* + \varphi_y(\tilde{\alpha}_L) y_t + \phi_F F_t^* + l_t^* \end{aligned} \tag{2.7}$$

where $\varphi_y(\tilde{\alpha}_L) = \phi_y[\alpha_L(1-n)]$ depends on the country's relative population size and the degree of domestic openness of the credit markets. As a consequence, the following conditions hold: $\frac{\partial \varphi_y}{\partial \tilde{\alpha}_L} > 0$ and $\varphi_y = 0$ for $\tilde{\alpha}_L = 0$. We extend the specification of Woodford (2012) to an open economy setting, where the dynamics of the foreign countries affect the domestic level of aggregate credit[6]. This equation shows how the macroprudential policy controls the excess of the credit growth (see BCBS (2016) for an empirical validation of the ability of the macroprudential policy to affect the credit growth). The last components of equations (2.7) define a leverage shock.

A specification of the credit frictions completes the model. We assume a direct link with the aggregate credit in the economy:

$$\begin{aligned} \Omega_t &= \delta \mathbb{E}[\Omega_{t+1}] + \omega L_t + \varpi(\tilde{\alpha}_L) L_t^* \\ \Omega_t^* &= \delta \mathbb{E}[\Omega_{t+1}^*] + \omega L_t^* + \varpi(\tilde{\alpha}_L) L_t \end{aligned} \tag{2.8}$$

In the richer and fully micro-founded version of Curdia and Woodford (2016), Ω_t depends directly on the short-run credit spread (ω_t in the paper), which is basically (in the log-linearized version) a linear function of the current volume of privately intermediate credit and various exogenous factors. Therefore, even without modelling explicitly the interest rate spread determinations and representing a simplified relationship between the credit friction and credit growth, our specification is able to reproduce the main features of the original specification. Our approach is also close to Woodford (2012), where Ω_t is defined as a two-state regime-switching process where the probability to move from a low to high state depends on the aggregate level of credit. In order to perform a detailed analysis of the strategic interactions between the policy authorities, we prefer a fully linear specification.[7] Moreover, we assume that the country specific credit spread depends also on the evolution of the aggregate credit in the other country. This assumption implies that when the countries are fully integrated in the financial markets the two credit spreads become fully synchronized. The importance of this feature has been largely documented in Devereux and Sutherland (2011), Dedola and Lombardo (2012) and Dedola, Lombardo and Karadi (2013). The following conditions hold: $\frac{\partial \varpi}{\partial \tilde{\alpha}_L} > 0$ and $\varpi = 0$ for $\tilde{\alpha}_L = 0$.

The model dynamics are characterized by six exogenous shocks $\{g_t, g_t^*, u_t, u_t^*, l_t, l_t^*\}$. For all these shocks we assume a AR(1) specification of the form $x_t = \rho_s x_{t-1} + \sigma \varepsilon_t^x$ where in the baseline model we set the autoregressive coefficients equal to $\rho_s = 0.85$ and a unitary standard deviation.

2.3.1 Monetary and macroprudential policies

We assume that the central bank and the macroprudential authority in each country commit themselves to a linear reaction rule when they set their instruments. The monetary authorities set the nominal interest rate following a simple Taylor rule, accounting for inflation (single

[6]Some recent contributions, such as Ajello et al. (2016) and Gerdrup et al. (2017) specifies a more detailed credit block and test empirically the best specification.

[7]However, in section **??** we propose a non-linear extension of the model based on a regime switching approach, that relies on the methodology of Gerdrup et al. (2017).

mandate) and eventually for the output gap (dual mandate):

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi \pi_t + \psi_y y_t] + \varepsilon_t^M P \quad (2.9)$$
$$R_t^* = \rho_R R_{t-1}^* + (1-\rho_R)[\psi_\pi \pi_t^* + \psi_y y_t^*] + \varepsilon_t^{MP,*}$$

where $\rho_R \in (0,1)$ stands for the degree of policy inertia, while $\psi_\pi(\psi_y)$ stands for the elasticity with respect to inflation (output gap). The last term$\varepsilon_t^M P$ is a monetary disturbance. The macroprudential policy follows a similar rule:

$$F_t = \rho_F F_{t-1} + (1-\rho_F)\psi_L L_t + \varepsilon_t^F P \quad (2.10)$$
$$F_t^* = \rho_F F_{t-1}^* + (1-\rho_F)\psi_L L_t^* + \varepsilon_t^{FP,*}$$

Parameters in equations (2.10) have a similar interpretation of those described in the monetary policy rule. Since the seminal contribution of Taylor (1993) the use of a feedback rule describing the behavior of monetary policy rules has become a standard practice in academic and policy contributions. Moreover, a larger consensus for the use of the same approach in describing the macroprudential policy is emerging (see, for examples, Angelini, Neri and Panetta (2014), Davis and Presno (2017) and Carrillo et al. (2017) among various relevant contributions).

2.3.2 Calibration

The calibration of the model parameters follows the main theoretical and empirical contributions. Table 2.1, in the next page, summarizes the calibration results. When a parameter is affected by the degree of the financial and commercial openness, we display the autarky (lower bound) and the full integration (upper bound) values. The forth column of the table indicates the source used in the calibration process. Model dynamics coherent with this calibration are described in appendix B.1.[8]

2.4 Strategic interactions within the domestic economy

In this section we study the strategic interactions between the monetary and macroprudential authorities in the domestic economy. In doing so, we first characterize the concept of *"Within-Country Nash and Cooperative Equilibria"* and then perform several exercises to quantify the gains from cooperation and evaluate the authorities' behaviors in different policy regimes.

2.4.1 Payoffs, reaction functions and equilibria definition

Each policy authority's payoff is defined in terms of a loss function depending on the variance of her own policy instrument and targets. This is a common approach in the literature studying optimal macroeconomic policies and the interaction between different authorities (see for example Carrillo et al. (2017), Angelini, Neri and Panetta (2014) and Quint and Rabanal (2014)).

[8]In the model dynamics section we will show also that a monetary shocks in the closed economy benchmark produces comparable results with those of the fully microfounded model of Curdia and Woodford (2016))

Table 2.1: Values for the calibrated parameters

Parameter	Mnemonic	Value[1]	Source[2]
IS curve sensitivity to lending rate	σ_ρ	1	Standard
IS curve sensitivity to terms of trade	σ_s	[0,0.5]	Pappa (2004), Groll and Monacelli (2016)
IS curve sensitivity to credit spread	σ_Ω	1.25	Curdia and Woodford (2016)
Country's exposure to foreign demand shocks	γ	[0,0.5]	-
Sensitivity of the lending rate to financial policies	ξ	0.045	OMRTF (2017)
Discount factor	β	0.998	Standard
NKPC sensitivity to output gap	κ_y	0.0234	Standard
NKPC sensitivity to terms of trade	κ_s	[0,0.007]	Pappa (2004), Groll and Monacelli (2016)
NKPC sensitivity to credit spread	κ_Ω	0.0117	Curdia and Woodford (2016)
Terms of trade sensitivity to output gap	ζ	[1,0.5]	Pappa (2004), Groll and Monacelli (2016)
Aggregate credit persistence	ζ	0.95	Ajello et al. (2016)
Aggregate credit sensitivity to domestic output gap	ϕ_Y	1.14	Ajello et al. (2016)
Aggregate credit sensitivity to foreign output gap	φ_Y	[0,1.14]	-
Aggregate credit sensitivity to financial policy	ϕ_F	1.25	OMRTF (2017)
Credit spread sensitivity to future credit spread	δ	0.65	Curdia and Woodford (2016)
Credit spread sensitivity to domestic credit	ω	0.8	Curdia and Woodford (2016)
Credit spread sensitivity to domestic credit	ϖ	[0,0.8]	-

[1] *For the parameters affected by the degree of trade or financial integration the column shows a grid of values. The lower bound corresponds to the situation of full autarky ($\alpha_Y = 0$ or $\alpha_L = 0$), while the upper bound to the situation of complete integration ($\alpha_Y = 1$ or $\alpha_L = 1$).*

[2] *All the parameters' values are based on authors' calculations, starting from the values assigned in the cited papers.*

Moreover, this type of loss functions is also coherent with quantitative studies of monetary policy (refers to Taylor and Williams (2011) for more details).

In the baseline formulation, the loss function for the Central Bank is defined as:

$$L^{MP} = -[\text{Var}(\pi_t) + \omega_y \text{Var}(y_t) + \omega_R \text{Var}(R_t)] \tag{2.11}$$

where Var(x) stands for the unconditional volatility of the variable x. Including the variance of inflation in the loss function is justified by a general consensus on inflation targeting. Moreover, Angelini, Neri and Panetta (2014) argue that $\omega_y = 0.5$ and $\omega_R = 0.1$ are values coherent with the estimated results of the Taylor rule. In the next sections we are going to stress the role of ω_y in the strategic interactions among authorities. The loss function for the macroprudential authority is:

$$L^{FP} = -[\text{Var}(\Omega_t) + \omega_F \text{Var}(F_t)] \tag{2.12}$$

The above specification takes into account that in the model the main cost associated to aggregate credit fluctuations is summarized by Ω_t (see Svensson (2012) for the technical explanation). Both policy rules also account for the volatility of the respective instrument in order to avoid unreasonable equilibria where authorities reach the full stabilization leaving the instruments to fluctuate excessively. Setting $\omega_F = \omega_R = 0.1$ is enough to achieve this objective.

In order to study a non-cooperative equilibrium, we derive a reaction function for each policy authority. These functions define the optimal choice of an authority's policy rule elasticity for a given value of the other authority's policy rule elasticity.[9] The two reaction functions are defined on a grid of admissible values for each of the two rules' elasticity, given by $\boldsymbol{\Psi}_L = \{\psi_L^1, \psi_L^2, ..., \psi_L^N\}$ and $\boldsymbol{\Psi}_\pi = \{\psi_\pi^1, \psi_\pi^2, ..., \psi_\pi^N\}$. Moreover, the authorities' optimal decisions are subject to the model's system of equations $\boldsymbol{\Gamma}(\boldsymbol{X}_t; \boldsymbol{\theta}, \tilde{\boldsymbol{\alpha}})$ and thus depend on the model parameters $\boldsymbol{\theta}$. We make explicit the role of the parameters controlling the degree of trade and financial integration $\tilde{\boldsymbol{\alpha}} = \{\tilde{\alpha}_Y, \tilde{\alpha}_L\}$ because we are interested in understanding how the strategic interactions between the policies are affected by a different degree of international synchronization. In the within-country equilibria the set of model parameters contains also the rules' elasticities of the foreign authorities[10]. Now we are ready to introduce the two policy games.

Definition 2.4.1 (Within-Country Nash Equilibria)**.** *The reaction functions for the monetary and macroprudential authorities are defined as follows:*

$$\psi_\pi^{RF}(\psi_L; \tilde{\boldsymbol{\alpha}}) = \{(\psi_\pi^{RF}, \psi_L) : \psi_\pi^{RF} = \textit{argmax } L^{MP}, \textit{s.t. } \boldsymbol{\Gamma}(\boldsymbol{X}_t; \boldsymbol{\theta}, \tilde{\boldsymbol{\alpha}}), \psi_L \in \boldsymbol{\Psi}_L\}$$
$$\psi_L^{RF}(\psi_\pi; \tilde{\boldsymbol{\alpha}}) = \{(\psi_L^{RF}, \psi_\pi) : \psi_L^{RF} = \textit{argmax } L^{FP}, \textit{s.t. } \boldsymbol{\Gamma}(\boldsymbol{X}_t; \boldsymbol{\theta}, \tilde{\boldsymbol{\alpha}}), \psi_\pi \in \boldsymbol{\Psi}_\pi\}$$

[9] As already mentioned in the previous section, the baseline exercises assume that the monetary authority responses only to inflation (that means assuming $\psi_\pi = 0$) and abstract from the policy inertia $\rho_R = \rho_F = 0$. Two reasons justify these assumptions: (1) reduce the strategy space to two-dimensions in order to simplify the analysis; (2) guarantee the full coherence of the analysis, avoiding calibrated parameters in the optimal policy exercise.

[10] The model equations and parameters characterizing the foreign economy assume no role when the exercise is performed assuming a fully autarkic domestic economy. In this case also the foreign authorities' elasticities do not affect the results.

The intersection of the reaction functions identifies the Nash equilibrium. Therefore, it can be characterized as following:

$$E^N(\tilde{\boldsymbol{\alpha}}) = \left\{ \left(\psi_\pi^N(\tilde{\boldsymbol{\alpha}}), \psi_L^N(\tilde{\boldsymbol{\alpha}}) \right) : \psi_\pi^N = \psi_\pi^{RF}(\psi_L^N), \psi_L^N = \psi_L^{RF}(\psi_\pi^N) \right\}$$

Definition 2.4.2 (Within-Country Cooperative Equilibria)**.** *Given λ, the weight assigned to the monetary authority's payoff, the cooperative equilibrium is:*

$$E^C(\lambda, \tilde{\boldsymbol{\alpha}}) = \left\{ \left(\psi_\pi^C(\lambda, \tilde{\boldsymbol{\alpha}}), \psi_L^C(\lambda, \tilde{\boldsymbol{\alpha}}) \right) : \{\psi_\pi^C, \psi_L^C\} = argmax \, [\lambda L^{MP} + (1-\lambda) L^{FP}], \; s.t \, . \, \boldsymbol{\Gamma}(\boldsymbol{X}_t; \boldsymbol{\theta}, \tilde{\boldsymbol{\alpha}}) \right\}$$

The equilibrium is therefore given by the two rules' elasticities obtained by maximization of a weighted sum of the two authorities' payoffs.

2.4.2 The closed economy

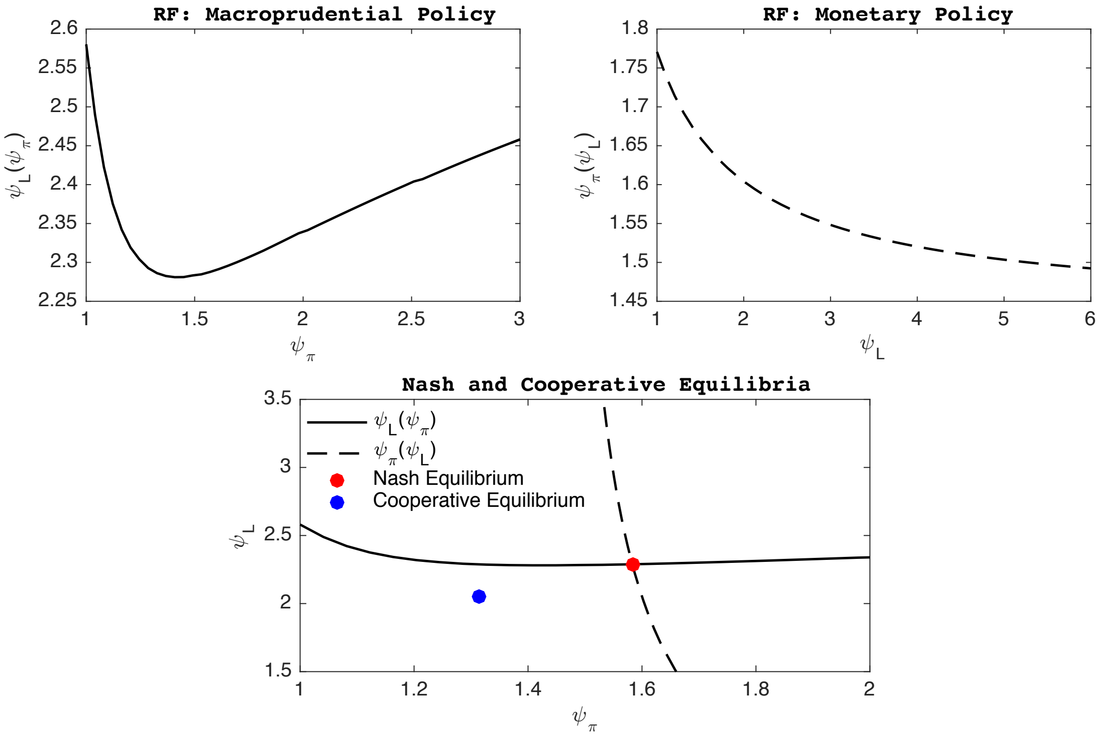
This section evaluates the Nash and cooperative equilibria assuming no trade and financial flows across the two economies, namely $\tilde{\alpha}_Y = \tilde{\alpha}_L = 0$[11]. The upper panels of Figure 2.1 display the reaction functions of the macroprudential authority (left panel) and the monetary policy authority (right panel). Moreover, the bottom panel shows the Nash (the intersection between the two reaction functions) and the cooperative equilibria where an equal weight ($\lambda = 0.5$) is assigned to the two authorities' payoffs.

The macroprudential authority's reaction curve defines its best elasticity choice as a strategic substitute for the choice of the monetary authority for low values of ψ_π but shifts to treat it as a strategic complement for higher values of ψ_π. The monetary policy reaction function, instead, shows strategic substitutability in the entire sample. Hence, as the elasticity of the macroprudential authority rule rises, the best choice of the elasticity of the monetary authority falls. The non-linearity of these reaction curves highlights how the authorities' incentive to strategic behaviors is strong. Indeed, each authority affects, at least indirectly, the objective of the other one. For example, the monetary policy, raising the interest rate affects also the aggregate credit through an output gap contraction, while the macroprudential policy decisions have a direct impact on the lending rate.

Regarding the comparison between the two equilibria, a clear outcome emerges: the Nash equilibrium features a higher inflation elasticity of the Taylor rule with respect to the cooperative equilibrium (1.583 vs 1.315 respectively) and a higher credit growth elasticity in the macroprudential policy rule (2.289 vs 2.054). This result is defined in Carrillo et al. (2017) as *"tight money-tight credit regime"*. When the authorities do not cooperate, each of them fails to internalize the contribution of the other authority in stabilizing its own objective and reacts too strong to business or financial developments. Moreover, the over-reaction of the two authorities produces a sub-optimal outcome with an excess of volatility in the system.

[11] This closed economy regime is characterized by the condition $\alpha_Y = \alpha_Y^* = \alpha_L = \alpha_L^* = 0$, implying that $\sigma_s = \kappa_s = \varphi_y = \gamma = 0$. In this case we can focus only on the domestic economy fully characterized by the system of price and allocations $\{y_t, \pi_t, \rho_t, L_t, \Omega_t, R_t, F_t\}$.

Figure 2.1: Nash vs Cooperative Equilibria



Note: the upper panels display the reaction functions for the macroprudential authority (left panel) and monetary policy (right panel). The lower panel displays the Nash and the Cooperative equilibria. The cooperative equilibrium is based on equal weights $\lambda = 0.5$

Table 2.2 summarizes the results of the analysis. The coordination failure characterizing the Nash equilibrium is clear. The overall loss function, indicated with L^{ToT} is 13.1% higher in the Nash equilibrium (0.780) with respect to the cooperative one (0.649). Moreover, looking at the distribution of the gains from cooperation we see that the macroprudential authority is the one that benefits more (16.4% vs 9.8% of the monetary authority). The latter result shows a clear contribution (*"helping hand"*) of the monetary authority in taming the financial cycle when the two policies cooperate.

We perform a further exercise and study how the weight associated to the output gap in the monetary authority's loss function affects the incentives to cooperation (technically we let ω_y varying between $[0, 0.25]$). The main conclusions highlighted above remain valid: the costs of strategic interactions are significant and the tight money-tight credit regime prevails. However, a set of new comments follow. First, the difference in terms of the overall loss function between the two equilibria is a decreasing function of the weight associated to the output (the gains from cooperation move from 11.7% when $\omega_y = 0$ to 3.9% when $\omega_y = 0.5$). Secondly, a positive weight to the output gap in the monetary policy loss function modifies the distribution of the gains from cooperation in favor of the monetary authority. An indirect alignment in the two authorities' objectives is the reason behind both of these results. Indeed, when the monetary policy assigns a positive weight to the output gap volatility, she indirectly contributes to the credit spread stabilization, given the link between the credit spread and the output gap trough the aggregate credit conditions (see equations 2.7 and 2.8). This indirect contribution of the monetary authority in taming the credit cycle reduces the loss function of the macroprudential policy and reduces the inefficiency of the non-cooperative equilibrium. Clearly the overall loss function increases given the new element in the monetary authority component.

From the above results, we can derive a first set of relevant policy implications. First, the cooperation among authorities is always beneficial, but it assumes a crucial importance when the monetary authority sets the policy rate in order to control inflation fluctuations only. Secondly, the monetary authority has an important role in helping the macroprudential authority in taming the financial cycle. In particular, when the monetary policy loss function assigns a positive weight to output gap fluctuations, the two authorities are forced to an *"indirect"* cooperation, making the Nash equilibrium less inefficient.

Table 2.2: Strategic interactions in the Closed-economy benchmark

	Weights		Loss Functions[1]			Elasticities	
Policy regime	λ	ω_y	L^{MP}	L^{FP}	L^{ToT}	ψ_π	ψ_L
Cooperation	0.5	0	1.131	0.167	**0.649**	1.315	2.054
Nash	0.5	0	1.229	0.330	**0.780**	1.583	2.289
Gains[2]	0.5	0	+9.8%	+16.4%	**+13.1%**	-	-
			The role of the dual mandate of the monetary authority				
Cooperation	0.5	0.25	1.380	0.091	**0.735**	1.105	1.824
Nash	0.5	0.25	1.439	0.170	**0.804**	1.155	2.349
Gains	0.5	0.25	+5.9%	+7.8%	**+6.9%**	-	-
Cooperation	0.5	0.5	1.522	0.064	**0.739**	1.003	1.658
Nash	0.5	0.5	1.570	0.096	**0.833**	1.016	2.545
Gains	0.5	0.5	+4.7%	+3.1%	**+3.9%**	-	-

[1] *The displayed values of the loss functions are computed as follows:* $L^{MP} = [Var(\pi_t) + \omega\, Var(y_t) + \omega_y\, Var(R_t)] * 100$, $L^{FP} = [Var(\Omega_t) + \omega_F\, Var(F_t)] * 100$ *(with* $\omega_R = \omega_F = 0.1$*) and* $L^{ToT} = \lambda L^{MP} + (1-\lambda) L^{FP}$.

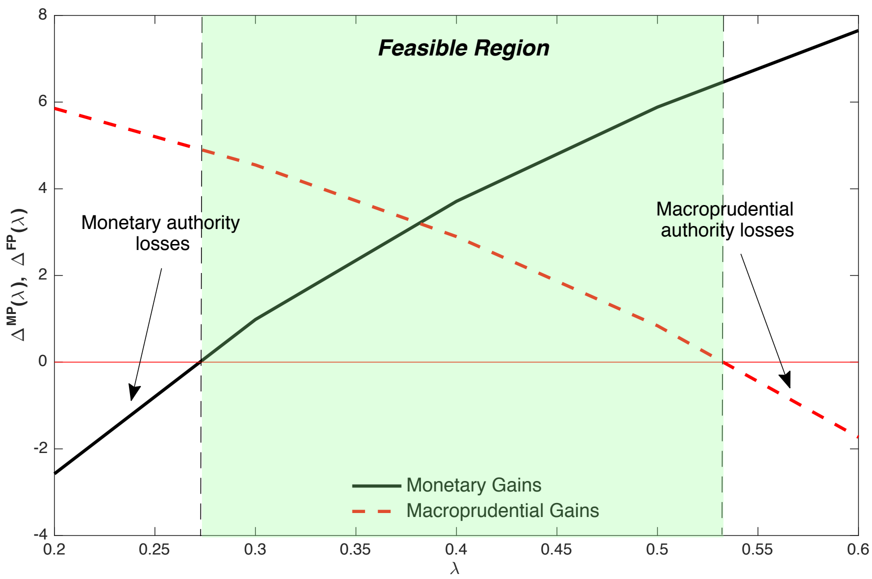
[2] *The gains from cooperation are are computed as follows:* $\Delta^{L,Nash} = [L^{j,Nash} - L^{j,Coop}] * 100$, *for* $j = \{MP, FP, ToT\}$.

Relative weight of the authorities and gains from cooperation

Usually, the assumption underlying a full cooperative equilibrium is that a social planner assumes the role of both monetary and macroprudential authorities. In that framework, the distribution of the gains from cooperation between the authorities does not matter in order to understand whenever the cooperative equilibrium is implementable. However, we rely on the importance of maintain the two authorities separated, because it better reflects the international institutional framework. But in this case, the analysis of the distribution of the cooperation gains between the two authorities becomes crucial. As a consequence, we study how the weight assigned to the monetary authority's component of the overall loss function ($\lambda \in [0,1]$) affects the strategic interactions and the distribution of the gains from cooperation.

Figure 2.2 shows the gains from cooperation of the monetary ($\Delta^{MP}(\lambda) = [L^{MP,Nash}(\lambda) - L^{MP,Coop}(\lambda)] * 100$) and the macroprudential ($\Delta^{FP}(\lambda) = [L^{FP,Nash}(\lambda) - L^{FP,Coop}(\lambda)] * 100$) authorities as a function of the weight λ. We can notice that the monetary (macroprudential) authority is better off in the cooperative regime only in the region $\lambda > 0.275$ ($\lambda < 0.58$). As a consequence, outside the region $[0.275, 0.58]$ the cooperative equilibrium is not implementable, because one of the two authorities would suffer an increase in the corresponding loss function with respect to the Nash equilibrium. Therefore, we call the region $\lambda \in [0.275, 0.58]$ as the feasible region (the green area in Figure 2.2). Finally, we can notice that $\lambda = 0.378$ identifies the optimal relative weight corresponding to the intersection of the two curves.

Figure 2.2: Distribution of the gains from cooperation



Note: The chart displays the gains from cooperation for the monetary (black line) and the macroprudential (red-dotted line) authorities given different values for the weight assigned to the monetary policy component of the total loss function ($\lambda \in [0.2, 0.6]$). The green area corresponds to the region in which both authorities are better off in the cooperative equilibrium and then where the cooperative equilibrium is implementable.

2.4.3 The role of international trade and financial flows

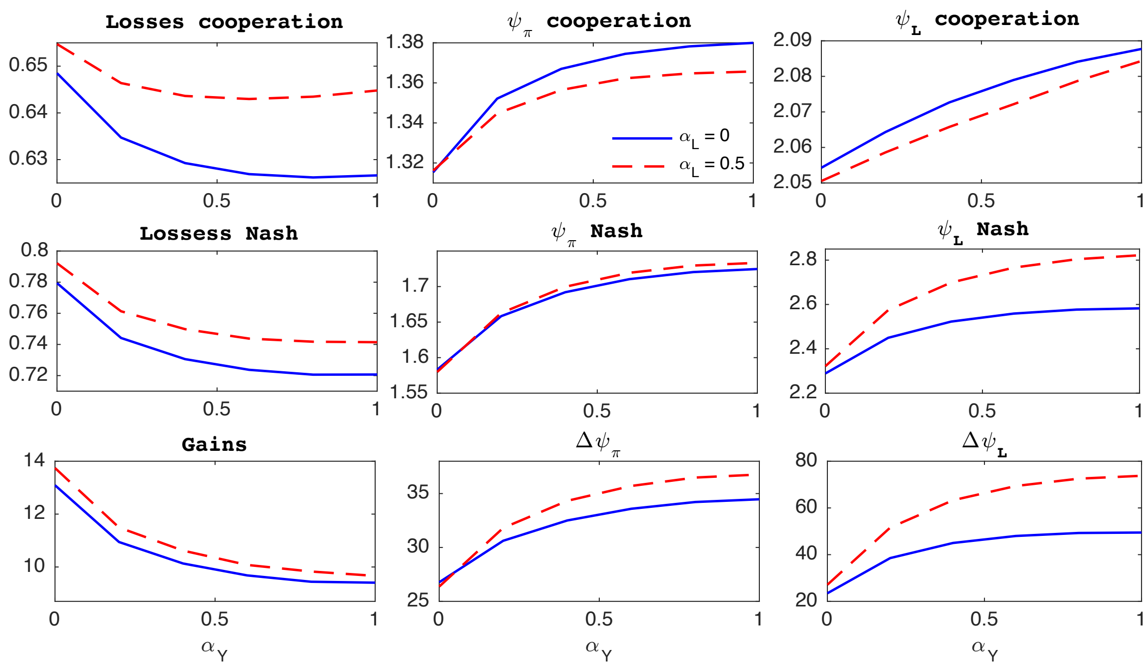
In this section we replicate the above exercise assuming different degrees of financial and commercial integration between the two countries. More technically, we compute and compare a set of *"Within-Country Nash and Cooperative Equilibria"* varying the values of the parameters controlling the degree of commercial and financial integration, $\tilde{\alpha}_Y$ and $\tilde{\alpha}_L$ respectively. These parameters are allowed to vary within the $[0, 1]$ interval, where a value close to zero means very low integration (due to an elevated home bias) while a value equal to 1 indicates a full integration. During this exercise we assume that the foreign authorities operate according to the autarkic cooperative equilibrium within the foreign country.

The analysis shows that the main characteristics highlighted in the previous section are preserved. All the within-country Nash equilibria are characterized by an over-reaction of the two authorities and by significant losses in terms of an excess of volatility of authorities' objectives. However, evaluating in details the role of trade and financial integration allows us to bring out new important insights concerning the strategic interactions between authorities.

The impact of terms of trade fluctuations

Figure 2.3 summarizes how the trade flows affect the strategic interactions between monetary and macroprudential policies. The upper (middle) panels display the overall loss function and the authorities' elasticities in the cooperative (Nash) equilibria as a function of the trade openness degree. The lower panel, instead, studies how the difference between the two equilibria is affected by the parameter $\tilde{\alpha}_Y$. In the charts, the blue curves correspond to the regime in which $\tilde{\alpha}_L = 0$, namely the financial flows are absent, while the red-dotted curves describe a regime in which $\tilde{\alpha}_L = 0.5$.

Figure 2.3: Strategic interactions and trade openness



Note: the upper panels display how the loss functions and the policy rules' elasticities in the cooperative equilibrium are affected by the degree of trade openness (α_Y). The middle panels replicate the exercise for the Nash equilibrium. The lower panels show how the percentage difference between the two equilibria changes with respect to the degree of openness. The percentage difference between the authorities' elasticities is computed as follows: $\Delta\psi_j = [\psi_j^{Nash} - \psi_j^{Coop}] * 100$, for $j \in \{\pi, L\}$.

The following results are worth mentioning. Firstly, the optimal policies' elasticities are an increasing function of the degree of trade openness. Secondly, costs associate to the coordination failure remain significant for the entire set of trade openness degrees, but the size is a decreasing function of α_Y. Moreover, these results are independent on the degree of financial openness. Indeed, the overall loss functions (authorities' elasticities) remain decreasing (increasing) functions of the parameter $\tilde{\alpha}_Y$, when we assume $\tilde{\alpha}_Y = 0.5$ (the red curves in Figure 2.3). Finally, both components of the overall loss functions (displayed in Figure B.6 in appendix B.2.1) are a decreasing function of the degree of trade openness.

These results can be explained as follows. A larger exposure to terms of trade fluctuations forces the two authorities to choose a stronger responsiveness to deviation of domestic variables from the respective targets, given their inability to control directly the terms of trade and/or the foreign variables. In other words, the domestic economy is exposed to another source of volatility coming from abroad and the two authorities react increasing the optimal elasticities in their policy rules. Moreover, the trade openness produces a stronger alignment of the domestic authorities' objectives even if the monetary authority does not assign any weight to the output gap; as a consequence, the gains from cooperation become weaker. The following example can help in understanding the underlying mechanism. When the domestic economy experiences a real depreciation, namely a rise in the terms of trade s_t, both the domestic output gap and inflation responses positively. As a consequence, the co-movement of output gap and inflation increases and then the trade-off inner in the monetary policy optimal decision become weaker. The result is an indirect alignment of the monetary authority's payoff to the one of the macroprudential authority.

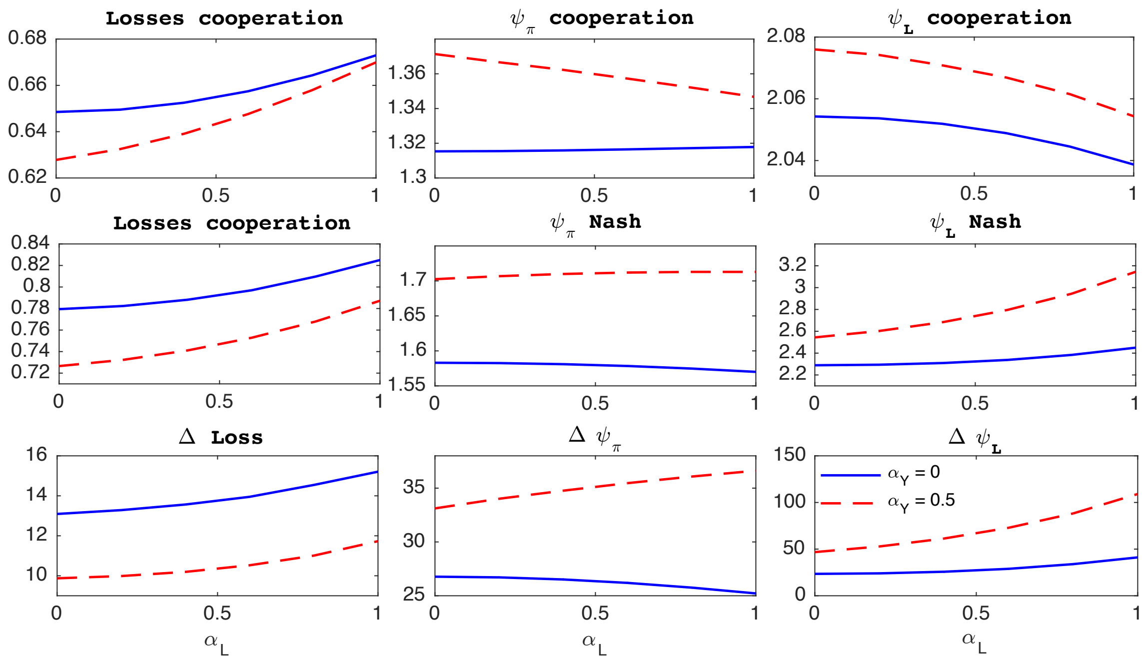
The impact of international financial integration

In this section we study how the degree of financial integration affects the strategic interactions between monetary and macroprudential policies. The quantitative results only partially mirror those of the previous exercise. As for the closed economy and the trade integration, the gains from policy cooperation are significant and both authorities under the Nash equilibrium response more aggressively to the business and financial cycles. However, differently from the trade integration case, the size of these gains responses positively to the increase of the degree of financial openness.

Figure 2.4 summarizes the results. In this case, the blue curves correspond to the regime in which $\tilde{\alpha}_Y = 0$, namely the trade flows are absent, while the red-dotted lines describe a regime in which $\tilde{\alpha}_Y = 0.5$. The following results are worth noticing. First, the monetary policy responsiveness is weakly affected by the degree of financial integration, but largely dependent on the trade integration as the distance between the blue and red-dotted line documents. Second, the results concerning the macroprudential authority's elasticity depend on the policy regime. Indeed, the optimal level of ψ_L switch to be a decreasing function of $\tilde{\alpha}_L$ in the cooperative equilibrium to be an increasing function in the Nash equilibrium. However, the difference between the Nash and the cooperative equilibria elasticities unambiguously increases with $\tilde{\alpha_L}$. Finally, the overall loss functions increases with the degree of financial openness, as well as the gains from cooperation. Figure B.7 in appendix B.2.1 replicates the analysis for the authorities' specific loss functions.

The stronger incentive to cooperate generated by a larger financial openness can be explained by the amplified fluctuation in the domestic credit cycle, due to foreign factors. When the two authorities do not cooperate and hence the monetary authority accounts only for the domestic inflation volatility, the macroprudential authority has to choose stronger response to domestic credit in order to deal with the foreign factors. Contrarily, when the two authorities cooperate the monetary policy implicitly set the optimal policy accounting also for the credit spread fluctuations and hence the macroprudential authority optimally chooses a lower value for ψ_L. The contribution of the monetary policy in taming the domestic credit cycle - when international financial factors matter - becomes more and more important for a larger degree of financial openness. The same explanation is behind the fact that the gains from cooperation are increasing functions of $\tilde{\alpha}_L$ (the lower panel on the left).

Figure 2.4: Strategic interactions and financial openness



Note: the upper panels display how the loss functions and the policy rules' elasticities in the cooperative equilibrium are affected by the degree of trade openness (α_L)*. The middle panels replicate the exercise for the Nash equilibrium. The lower panels show how the percentage difference between the two equilibria changes with respect to the degree of openness. The percentage difference between the authorities' elasticities is computed as follows:* $\Delta\psi_j = [\psi_j^{Nash} - \psi_j^{Coop}] * 100$, for $j \in \{\pi, L\}$.

2.5 Strategic interactions between integrated countries

In this section we extend the analysis to the strategic interactions between authorities located in different countries, introducing the concept of *"Between-Countries Nash and Cooperative Equilibria"*. Endowed with this theoretical background, we investigate the gains associated to an institutional setup in which the two macroprudential authorities (domestic and foreign) fully cooperate on the basis of a global measure of policy efficiency (global loss function) with respect to the case in which they behave accordingly to a country-specific payoff maximization.

2.5.1 Strategic interactions between countries

Before characterizing the *"Between-Countries Nash and Cooperative Equilibria"*, we need to introduce two new concepts. First of all, the foreign authorities' loss functions (the equivalent of (2.11) and (2.12) for the foreign economy) are defined as:

$$L^{*,MP} = -[\mathrm{Var}(\pi_t^*) + \omega_y \mathrm{Var}(y_t^*) + \omega_R \mathrm{Var}(R_t^*)] \tag{2.13}$$

$$L^{*,FP} = -[\mathrm{Var}(\Omega_t^*) + \omega_F \mathrm{Var}(F_t^*)] \tag{2.14}$$

Secondly, we define a *"global loss function"* for each type of authority. This measure of policy efficiency is characterized as a weighted sum of the country-specific loss functions of the considered authority:

$$L^{W,I} = [\Lambda L^I + (1-\Lambda)L^{*,I}] \qquad \text{with} \qquad I = MP, FP \tag{2.15}$$

where Λ is the relative weight assigned by the global planner to the domestic component of the world loss function. Endowed with all these elements, we can define the between-countries equilibria.

Definition 2.5.1 (Between-Countries Nash Equilibrium)**.** *The reaction functions for the foreign and domestic authorities of type i are defined as follows:*

$$\psi_{J(i)}^{RF}\left(\psi_{J(i)}^{*};\Lambda,\tilde{\alpha}\right) = \left\{\left(\psi_{J(i)}^{RF},\psi_{J(i)}^{*}\right) : \psi_{J(i)}^{RF} = \mathit{argmax}\ L^{J(i)},\ \ s.t.\ \mathbf{\Gamma}(\boldsymbol{X}_t;\boldsymbol{\theta},\tilde{\boldsymbol{\alpha}}), \psi_{J(i)}^{*} \in \mathbf{\Psi}_{J(i)}^{*}\right\}$$
$$\psi_{J(i)}^{*,RF}\left(\psi_{J(i)};\Lambda,\tilde{\alpha}\right) = \left\{\left(\psi_{J(i)}^{*RF},\psi_{J(i)}\right) : \psi_{J(i)}^{*,RF} = \mathit{argmax}\ L^{J(i)},\ \ s.t.\ \mathbf{\Gamma}(\boldsymbol{X}_t;\boldsymbol{\theta},\tilde{\boldsymbol{\alpha}}), \psi_{J(i)} \in \mathbf{\Psi}_{J(i)}\right\}$$

where $\mathbf{\Psi}_{J(i)} = \{\psi_{J(i)}^1, \psi_{J(i)}^2, ..., \psi_{J(i)}^N\}$ and $\mathbf{\Psi}_{J(i)}^* = \{\psi_{J(i)}^{*,1}, \psi_{J(i)}^{*,2}, ..., \psi_{J(i)}^{*,M}\}$. Moreover, $J \in \{\pi, L\}$ and $I \in \{MP, FP\}$ i is the index that identifies the type of authority considered ($i = 1$ indicates the monetary authority, while $i = 2$ stands for the macroprudential authority). The intersection of the two reaction curves defines the Nash equilibrium:

$$E^{N,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}}) = \left\{ \begin{array}{c} \left(\psi_{J(i)}^{N,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}}), \psi_{J(i)}^{*,N,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}})\right) : \\ \psi_{J(i)}^{N,I(i)} = \psi_{J(i)}^{RF}(\psi_{J(i)}^{*,N,I(i)}), \psi^{*,N,I(i)} = \psi_{J(i)}^{*,RF}(\psi^{N,I(i)}) \end{array} \right\}$$

Definition 2.5.2 (Between-Countries Cooperative Equilibrium)**.** *Given Λ the weight assigned to domestic component of the world payoff, the cooperative equilibrium is defined as following:*

$$E^{C,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}}) = \left\{ \begin{array}{c} \left(\psi_{J(i)}^{C,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}}), \psi_{J(i)}^{*,C,I(i)}(\Lambda,\tilde{\boldsymbol{\alpha}})\right) : \\ \{\psi_{J(i)}^{C,I(i)}, \psi_{J(i)}^{*,C,I(i)}\} = \ \mathit{argmax}\ [\Lambda L^{I(i)} + (1-\Lambda)L^{*,I(i)}],\ \ s.t.\ \mathbf{\Gamma}(\boldsymbol{X}_t;\boldsymbol{\theta},\tilde{\boldsymbol{\alpha}}) \end{array} \right\}$$

Therefore, the cooperative equilibrium is characterized by a pair of policy rules' elasticities obtained by maximization of the linear combination of the two country-specific payoffs.

During this exercise, we assume that the other authorities behave according to a global cooperation regime (defined below). For example, if we study the interaction among macroprudential authorities, looking for the optimal values of $\{\psi_L, \psi_L^*\}$, we assume $\psi_\pi = \psi_\pi^* = \psi_\pi^{GC}$. In doing so, we isolate the costs of the strategic interactions to the macroprudential policy dimension of the international policy game. The global social planner, accounting for both monetary and macroprudential policies, can be defined as follows:

$$L^G = \Lambda[\lambda L^{MP} + (1-\lambda)L^{FP}] + (1-\Lambda)[\lambda L^{*MP} + (1-\lambda)L^{*FP}] \tag{2.16}$$

where $\lambda(\lambda^*)$ is the relative weigh assigned to the monetary policy component of the domestic (foreign) loss functions.

Definition 2.5.3 (Global Cooperative Equilibrium)**.** *Given L^G the global loss function, the global cooperative equilibrium is defined as following:*

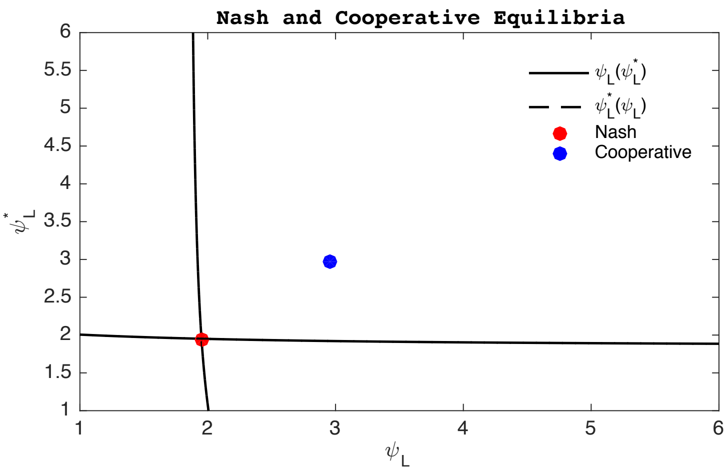
$$E^{GC}(\mathbf{\Lambda}, \tilde{\boldsymbol{\alpha}}) = \left\{ \begin{array}{c} \left(\psi_\pi^{GC}(\mathbf{\Lambda}, \tilde{\boldsymbol{\alpha}}), \psi_L^{GC}(\mathbf{\Lambda}, \tilde{\boldsymbol{\alpha}}), \psi_\pi^{*,GC}(\mathbf{\Lambda}, \tilde{\boldsymbol{\alpha}}), \psi_L^{*,GC}(\mathbf{\Lambda}, \tilde{\boldsymbol{\alpha}})\right) : \\ \{\psi_\pi, \psi_L, \psi_\pi^*, \psi_L^*\} = \ argmax\ L^G, \ \ s.t.\ \mathbf{\Gamma}(\boldsymbol{X}_t; \boldsymbol{\theta}, \tilde{\boldsymbol{\alpha}}) \end{array} \right\}$$

where $\mathbf{\Lambda} = [\lambda, \lambda, \lambda^*]$ stands for the vector of weights defining the preferences of the global planner.

2.5.2 Strategic interactions between macroprudential authorities

Figure 2.5 provides graphical representation of the *"Between-Countries Nash and Cooperative Equilibria"* for the macroprudential authorities. Two main comments follow. First, the reaction functions show strategic substitutability. Second and most important, differently from the case of the *"Within-Country Nash Equilibrium"*, the non-cooperative regime is characterized by an under-reaction of the two authorities.

Figure 2.5: Strategic interactions and trade openness



These results can be explained looking at the international macroprudential spillovers spreading through the international financial flows that are not fully internalized by the authorities in the non-cooperative set-up. When a country-specific macroprudential authority react to the country-specific credit cycle stabilizing the credit spread, positive effects are partially or fully (depending on the degree of financial integration) transmitted to the other country. As a consequence, the macroprudential authority operating in the other country anticipates these effects and chooses an inefficiently low responsiveness to the aggregate credit condition. The resulting equilibrium is characterized by sub-optimal too much weak macroprudential policies and by an excess of volatility in the credit cycles, even if both authorities choose the optimal response from the country-specific measure of efficiency. In other words, both authorities try to maximize the benefits from the contribution of the other authority's policy, but in doing so they set a too weak policy, leaving the country-specific and global credit conditions to fluctuate in an inefficient way. In the cooperative equilibrium, instead, the spillover effects are internalized by the two authorities and they become more efficient in taming the country-specific and international credit cycles.

Table 2.3: Strategic interactions between macroprudential authorities

Integration		**Loss Functions**[1]			**Elasticities**[2]		
α_Y	α_L	**Coop**	**Nash**	**Gains**	ψ_L^{Coop}	ψ_L^{Nash}	ψ_π
			The role of financial integration				
0.5	0.25	0.1855	0.2584	**+7.29%**	2.693	1.912	ψ_π^{GC}
0.5	0.5	0.1814	0.2571	**+7.58%**	2.971	1.958	ψ_π^{GC}
0.5	0.75	0.1875	0.2687	**+8.12%**	3.160	2.084	ψ_π^{GC}
0.5	1	0.1968	0.2880	**+9.12%**	3.337	2.256	ψ_π^{GC}
			The role of trade integration				
0.25	0.5	0.2129	0.2913	**+7.84%**	2.858	1.862	ψ_π^{GC}
0.5	0.5	0.1814	0.2571	**+7.58%**	2.971	1.958	ψ_π^{GC}
0.75	0.5	0.1830	0.2584	**+7.54%**	3.020	2.008	ψ_π^{GC}
1	0.5	0.1756	0.2504	**+7.48%**	3.040	2.030	ψ_π^{GC}

[1] *The loss functions are computed as follows:* $L^j = [\Lambda L^{FP,j} + (1-\Lambda)L^{*,FP,j}] * 100$ *for* $j = \{Coop, Nash\}$. *The gains from cooperation are compute as follows:* $[L^{FP,Nash} - L^{FP,Coop}] * 100$. *The country-specific weight is set at* $\Lambda = 0.5$ *given the assumption of symmetric countries.*

[2] *The monetary policy elasticities based on the global cooperation are around 1.32.*

Table 2.3 summarizes the results. First of all, the differences between the cooperative and non-cooperative equilibria are significant in terms of both the loss functions and the rules' elasticities. For example, in case of moderate integration between countries $\tilde{\alpha}_Y = \tilde{\alpha}_L = 0.5$, significant gains from cooperation (7.58%) and an even stronger difference in the responsiveness of the two authorities (the elasticity is equal to 2.971 in the cooperative regime and 1.958 in the Nash equilibrium) are displayed. Exploring the role of the two international channels, two main results are worth noticing.

First of all, the gains from cooperation are an increasing function also of the degree of financial openness. When the two countries become more integrated, the macroprudential spillovers and the associated externalities acquire a more prominent role, widening the gains from cooperation. Secondly, gains from cooperation decline when the degree of trade openness increases. This result points out the crucial role of the terms of trade in disciplining the authorities' non-cooperative interactions, when the two economies are financial integrated. When countries are highly exposed, to the fluctuations in international relative price fluctuation the two authorities are pushed to choose policies closer to the cooperative regime, due to a reduced level of synchronization in the credit conditions (this effect come from equation (2.6). This result mirrors the one of the within-country analysis. However, the gains from cooperation remain large.

2.6 Conclusion

This chapter studies the strategic interactions between monetary and macroprudential authorities through the lens of an open-economy monetary model with trade and financial flows across two symmetric countries. The demand costs associated to an excess in aggregate credit growth justifies the introduction of a macroprudential policy targeting the credit spread fluctuations. In this setup, the interest in the strategic interactions between the two policies comes from the fact that each authority's objective is affected by the other authority's instrument; as a consequence, policy spillovers produce strong incentives to strategic behaviors. Moreover, the presence of trade and financial flows between the two countries imply that the strategic interactions are strongly affected by the degree of countries' integration. Within this theoretical framework, we characterize two different policy games: first, we develop the concept of *"Within-Country Nash and Cooperative Equilibria"* in order to evaluate how trade and financial flows affect the strategic interactions between monetary and macroprudential authorities within the domestic country. Second, we setup a set of *"Between-Countries Nash and Cooperative Equilibria"* for each policy domain and evaluate the gains from cooperation when macroprudential authorities sited in different countries interact.

The autarkic within-country analysis produces three key results. First, the gains from cooperation between the domestic authorities are large. The Nash equilibrium is indeed characterized by an inefficient overreaction of both authorities to fluctuations in the target variables, and higher overall loss function (the sum of the two quadratic loss functions). Second, the macroprudential policy massively benefits from cooperation; with the support of a coordinated monetary policy, the macroprudential authority is extremely able to smooth the fluctuations in the aggregate credit and reduce the subsequent demand costs. Finally, looking at the distributions of the gains from cooperation among authorities, we identify the region in which the shift form a non-cooperative to a cooperative setting is desirable for both authorities on a basis of Pareto improvement criterion.

Then we move to the open economy within-country analysis, computing a set of Nash and cooperative equilibria for increasing degrees of trade/financial integration. The open economy extension confirms the gains from the cooperation remain positive and significant for the entire set of integration levels. However, the size depends significantly on which kind of integration - financial or commercial - is introduced: trade integration reduces the gains from cooperation, while higher globalization of credit markets makes cooperation more valuable. The former result is explained by a higher alignment of the two authorities' objective when the country is more exposed to terms of trade fluctuations. The latter, instead, depends on the increasing value of the monetary policy in taming the credit cycle when it depends also on external factors.

Finally, the between-countries analysis of the interactions among macroprudential authorities yield the following two main results. First, the cooperation is still beneficial from both the country-specific and global perspective. Second, the Nash equilibria, differently from the case of within-country case, are characterized by an under-reaction of the authorities with respect to the cooperative equilibria. This feature can be explained looking at the international policy spillovers spreading across countries that are not fully internalized by the authorities in the non-cooperative setup.

Chapter 3

MacroFin Copula: a Probabilistic Approach for Countercyclical Scenarios Calibration

3.1 Introduction

This chapter presents an innovative methodology for the calibration of countercyclical adverse scenarios suitable for calibration and impact assessment of macroprudential policies aimed at addressing externalities caused by strategic complementarities. The design of simulations for the macroprudential policies calibration is a particularly relevant in this historical moment as, due to the positive economic outlook and increased momentum in the business and financial cycles, many policy institutions are facing the issue of whether countercyclical tools should be activated[1]. Scenario-based simulations relying on top-down stress test models are a very promising approach as, thanks to their granular description of the banking system, they allow distinguishing the pass-through of different policies on the banking system and the real economy.

Our methodology allows building tailored scenarios that are characterized by two main innovative features. First, there is a stable and transparent mapping between the level of cyclical systemic risk with the path of the scenario's *target variables*, which determine the overall scenario's severity. The link with the cyclical risk assessment guarantees a procyclical severity, namely the severity is stronger when the cyclical systemic risk is higher, and allows us to reconcile the scenario design with the ultimate goal of the macroprudential policy objective of the calibration. Indeed, the amplification of the financial cycle generated by the strategic complementarities is linked to the build-up of cyclical systemic risk. Second, we propose a coherent calibration of the scenario's *complementary variables* based on a multivariate copula model (see Nelsen (2006) and Joe (2014) for an introduction of copulas). As far as we know, this is the first attempt to apply a multivariate copula model to a mixture of macroeconomic and financial data

[1] For example, up to date, several countries have activated the CCyB (Countercyclical Capital Buffer): Bulgaria (0.5%), Czech Republic (1.5%), Denmark (1.0%), France (0.25%), Hong Kong (1.875%), Iceland (1.75%), Ireland (1.0%), Lithuania (1.0 %), Norway (2.0%), Slovakia (1.5%), Sweden (2.5%) and United Kingdom (1.0%).

(MacroFin Copula) and, with this aim, the original methodology has been adapted. The multivariate copulas is commonly used in the banking sector and in institutions for the calibration of financial scenarios, as it is particularly well suited to deal with asymmetric distributions and capture correlations in the tails.[2] However, there are no examples of application of the copula to macro scenarios due to the restricted number of observations available and the difficulty then of relying on their multivariate empirical distribution. We believe, that the ability of the copula to properly estimate the co-movement of the variables conditional to the materialization of a tail event can significantly improve the economic coherence of the scenarios.

Conceptually, the proposed methodology is close to the Growth-at-Risk (GaR) approach (see Adrian, Boyarchenko and Giannone (2016) and Adrian et al. (2018) for references), but it differs over two relevant dimensions: first, it does not focus on the real GDP (which does not necessarily represent the most relevant variable in a stress test scenario) but allows the calibration of the path of the full set of scenarios' variables; second, the scenario calibration is based on the concept of Expected Shortfall rather than Value at Risk, as it better captures tail risks.[3] Therefore, these characteristics make the MacroFin Copula more suitable for the calibration of a scenario characterized by rich interactions between macro and financial variables. These scenarios could be fed into different types of models for macroprudential policy calibration (e.g. DSGE, VAR, GVAR, panel regressions etc.), but are particularly useful for top-down stress test models, which represent a new avenue for the calibration and the impact assessment of macroprudential policies.[4] In this framework, the scenario plays a key role in determining the results, thus the strong economic coherence granted by our methodology clearly plays a significant role.

Although this aspect is not widely discussed in the literature yet, the scenario calibration strategy for top-down or bottom-up stress tests should be thought in a way to match the final goal of these simulations. Calibration exercises are usually focused on specific instruments (e.g. a specific capital buffer such as the CCyB) with the risk of a simultaneous activation of multiple instruments with overlapping objectives. This depends also on the fact that a plethora of macroprudential instruments have been created by regulators in the aftermath of the financial crisis and, several of them have partially overlapping objectives.[5] The complexity of the frame-

[2]ESRB (2015) briefly describes the copula model used by the European Central Bank (ECB) for the European Insurance and Occupational Pensions Authority (EIOPA) insurance stress test and for the calibration of the financial side of the adverse macro-financial scenario for the 2018 EU-wide banking sector stress test.

[3]Indeed, the Value at Risk is the value of a random variable at a certain percentile of the distribution, while the Expected Shortfall is the mean value of the variable beyond the same percentile.

[4]Anderson et al. (2018) discuss the use of stress testing for macroprudential policy calibration. Moreover, an increasing number of institutions is using either bottom-up or top-down stress test results to inform macroprudential policy decisions (BoE (2016), BoE (2017), NBB (2016), BNM (2016), LB (2017), CNB (2017)). Finally, some other institutions and authors recognize the role that could be played by top-down stress test models in providing inputs for macroprudential policy decisions although do not yet implement them (Klacso (2014), Daniëls et al. (2017), Peréz Montes and Artigas (2013), Hristev (2018)) and some of them have published concrete proposals (Bennani et al. (2017)).

[5]The Basel Committee on Banking Supervision (BCBS) revised the Basel II framework starting from 2009 (BCBS (2009a), BCBS (2009b), BCBS (2009c), BCBS (2009d), BCBS (2009e)), and introduced the Basel III in 2011, which has being further revised till nowadays (BCBS (2011), BCBS (2014), BCBS (2016), BCBS (2017a), BCBS (2017b)). In European countries, the Capital Requirements Regulation (CRR) (Regulation (EU) No 575/2013) and the Capital Requirements Directives IV (CRD IV) (Directive 2013/36/EU) of the European Parliament were introduced in 2013 in order to regulate the implementation of macroprudential policies and further revisions of these regulations are currently ongoing.

work is also increased by the large number of institutions with a micro- and a macro-prudential policy mandate.[6] Despite this regulatory and institutional complexity, macroprudential policy instruments should aim at correcting only three groups of externalities and the scenario-based simulations should then be focused on the associated risks: externalities caused by *strategic complementarities*, *fire sales* or *interconnectedness* (see De Nicolo, Favara and Ratnovski (2014)).

In this paper we define a scenarios' calibration strategy that aims at re-conciliating the simulation design with the ultimate objective of the macroprudential policy, namely addressing the connected externality. More in details, the methodology focuses on macroprudential policies related to externalities caused by strategic complementarities. The stable and transparent mapping between the cyclical systemic risk and the scenario's projections guarantees an strong link between the macroprudential policy and the externality. Indeed, the externalities caused by strategic complementarities lead an amplification of the business and financial cycles and thus are specifically connected with the build-up of cyclical systemic risk (ESRB (2014), Borio (2009)). These scenarios are though to assess the bank capital need in order to face the materialization of the cyclical risks, while we leave to the policy and institutional debate the discussion on how to use the scenarios for the evaluation and comparison of different macroprudential policies aiming at correcting these externalities.

Our strategy can be described as following. First of all, the scenario's variables are divided into two different groups: the *target variables*, those have a crucial role in terms of narrative and severity of the scenario, and *complementary variables*. Coherently with the best practices, we define the real GDP and the unemployment rate as target variables.[7] Then, the calibration of the variables' projections during the downturn follows a two-step approach. In the first step, we define the severity of the overall scenario by mapping the level of cyclical systemic risk into a tail percentile of the joint distribution of the target and complementary variables. This mapping follows the rule that a lower percentile is associated with an higher level of cyclical risk. The size of these percentiles reflects the frequency of historical crises. Then, in the second step, the projections of the complementary variables are computed exploiting the MacroFin Copula. In order to deal with the risk of model misspecification coming from the scarcity of macroeconomic data, we exploit two different specifications for the copula: a Gaussian parametric copula and a non-parametric variation that relies on a kernel smoothing function. Although the imposed Gaussian distribution alleviates the data scarcity issue, its symmetry goes against the empirical evidence of a negative skewness characterizing the financial cycle fluctuations. The non-parametric approach, instead, is particularly suitable for adapting the copula to macroeconomic data not only because overcome the data scarcity issue, but also because has enough flexible to be able to replicate the complexity of data around the unfolding

[6]The institutional setup in SSM countries is even more complex than in other jurisdictions as the organisation of macroprudential policymaking is rather decentralized reflecting still the incomplete integration of the European financial system and the heterogeneity of the credit cycles. Usually, more than one national authority has a macroprudential mandate (so called National Competent Authorities, NCAs), but shares some competencies also with supranational authorities.

[7]For example, the Bank of Japan define the scenario's severity in terms of GDP (BoJ (2015)), the FED target the unemployment rate (Fed, 12 CFR 252), and finally, the IMF has started to using the GDP projections based on the GaR approach as main target.

of a systemic crises. Finally, the projections of the complementary variables over the scenario horizon are derived as the conditional expected shortfall (CoES) of both types of copulas. The concept of CoES allows us to properly replicate the co-movement of the key macro-financial variables observed historically during the systemic crises, granting a fully economic coherence of the scenario. In the last section we show how our methodology is able to calibrate adverse scenarios for Euro Area (EA) countries that fully replicate the global financial crises dynamics in terms of severity and comovement between the main macroeconomic and financial variables.

Even if the methodology is tailored for scenario-based calibration and impact assessment of macroprudential policies, the potential application of the MacroFin Copula is wider. Indeed, our approach is suitable for designing generic adverse macro-financial scenarios with stress testing purpose. Although in the last years stress test exercises have become a widely used tool for resilience assessment of the banking system, a clear methodology for scenarios' calibration is not available yet. This practice has generated a lack of transparency in the interpretation of the scenario's projections and often the economic coherence of the scenarios has been questioned. This paper aims at overcoming these challenges, proposing a transparent methodology strictly connected with the risk assessment process and characterized by a strong economic coherence in the variables' projections.

The chapter proceeds as follows: section 3.2 summarizes the most common methodologies for the scenario design; section 3.3 illustrates our methodology, while a practical application of the methodology for EA cyclical scenarios are contained in section 3.4; section 3.5 concludes.

3.2 Scenarios' calibration methodologies

3.2.1 Review of current approaches

Before describing the methodology proposed in this paper, it is relevant to have an overview of the current state of the art in scenarios design employed by central banks and institutions.[8] In this section, we provide a detailed review of the methodologies and the country experiences in terms of scenarios calibration for stress testing and macroprudential policies calibration based on the publicly available literature.

Simulations of adverse scenarios are typically used for the calibration of the capital needs of banks, i.e. for top-down or bottom-up stress test exercises. The scenario calibration can be performed for this purpose in three main ways:

1. *Historical scenarios*: the path of the variables replicates crisis which have been observed.

2. *Synthetic scenarios*: the path of the variables describes hypothetical conditions that have not been observed and that can be tailored to a specific situation of interest. The calibration of these scenarios is usually based on models estimated on historical data.

[8] In this review we intentionally don't summarize the current methodologies used by private banks for designing stress scenarios for two reasons. First scenarios designed at banks level are usually tailored on the specific vulnerabilities of each bank. Second,there is limited publicly available information on the scenario calibration.

3. *Reverse scenarios*: the path of the variables are calibrated targeting some particular output of the simulations (e.g. credit losses on particular exposures).

4. *Scenarios based on the forecast density*: the path of the scenario variables is derived from the tail of the forecast density distribution.

The literature on the methodologies to calibrate adverse scenarios is very limited. Most of the institutions that have public documents on the design of scenarios adopted a mixture of historical and synthetic scenarios. The calibration of adverse scenarios present difficulties. First of all, scenarios should be severe but plausible, meaning that they should have a strong economic coherence in terms of the magnitude of variables' responses but conditioned to a sufficient severity. Moreover, for many countries data quality and scarcity issues are significant. Indeed, very often the available time series are short and capture, at maximum, a couple of recessions. As a consequence, extracting sufficient information from historical data in order to get the economic coherence to the scenario is a challenging task. An additional difficulty within the European Union is represented by the strong interlinkages across countries, which makes it particularly difficult to create adverse scenarios at country level that are coherent for the entire area. Our methodology aims at overcoming these challenges. Indeed, the MacroFin copula, in particular the non-parametric version based on a kernel distribution, seeks to overcome these data quality and quantity issues. In doing so, the copula-approach allows estimating the projections of the scenario's variables granting a strong economic coherence in terms of sign and magnitude.

In the context of the EU-wide banking sector stress test, the ECB, in collaboration with the ESRB, develops the narrative and methodology and calibrates the adverse macro-financial scenario for each exercise (see ESRB (2014), ESRB (2016) and ESRB (2018a)). The scenario includes variables such as GDP, inflation, unemployment, asset prices and interest rates and covers a three years horizon. The narrative of the adverse scenario reflects the four systemic risks identified by the ESRB General Board as representing the most material threats to the stability of the EU financial sector and then the main risks are mapped into macro-financial shocks. However, how the risk assessment outcomes is traduced into shocks's severity is largely based on the expert judgement and a clear explanation is missing. For a review of the models used for the scenario calibration see Henry (2015). We depart from this approach defining a clear and stable link that maps the cyclical systemic risk into the overall scenario's severity.

The Federal Reserve (Fed) conducts annually two supervisory stress test exercises: the Dodd-Frank Act Stress Test (DFAST) and the Comprehensive Capital Analysis and Review (CCAR).[9] Both exercises are based on two adverse scenarios: the *adverse* and the *severely adverse* scenarios. The scenarios are not accompanied by a narrative describing the main

[9]The Dodd-Frank Act Stress Test (DFAST) is a forward-looking component conducted by the Federal Reserve and financial companies supervised by the Federal Reserve to help assess whether institutions have sufficient capital to absorb losses and support operations during adverse economic conditions. The Comprehensive Capital Analysis and Review (CCAR), instead, is an annual exercise by the Federal Reserve to assess whether the largest bank holding companies operating in the United States have sufficient capital to continue operations throughout times of economic and financial stress and that they have robust, forward-looking capital-planning processes that account for their unique risks.

events triggering the crisis and there is not a clearly specified link with the risk assessment. The calibration of the path for the macroeconomic variable of these scenarios begins with the calibration of the unemployment path for the *severely adverse* and then of the other variables of this scenario. The *severely adverse* scenario should feature an unemployment rate increase between 3 to 5 percentage points from its initial level over the course of 6 to 8 calendar quarters (see Fed, 12 CFR 252). However, if a 3 to 5 percentage point increase in the unemployment rate does not raise the level of the unemployment rate to at least 10 percent[10] the path of the unemployment rate will be set at least at 10 percent. Other variables such as the real GDP and inflation rate are derived using standard economic equations (e.g. Okun's Law, the Phillips Curve, and interest rate feedback rules). The market risk parameters of the *severely adverse* scenario are calibrated mainly using historical paths. In particular, the main variables reflect the developments in the credit markets during the second half of 2008. In some cases, the movement in particular risk factors may be amplified based on theoretical relationships, market observations, or the saliency to company trading books. Our methodology shares with the FED's approach the key role assigned to the unemployment in determining the overall severity of the scenario but it extends this concept in two directions. First, we consider also the real GDP growth as a target variable and the scenario severity is defined in terms of the joint distribution of real GDP and unemployment. Second, the severity is calibrated in a more structured way. Indeed, we define transparent mapping with the risk assessment and exploit a statistical model, the MacroFin copula, in order to extract historical information.

The Fed specifies that the approach for the calibration of the *adverse scenario* may vary. The simplest method to specify the adverse scenario is to develop a less severe version of the severely adverse scenario. For example, the adverse scenario could be formulated such that the deviations of the paths of the variables relative to the baseline were simply one-half of or two-thirds of the deviations of the paths of the variables relative to the baseline in the severely adverse scenario. Another method to specify the adverse scenario is to capture risks in the adverse scenario that the Board believes should be understood better or should be monitored, but does not believe should be included in the severely adverse scenario, perhaps because these risks would render the scenario implausibly severe. Finally, the Board may consider specifying the adverse scenario using the probabilistic approach (that is, with a specified lower probability of occurring than the severely adverse scenario but a greater probability of occurring than the baseline scenario).

The Bank of England (BoE) (see BoE (2015)) runs two types of scenarios for the bottom-up stress test exercise. The first type of scenario, called *Annual Cyclical Scenario (ACS)*, is calibrated in order to reflect the policymakers' assessment of the state of the financial cycle. Indeed, the severity of this scenario increases as risks build up and decrease after those risks materialize. The starting point for the calibration of the annual cyclical scenario is a systematic review of a range of indicators, in order to identify the prevailing imbalances. However, there is no specific explanation on how the mapping between the indicators and the severity is implemented or a

[10]This corresponds to the average level to which the unemployment rate has increased in the most recent three severe recessions

technical description of the methodology for the calibration of the variables' projections. The second type of scenario, called *Biennial Exploratory Scenario (BES)* is designed in order to test the resilience of the banking system to a wider range of risks. The risks involved might therefore be unusual from a historical perspective. Also for this second scenario, the transparency of the methodology is very limited. We borrow extensively from the *ACS* for what concerns the idea to link the position in the credit cycle with the severity of the scenario. The idea underlining the procyclical severity is the same, namely the higher the level of cyclical risk the stronger the potential downturn triggered by the materialization of this risk. However, we depart from this approach in two directions. First, we define a transparent mapping between the cyclical risk assessment and the overall severity of the scenario, based on the tail percentiles of the target variables' distribution. Second, we apply a clear probabilistic approach in order to derive projected paths of the scenario's variables. In doing this, we improve the transparency and economic coherency of the scenario calibration.

The International Monetary Fund (IMF) designs stress test scenarios in the framework of the Financial Sector Assessment Programs (FSAPs), with the objective of conducting top-down stress test for monitoring the stability of the banking sectors in the country under analysis (source). There is a clear link between risk assessment and scenario calibration as the narrative of the scenario is derived from the main financial stability risks as identified in the Global Risk Assessment Matrix (G-RAM). The scenario is then calibrated by means either of a panel dynamic stochastic general equilibrium model disaggregated into forty national economies, as documented in Vitek (2015), or by the so called Flexible System of Global Models (FSGM) which is a multi-region, forward-looking semi-structural global model consisting of 24 regions (see Andrle et al. (2015)). The scenario severity usually is calibrated looking at the historical GDP distribution and is targeted at the 2 standard deviations of real GDP deviation form the trend.

Similarly to the Fed, the Bank of Japan (BoJ) designs two adverse scenarios for the supervisory stress test (see BoJ (2015)). A *tailored event scenario* reflects the current risk assessment and is mainly an historical scenario mimicking previous crises. A *tail event scenario*, instead, is developed to assess the change in the financial stability resilience over time under equal severe economic and financial environment developments (similarly to the *severely adverse scenario* of the Fed). This latter scenario is designed such that Japan's output gap deteriorates to around minus 7 to minus 8 percent, as experienced at the trough of the Lehman shock. Assumption that also implies a countercyclical scenario severity. Other financial and economic variables are then calibrated so that they are generally consistent with an economic downturn of such a magnitude. The main trigger of the scenario is a rise in long-term interest rates in the U.S. (about 200 bps) the implied decline in economic growth across different regions in the world is projected via a VAR model.

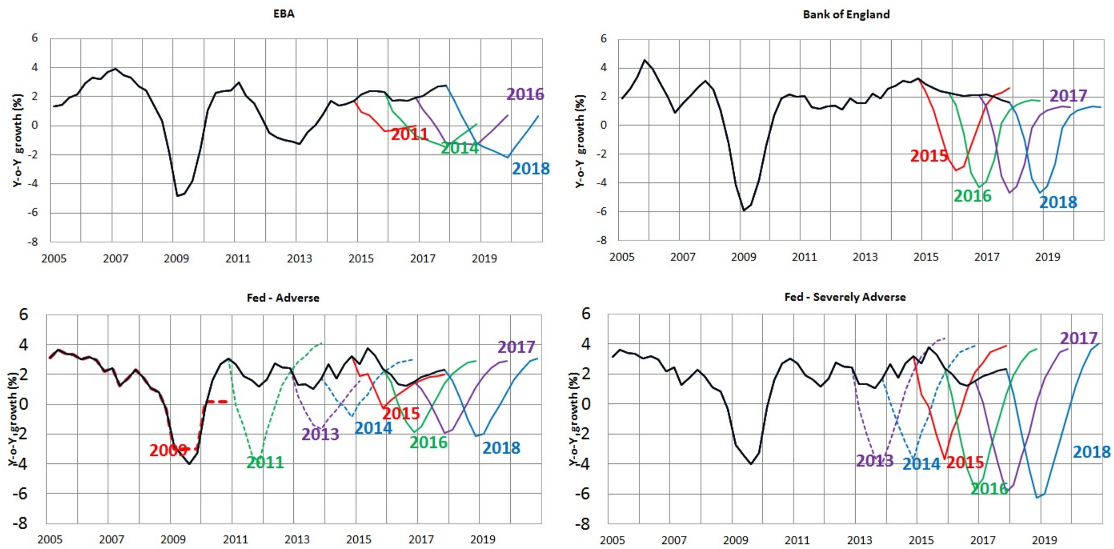
A model-based approach for the scenarios calibration is proposed by Bennani et al. (2017). The scenarios consist in 2 years of baseline plus one year of adverse. The baseline projections are simulated with dynamic macro models (DSGEs or VAR) conditional on the Broad Macroeconomic Projection Exercise (BMPE) and potentially other relevant information or projections available to the macroprudential authority (e.g. growth rates of credit). Standard Kalman smoother techniques are employed in order to recover the sequence of structural shocks which are consistent with these extra-information. The adverse, instead, consists in a linear combination of impulse responses to the projected series after the simulation of a set of shocks with a clear economic interpretation. The coherence with the current situation of risks and vulnerabilities is guarantee by a preliminary systemic risks assessment based on early warning systems (Coudert and Idier (2016)). In order to generate the shocks triggering the adverse event, they rely on two methodologies: either a simple recursive-ordering approach based on a Cholesky decomposition of the covariance matrix of the VAR innovations or a dynamic general equilibrium models (DSGE), in which structural shocks have a more straightforward economic interpretation. An other model-based approach is implemented for the stress-testing framework of the Czech National Bank (CNB, see Geral et al. (2012) for further details). The *Alternative Macroeconomic Scenarios* are designed using the CNB's official g3 prediction DSGE model (Brazdik, Hlavacek and Marsal (2012)). Foreign variables, crucial for the small open Czech economy are imposed exogenously in the model; therefore, their trajectories (3M Euribor, effective euro-area GDP and PPI, the USD/EUR exchange rate and selected commodity prices) are obtained trough the NiGEM DSGE model for the global economy. In a nutshell, these foreign variables trajectories enter in the g3 model, which provides quarterly projections for the main domestic variables. Furthermore, the stress testing variables, not derived with the 3g model (unemployment rate and the yield curve), are estimated with supplementary models (Okun's law and models for the yield curve). Finally the size of the shocks is fine-tuned with a combination of expert judgement and statistical analysis based on historical data. There is a tradeoff between a model-based and a non-formalized approach. The former guarantees a higher level of economic coherence but at the cost of introducing a new layer of model uncertainty and noise in the exercise. Contrary, a non-formalized approach, based on the expert judgement, does not introduce additional uncertainty but lacks of transparency and economic coherence. Our methodology beyond between these two extreme approaches. The probabilistic approach that we propose has an enough level of robustness and flexibility. The scenarios calibrated with the MacroFin copula are transparent and with a clear economic interpretation.

3.2.2 Unconscious developments in supervisory stress test scenario design

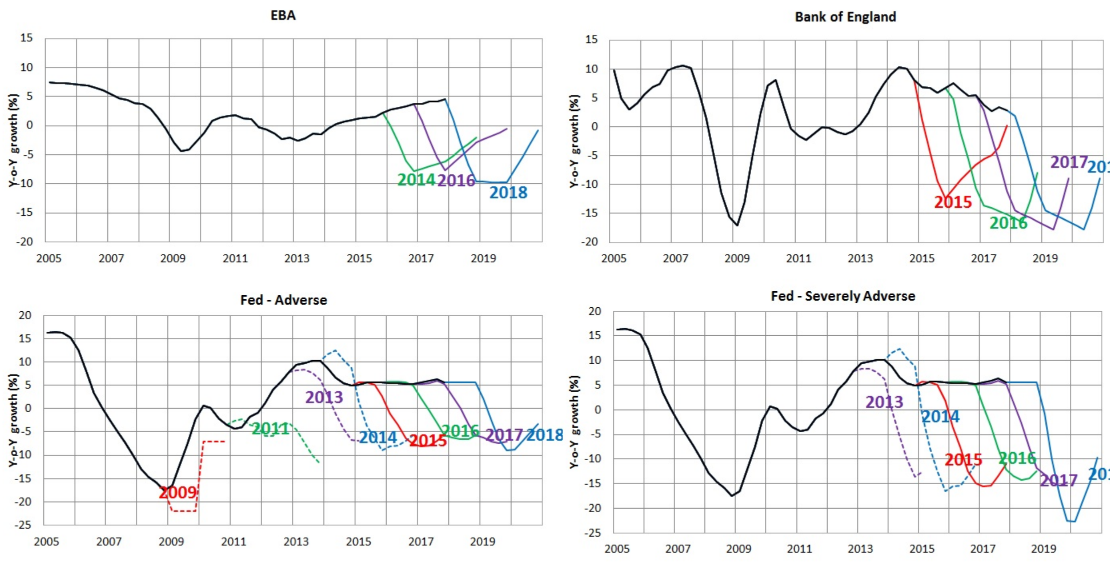
From the review of the scenarios calibration methodologies we could conclude two main facts. First, the definition of scenario' s severity is ambiguous. In terms of variables, some institutions look at the real GDP, while others at the unemployment rate and house prices. In terms of metric, the adverse growth rate or the deviation from the trend are the more diffuse. Second, the calibration of the scenario severity has evolved in a way that the scenario severity has become higher with the improvement of the economic cycle, leading to an unconscious scenario countercyclicality. However, only the BoE has introduced an explicit cyclical target connecting it with the macroprudential purpose of the stress test based on the ACS scenario. Across the institutions, the change toward more countercyclical scenarios has been gradual: BOJ introduced in 2015 the current methodology where one scenario is countercyclical, BoE introduced in

Figure 3.1: Different vintages of supervisory stress test scenarios

(a) Real GDP path



(b) House rices path



2016 an explicit countercyclical target, the Fed has changed in 2017 the definition of its severity targets of the severely adverse scenario (Fed, 12 CFR 252), in the EBA 2018 scenario an explicit condition on the cumulative real GDP growth was introduced in order to grant a sufficient real GDP decline (ESRB (2018a)). To show the increased countercyclicality (i.e. a deeper downturn is calibrated in an upswing of the cycle) of the scenario, in Figure 3.1 we compare the path of GDP and house prices of supervisory stress test scenarios from some institutions across different vintages. This evolution has been lead by the necessity to grant enough scenario severity in an historical period where the economic conditions improved significantly. However, an explicit reflection on the use of these scenarios and the potential overlap with macroprudential objectives should be done.

3.2.3 A comparison with the Growth-at-Risk

To some extent, our method is close to the quantile regression of the Growth-at-Risk (GaR) approach, initially developed by Adrian, Boyarchenko and Giannone (2016) and Adrian et al. (2018). The GaR produces a conditional density forecast of the real GDP approach following a two-step approach. During the first step a quantile regression is run in order to predict the future GDP q-quarters ahead based on current GDP and some univariate financial condition index (FCI). Then, a second steps fits the forecast from previous regression to a skewed-t distribution. Therefore some limitations are implicit in this approach: first, the outcomes depend on the specification FCI; second, a measurement error depending on the statistical approach for the indicator can emerge; third, the correlation between the FCI and the current GDP could arise problems of multicollinearity; finally, the two-step approach introduces another source of measurement error that can make the estimation on the tail of the distribution noisy.

Our approach, similarly to the GaR, gathers in an explicit way a link between the cyclical systemic risk as identified by a composite indicator with the severity of a downturn. However, contrarily to GaR, this mapping does not include an additional statistical/econometric step that adds noise to the results. Moreover, the MacroFin Copula exploits the multivariate dimension of the historical data and the triggering event considers worse scenarios compared to a quantile regression, because we are concerned about all the possible outcomes below a certain quantile. Finally, our approach is employing the joint distribution between the triggering variable and the response variables to create consistent scenarios for all the variables, without measurement errors or endogeneity issues.

3.3 Design Cyclical Scenarios with MacroFin Copula

This section presents our methodology for the design of scenarios belonging to the category of synthetic scenarios (i.e. representing hypothetical events where the relations among the scenario's variables are designed on the basis of historical crises data) and that can be used for the calibration of countercyclical macroprudential policies. In order to estimate the macro-financial variables' reactions and their co-movements during the downturn we employed a macro-financial copula (MacroFin Copula) approach that draws the variables' responses from their joint distribution. In this way, historical information are used in order to derive plausible scenarios that are coherent with historical crises in terms of severity and dynamics. Moreover, our methodology relies on a stable and transparent mapping between the current level of cyclical risk, as identified by the risk assessment process, and the scenarios' severity. In doing so, we build scenarios that fully reflect the current position in the financial cycle.

Figure 3.2: Cyclical Scenario Design: a Graphical Illustration

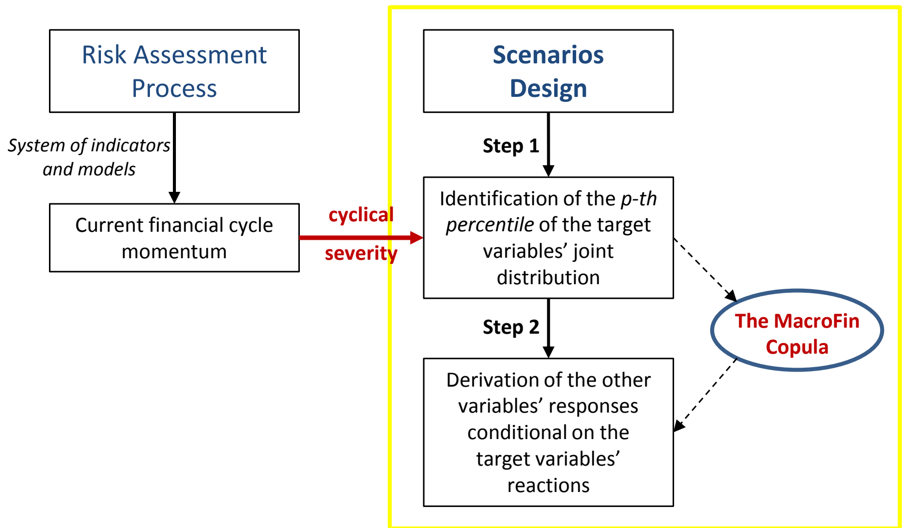


Figure 3.2 summarizes the main steps of our strategy. The starting point of the scenario design is the risk assessment which, through a system of indicators and models, identifies the current level of financial imbalances and risks. In order to incorporate this information into the scenarios, we define a stable mapping between the risk assessment and a key metric of the scenario severity (*step 1*), defined as the percentile on the tail of the joint distribution of the real GDP growth and unemployment rates. We define these latter as *target variables*, given their relevance in terms of the narrative and severity of the scenario. Instead, we define *complementary variables* all the other variables that complete the scenario, such as real estate prices, stock prices and interest rates.[11] In this way, the scenario's severity fully reflects the current financial conditions and developments. After the identification of the overall scenario severity, we derive the growth rates of the complementary variables with the MacroFin Copula, which identifies the reaction of the complementary variables to the realization of tail events of target variables' distribution (*step 2*).

The methodology presents two innovative elements. The first consists in the application for the first time of a multivariate copula to macroeconomic data. A copula is a function that gathers the dependence between the distributions of random variables (for a general description of copulas, see Nelsen (2006) and Joe (2014) among others). Although the copula is usually applied to high-frequency financial data, we propose to adapt the methodology to low-frequency macroeconomic data. The application of copula to the latter type of data is associated to

[11]The target variables do not depends on the type of scenario object of the calibration process, while the complementary variables change in number and types on the the basis of the scenario.

model misspecification due to the scarcity of data. In order to deal with this issue, we propose two approaches. First, we employ a parametric copula that solve the data scarcity imposing a Gaussian type of distribution. However, the symmetry of the Gaussian distribution goes against with the negative skewness characterizing the business and financial cycle. Then we propose a second non-parametric approach that relies on a kernel smoothing function. This second method is particularly suitable for adapting the copula to macroeconomic data not only because overcome the data scarcity issue, but also because has enough flexible to be able to replicate the complexity of data around the unfolding of a systemic crises. For both of the approaches, the projections of the complementary variables over the scenario horizon is derived as the conditional expected shortfall (CoES) of the copula.[12] The application of the concept of the CoES to the scenario's calibration allows us to properly replicate the co-movement of the key macro-financial variables observed historically during the systemic crises.

The second innovative feature of the calibrated scenarios is the procyclical severity, namely the downturn is stronger when the cyclical systemic risk as identified by the risk assessment is higher. This procyclical severity relies on the simple empirical fact of a positive correlation between the financial cycle momenta and the severity of the potential downturn triggered by the materialization of the risk accumulated during the upswing of the cycle. As already mentioned and described in section 3.2, there is a tendency to have procyclical severity in many scenarios employed for supervisory stress testing and macroprudential policy assessment, even if not always in an explicit way. Contrary, our methodology makes the cyclicality explicit and based on a transparent and stable mapping between the level of systemic risk and the severity of the scenario.

3.3.1 Risk assessment process

One of the key elements of our methodology consists in the systemic mapping of the cyclical risk assessment outcomes into the design of the scenario and in particular into the downturn's severity. This stable link is motivated by the objective of reflecting the evolution of the financial cycle, and in particular the build up of the cyclical risks, in the simulation of the hypothetical downturns. Moreover, when the scenarios are applied to the macroprudential policies calibration or impact assessment, the strong link with the cyclical risk assessment implies an indirect connection between the macroprudential policy objective of the exercise and and the externality caused by strategic complementarities. Indeed, the amplification of the financial cycle generated by this externality is well reflected by the cyclical risk build-up.

Monitoring the systemic risks is a challenging process given the several elements that must be considered, such as imbalances at global and national level, the channels of transmission between the different players in the financial system, the propagation and amplification mechanism of individual shocks and the feedbacks between the real economy and the financial system. Although, the definition of a framework for systemic risks goes beyond the objective of this

[12] As we are going to discuss later in section 3.3.4, the CoES of a random variable is the expected shortfall, namely the expected value of this variable beyond a certain percentile, given that the triggering variable is in a distress scenario defined as the value corresponding to a certain tail percentile of its distribution.

paper, a short review of the most common methodologies clarifies the link with the scenario design. Most of the financial stability departments of central banks and other international institutions conduct regularly (quarterly or semi-annually) risk assessment analysis, which is generally published in official documents, see for example the Financial Stability Review of the ECB (ECB (2018)), the Global Financial Stability Report of the IMF (IMF (2018)), the Annual Report of the Fed (FED (2016)), the Financial Stability Report of the Bank of England (BoE (2018)), the Financial Stability Report of the Sveriges Riksbank (Riksbank (2018)), the Financial System Review of the Bank of Canada (BoC (2017)), the Annual Report of the Bank for International Settlements (BIS) (BCBS (2017c)), among many others.

Usually, the risk assessment process starts with the monitoring of a large set of indicators measuring the financial conditions. Then, a synthetic and exhaustive measure of systemic risks is derived trough either the selection of limited core indicators or the derivation of composite indicator. The selection of the core indicators and/or the aggregation process normally is based on forecasting or early warning performances. The methodology relying on the forecasting performance of the indicators, generally produces the so-called *Financial Condition Indices*[13] (FCIs), which have the main aim of evaluating the tightness of the financial conditions and the associated level of risks for a stable economic growth. Usually, this class of indicators takes into account both the cyclical and the structural component of the systemic risks. A prominent example is the *Global Financial Conditions Index* provided by the IMF for 43 advanced and emerging market economies[14] (see the chapter 3 in IMF (2017) for a description of the methodology). The *Financial Stress Index* of the Federal Reserve Bank of Kansas City (see Craig and Keeton (2009) for reference) and the *Index of Financial Stress* of the Bank of Canada (Illing and Liu (2003)) are other relevant examples.

The second main methodology to create a summary measure of systemic risk is based on *Early Warning Systems* (EWSs) and is more focused on the time-varying, hence cyclical, component of the systemic risk. The EWSs want to detect the risk of future crises on an empirical basis by considering the evolution of the fundamental (real and financial) variables in the economy. In few words, these systems aim at identifying a set of indicators and a series of thresholds that are able to signal before the crises the rise of the financial imbalances and sub-sequential vulnerabilities[15]. An example of the use of EWSs for regular risk assessment can be found

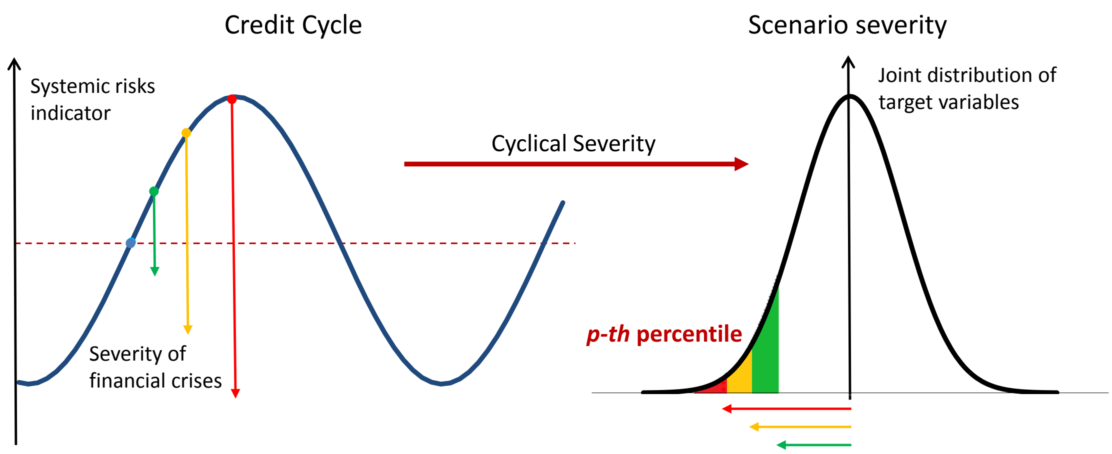
[13]The methodologies for constructing FCIs tend to fall into two broad categories: a weighted sum approach and a principal component approach In the weighted-sum approach, the weights on each financial variable are generally assigned based on estimates of the relative impacts of changes in the variables on real GDP. These estimates or weights have been generated in a variety of ways, including simulations with large-scale macroeconomic models, vector autoregression (VAR) models, or reduced-form demand equations.

[14]This index is estimated exploiting the information of 10 indicators trough a factor augmented vector autoregression with time-varying coefficients and stochastic volatility. The underlining methodology was developed by Koop and Korobilis (2014)

[15]Starting from the seminal contributions of Frenkel and Rose (1996), Kaminsky and Reinhart (1999), and Demirgüç-Kunt and Detragiache (1998), they have been developed for long to predict the financial crises in the emerging countries but after the 2008 global financial crisis, a renewed interest pushes the methodology to be applied to advanced countries (Borio and Lowe (2002), Barrell et al. (2010), Lo Duca and Peltonen (2013), Aldasoro, Borio and Drehmann (2018) and Davis and Karim (2008)) and to the risks assessment for macroprudential instruments calibration (see for example Detken et al. (2014), Drehmann and Juselius (2012), Ferrari and Pirovano (2015) review this quote, Tölö, Laakkonen and Kalatie (2018)).

in the third chapter of the BIS's Annual Report (BCBS (2017c)) in which the evolution of a set of early warning indicators of financial distress (credit-to-GDP gap, debt service ratio and property price gap) are monitored (Drehmann, Borio and Tsatsaronis (2011), Drehmann and Juselius (2014) and Aldasoro, Borio and Drehmann (2018) provide empirical support to the selection of these indicators). Other examples of EWSs applied can be found in Detken et al. (2014), where EWSs are used for the calibration of the CCyB, and Coudert and Idier (2016), who developed an EWS for the the risks assessment of the banking sector in France. Finally, also the BoE conducts a risk assessment based based on early warning indicators both on core indicators and expert judgement (see BoE (2016)).

Figure 3.3: Shocks' severity and level of cyclical systemic risks



3.3.2 The procyclical severity

Empirical evidence shows that there is a positive correlation between the severity of the crisis that can hit the economy and the current position in the financial cycle. For example the *"Special feature B"* of the May 2018 Financial Stability Review (ECB (2018)) shows that there is high correlation between the level of cyclical systemic risk (measured by a cyclical indicator) around the start of systemic financial crises and the drop in real GDP that materialised during those crises. This result is motivated by the fact that larger financial imbalances before the crises are associated with more severe financial crises. Based on this simple evidence, our strategy consists in a stable and transparent mapping between the level of the systemic risk assessed by the risk assessment process and a metric measuring the scenario' severity in a way that the level of systemic risk is positively correlated with the severity of the scenario. This metric is defined in terms of a tail percentile of the target variables' joint distribution, namely the real GDP growth and the unemployment rates. As already explained in section 3.2, this feature is shared with the approach implemented by the Bank of England for the Annual Cyclical Scenario (see Box 2 of BoE (2016)) and indirectly also with the Fed in the *severely adverse scenario* (see Fed, 12 CFR 252).

Figure 3.3 shows graphically the idea of the procyclical severity. The left panel shows a cyclical systemic risk indicator that quantifies the build-up of cyclical risk (e.g. the credit-to-

GDP gap or a composite indicator) and four different cycle momenta: no build-up of cyclical systemic risks (blue dot), build-up of low level vulnerabilities (green dot), build-up of medium level vulnerabilities (orange dot) and peak of the cycle (red dot). The arrows, instead, represent the magnitude of hypothetical systemic crises triggered by the materialization of the systemic risk corresponding to each dot. As we can notice, no downturn is associated to the blue dot while, gradually harsher downturns are associated to larger level of financial imbalances identified by an higher momenta of the financial cycle. The stronger downturn is associated to the peak of the cycle. The right panel shows the joint distribution of the target variables of the scenario and how the percentiles selection is related to risk indicator of the left panel. The higher the current level of financial imbalances, the lower the percentile selected from the joint distribution of the target variables, thus the stronger the scenario severity.

Table 3.1: The mapping between level of risk and severity of the scenario

Level of Cyclical Systemic Risk	Percentiles	Economic Interpretation – Frequency (see Borio 2012)
No risk	-	-
Low risk	14%	**One crisis every 7 years:** Regular downturns of the financial cycle
Medium risk	5%	**One crisis every 20 years:** Exceptional crises (e.g. 2001 dot-com bubble burst)
Highest risk	1%	**One crisis approximately every 100 years:** Great Depression 1929, Global Financial Crises 2007

3.3.3 Stable mapping between the risk assessment and scenario's severity

The calibration of the scenario starts mapping the levels of financial imbalances, as identified by the risk assessment process, into a percentile of the target variables' joint distribution. Indeed, we assume that during the downturn, the target variables growth at a rate in the tail of the distribution delimited by the percentile. We consider real GDP and the unemployment rate as the target variables. The choice is motivated by the fact that the real GDP is usually considered as the main variable representing the severity of crisis. In addition, for the evaluation of credit risk losses (which are one of the main variables affecting capital depletion) the unemployment rate is very significant. Looking for example at the Fed's CCAR scenario the increase of the unemployment rate is, jointly with the decline of house prices, the starting point of the scenario calibration.[16]

In order to avoid an unstable/arbitrary calibration and make also results comparable between different vintages, the mapping between risk level and the metric of the scenario severity is stable and reported in Table 3.1. The idea is to select the percentiles on the basis of the

[16]Several others leading institutions select the same target variables. For example, The FED calibrates the severely adverse scenario in order to have am unemployment rate increase between 3 to 5 percentage points from its initial level. The scenario severity of the IMF used for the FSAPs is usually is calibrated looking at the historical GDP distribution and is targeted at the 2 standard deviations of real GDP deviation from the trend.

historical frequency of the crisis: the most severe crisis is associated to the 1% tail of the joint distribution of the target variables (i.e. one crisis every 100 years); the least severe crisis, namely the regular downturn of the credit cycle, is associated to the 14% tail of the joint distribution of the target variables (i.e. one downturn every 7 years).

3.3.4 The MacroFin Copula

The second step of the scenario calibration deals with the estimation of the complementary variables' responses in a joint distress event with the target variables. Technically, a scenario has the aim to modelling the randomness concerning key variables in the economy, taking into account the joint dependence across macroeconomic indicators. Ideally, a good scenario should be an extreme realization such that it is not expected to happen but it is statistically possible and plausible. For instance, in a scenario where the GDP deals with a crisis, we do not only want a negative and extreme realization of GDP (i.e. a severe drop in the GDP growth), but we also want that the responses of other variables, such as the unemployment rate, house prices and interest rates have economic sense in the magnitude and the sign. As a consequence, modelling scenarios calibration strategies means taking into account marginal and joint features of the random variables forming the scenario. To cope with the challenge of generating extreme but plausible scenarios for a large set of stochastic variables and for several countries we rely on a copula approach. This statistical tool helps us to understand the stochastic dependence between the target and complementary variables then, to produce coherent responses conditioned to a stress event for the target variables. While the marginal distributions help us to produce the proper degree of distress in the complementary variables, the copula, acting as a bridge across marginal distributions, links the responses of the complementary variables to the distress of the target ones.

Copulas allow us to model separately the marginal distribution and the joint dependence structure enabling to divide the generation of scenarios in three stages. A first stage focuses on the distress event of the target variables,[17] which generate the scenario. In other words, the response of the target variables is calibrated from the marginal distribution coherently with the percentile identified as representative of the current level of the cyclical systemic risk. In the second stage we capture the dependence between the target and complementary variables, focusing on the percentile of their distributions conditioned to the triggering event defined in the first stage. Finally, the inverse marginal distribution of the complementary variables is employed to transform the uniform representation (in terms of percentiles) into a value of the magnitude of the response. This characteristic to untangle the joint co-movement across variables in marginal and dependence structure has boosted in the latest years the interest on copulas in several areas, including actuarial science (Frees, Carriere and Valdez (1996)), finance (Cherubini, Luciano and Vecchiato (2004), McNeil, Frey and Embrechts (2005)), neuroscience (Onken and Obermayer (2009)), weather and water research (Schoelzel and Friederichs (2008),

[17] The target variables are those lead the scenario severity and that have a central interpretation in terms of the scenario's interpretation. In purely financial scenarios these variables likely are those trigger the scenario and indeed are often called *triggering variables*.

Favre et al. (2004)) or biology (Kim et al. (2008)).

The copula is a function that gathers the dependence between random variables. The Sklar (1959)'s theorem states that the joint cumulative distribution function of a random vector can be rewritten in terms of marginal distribution functions of each random variable of the vector and a copula that describes the dependence structure across variables[18]. The copula methodology is widely employed in the financial literature to gather complex features as asymmetric relationship, non-linearities and strong dependence in extreme quantiles (see for instance Rodriguez (2007), Nikoloulopoulos, Joe and Li (2012), (Lucas, Schwaab and Zhang, 2017)). However the application of copula models to low-frequency data such as the macroeconomic variables is not extended yet due to the risk model presented in this methodology when the data is scarce. I order to face these challenges, we rely on two approaches: a Gaussian copula and a non-parametric variation that relies on a kernel smoothing function. The former model even if alleviate the data scarcity imposing a theoretical distribution, is not able to capture the complexity of the data and, in particular, those features above the second moment, such as kurtosis and skewness, which are related to the probability of extreme scenarios and its asymmetric behaviour. The second method, instead, has enough flexibility to replicate asymmetries in the data around the crises materialization.

Gaussian MacroFin Copula

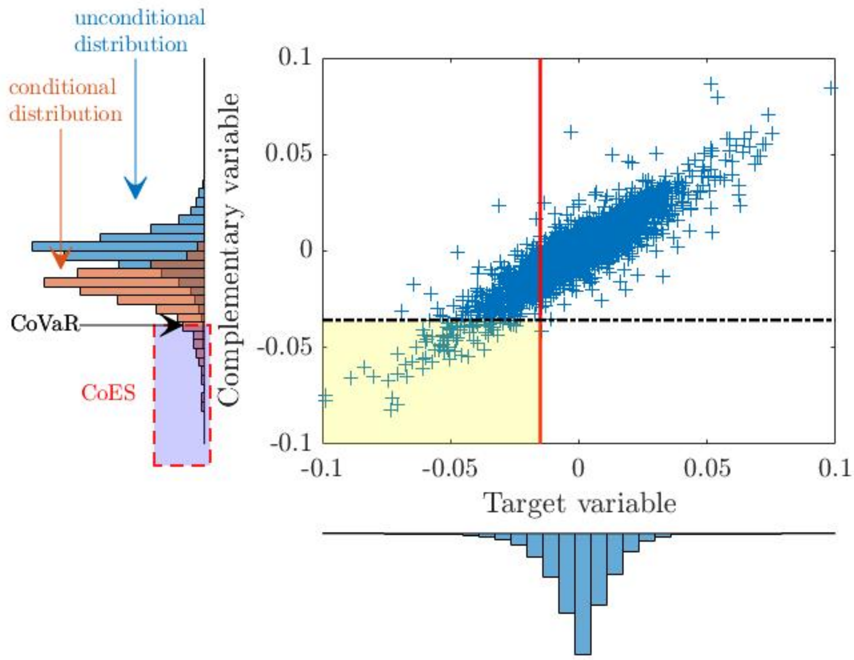
To obtain the scenario's responses with the Gaussian copula, we estimate the mean and the variance of the variables (once employed a stationary transformation) and the correlation between the uniform representation of each variable and the joint distribution of the target variables. The projections of the complementary variables are estimated with the Conditional Expected Shortfall (CoES):

$$CoES_{y|x}(\alpha, \beta) = \mu_y - \sigma_y \left(\frac{\sqrt{1-\rho_{x,y}^2}\,\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \frac{\rho_{x,y}\phi(\Phi(\alpha))}{\alpha} \right), \tag{3.1}$$

The derivation of this formula is contained in Appendix C.1. In the above equation, y (x) stands for the complementary (target) variable, μ_y and σ_y indicate the mean and the standard deviation of the complementary variable, while $\rho_{x,y}^2$ defines the correlation between the two types of variables. Finally, Φ is the cumulative Gaussian distribution function. In few words, the CoES is the the expected shortfall, namely the mean value beyond a certain percentile, conditional on the fact that the target variables are in a certain distress scenario. The condition on the target variables are defined in terms of a threshold which corresponds to a percentile of the target variables joint distribution. More specifically, equation 3.1 indicates the mean return of the complementary variable y below its $\beta 100\%$ percentile given that the target variables x are in the $\alpha 100\%$ of its joint distribution.

[18]For a general description of copulas, see for example Nelsen (2010) or Joe (2014)

Figure 3.4: The CoES for a bivariate example



The scatter plot in figure 3.4 shows the dependence relation between the target (x-axis) and the complementary variables (y-axis). Blue histograms represent the marginal distributions for target and complementary variables respectively, while the orange histogram shows the distribution of the complementary variables given the extreme scenario for the target variables. The realizations on the left of the vertical red line correspond to the $VaR(\alpha)$ for the target variables. The dash-dotted black line indicate the threshold that leaves the $\beta 100\%$ of the distribution of the complementary variables given the distress scenario for the target variable. The mean value of the realizations within the black-dotted and the red lines is the CoES.

Non-parametric MacroFin Copula

The MacroFin copula estimated with the kernel smooth function[19] is based on the following steps.

1. For each variable once employed a stationary transformation, the kernel smoothing function, is estimated. Then, the realizations of the variables are transformed into uniform representations, that links the realization to a specific percentile:

$$u_j^k = \hat{F}_j(x_j^k, s_j^k) \quad \text{where} \quad k = \{t, c\} \tag{3.2}$$

[19]The kernel distribution is a non-parametric estimation of the probability distribution function of a random vector. The kernel distribution is given by a smoothing function and a bandwidth value, which are employed as smoothness factors. The normal smoothing function and its optimal width according to Bowman and Azzalini (1997) has been employed to obtain the results presented in this article. The kernel distribution allows for taking the most from the data sample without imposing any constraint about distribution or uncertainty about estimated parameters, which due to the limited data could be quite significant

where x_j^k is the stationary transformation for the variable X_j^k and $\hat{F}_j$ is the marginal cumulative distribution function given by the kernel smoothing function. The index k indicates the type of variable: t stands for *target* and c for complementary. In the above equation, u_j indicates the uniform transformation of the variables realization, while s_k is index that values 1 or -1 depending if low or high percentile corresponds to a negative realization. For example, the worst realization for the unemployment rate corresponds to the 99th percentile of its distributions and in this case an index $s_j = 1$ is applied. The opposite happens for the real GDP growth. As already mentioned, this procedure allows us to calculate the joint distribution of variable that differs in terms of interpretation of the realizations' ordering.

2. The uniform transformations of the target variables u_j^t are merged using either the simple average, which would correspond to a joint distribution, or a weighted average, which would allow us to modulate the scenario event. In this way we derive the transformations of the joint distribution of the target variables, defined as U^t. The same procedure is applied to the transformations of the complementary variables in order to obtain U^c. Then the bivariate kernel smoothing function is estimated between the target and complementary joint transformation, i.e. $F_{t,c}(U^t, U^c)$.

3. We simulate W values between 0 and α where α could be 0.01, 0.05 or 0.14 depending on the degree of distress chosen for the scenarios. For each draw $\omega_w \in W$, we calculate the conditional distribution of U^c, i.e. $P(U^c < U^*|U^{target} = \omega_w)$. Then we simulate a uniformly distributed random number q between 0 and 1 getting the value of U^* such that $q = P(U^c < U^*|U^{target} = \omega_w)$. In other words, we use the inverse conditional copula to obtain the conditional percentile of variable c given a shock in the target variables.

4. For the vector of U^*, we recover the realizations of the complementary variables using the inverse cumulative distribution function, i.e. $x_j^c = \hat{F}^{-1}(U^*)s_j$. Multiplying for s_j guarantees the right sign to the variable. Assessing the mean value of x_k^W would give us the mean response of the complementary variables, while the mean below the percentile $\beta100$ would give us the conditional mean[20].

The kernel smoothing function helps us to alleviate the scarce data while gathering the marginal features and the characteristics of dependence between the complementary variables and the target variables. This approach provides us with a flexible and at the same time historic perspective to generate coherent scenarios from a probabilistic view (see Figure 3.5).

3.4 The MacroFin Copula in practice (in process)

3.4.1 Data source and assumptions

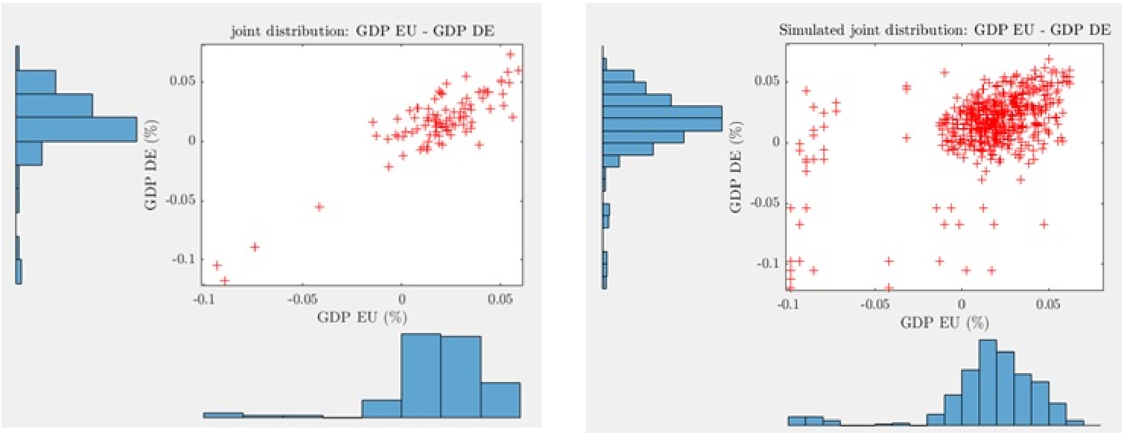
In this section, we presents the calibrated scenarios for the 19 euro area countries using the parametric and non-parametric approaches. For each country separately, a scenario was cali-

[20]if s_k is -1 we should compute instead the mean above the highest $\beta100$ percentile

brated using quarterly data starting in 1980 from Eurostat and Bloomberg (see Appendix C.2). As in the EBAs' scenarios, we calibrate the path of real GDP, stock prices, unemployment rates, residential and commercial property prices and government bond yields. Real GDP, real estate prices and stock prices were transformed into stationary series using the year-over-year growth rate, while for the unemployment rate and the government bond yields we use the yearly difference.

Figure 3.5: Scatter plot and histogram of the GDP distribution for EU and Germany

(a) Empirical Distribution   (b) Kernel Distribution



In both cases, the target variable coincides with the joint distribution of the quantiles of the real GDP and unemployment rate distributions. As mentioned in section xx, if the composite cyclical indicator would signal low level of risk, then the target and complementary variables would be assumed to be in the 14% tail of their joint distributions, if the cyclical risk indicator would signal medium level of risk, then the target and the complementary variables would be assumed to be in the 5% tail of their joint distribution. Finally, high level of cyclical risk would be associated with the 1% tail of the joint distribution of the target and complementary variables.

3.4.2 The parametric approach

Table 3.2 reports the adverse growth rates of real GDP and of the unemployment rate for all country-specific scenarios and for the three level of cyclical risk considered these rates are then compared with the historical observations during the financial crisis to provide an idea of the relative severity of the downturn.

As we can see, the scenarios calibrated for high level cyclical systemic risks would be closer to the 2009 values. Table 3.3 reports the results for stock price and government bond yields. As mentioned earlier in the paper, this is the first time that the copula model is used to calibrate the dependency in crisis times between macroeconomic and financial variables. Stock prices and government bond yields are accounted in the set of complementary variables. One of the major issues of traditional econometric approaches is that it is difficult to capture by means of normal regressions the relationship between financial and real variables. As the results show, our statistical approach allows to capture the high correlation of these variables in downturn times.

The cumulative growth of real estate prices under the first two years of the scenario is reported in Table 3.4. In this case, an additional step as added to the methodology described in the previous sections. The housing prices series for EA countries are very short and for some countries do not present any downturn. For this reason the final results have been calculated for some countries where the data quality was not sufficiently high, using the weighted average of the other countries'results.

3.4.3 The nonparametric approach

[TBD]

3.5 Conclusions

Due to the positive economic outlook and increased momentum in the business and financial cycles, several policy institutions are facing the issue of whether countercyclical macroprudential policies should be activated. Scenario-based simulations relying on top-down stress test models are a very promising approach for the calibration and impact assessment of macroprudential policies as, thanks to their granular description of the banking system. This kind of methodology, however, is strongly affected on the way in which the scenarios are calibrated. Motivated by these facts, we present an innovative methodology for designing countercyclical adverse scenarios suitable for calibration and impact assessment of macroprudential policies aimed at addressing externalities caused by strategic complementarities.

Our methodology allows building tailored scenarios that are characterized by two main innovative features. First, there is a stable and transparent mapping between the level of cyclical systemic risk and the path of the scenario's *target variables*, which are those variables crucial for determining the overall scenario's severity. The link with the cyclical risk assessment guarantees a procyclical severity, namely the severity is stronger when the cyclical systemic risk is higher, and allows us to reconcile the scenario design with the ultimate goal of the macroprudential policy objective of the calibration. Indeed, the amplification of the financial cycle generated by the strategic complementarities is linked to the build-up of cyclical systemic risk. Second, we propose a coherent calibration of a set of scenario's *complementary variables* based on a multivariate copula model on a mixture of macro and financial data (MacroFin Copula). The ability of the copula to properly estimate the co-movement of the variables conditional to the materialization of a tail event can significantly improve the economic coherence of the scenarios.

In order to adapt the copula to low-frequency macroeconomic data, we propose a nonparametric method that relies on a kernel smoothing function. This approach is particularly suitable for adapting the copula to macroeconomic data not only because overcome the data scarcity issue, but also because has enough flexible to be able to replicate the complexity of data around the unfolding of a systemic crises. As far as we know, this this the first time that this improvement of the non parametric copula has been developed.

Simulating scenarios for real GDP, unemployment, stock prices, long-term interest rates and real estate prices, we show that our methodology produces scenarios able to well reproduce the dynamics observed during the global financial cycle in terms of the magnitude of the downturn and the co-movements between the macro-financial variables. As a consequence, the scenarios have a strong economic coherence, a clear historical interpretation and abstracting from expert judgement. The latter features significantly increase the reliability of the scenario-based simulations, in particular if applied to calibration and impact assessment of macroprudential policies.

Table 3.2: Parametric scenario: Real GDP and Unemployment rate

	GDP [%]				Unemployment [pp]			
	1%	5%	14%	2009	1%	5%	14%	2009
EA	-3.7	-2.4	-1.4	-5.1	1.8	1.4	1.0	2.2
AT	-4.0	-2.6	-1.5	-4.1	1.4	1.1	0.8	1.5
BE	-3.3	-2.1	-1.2	-3.9	1.9	1.4	1.0	1.2
CY	-6.4	-4.4	-2.8	-2.6	4.7	3.6	2.8	2.0
DE	-4.8	-3.3	-2.2	-6.4	1.8	1.4	1.0	0.2
EE	-8.5	-5.7	-3.4	-17.3	2.2	1.4	0.8	8.8
ES	-4.0	-2.5	-1.4	-3.9	4.2	3.1	2.3	7.4
FI	-10.4	-7.7	-5.5	-7.8	4.2	3.3	2.5	1.7
FR	-3.5	-2.3	-1.3	-4.0	1.5	1.2	0.9	1.8
GR	-8.9	-6.6	-4.8	-0.9	4.2	3.2	2.5	1.7
IE	-9.2	-5.8	-3.1	-8.5	3.3	2.4	1.7	6.6
IT	-5.6	-4.1	-2.9	-5.6	2.0	1.5	1.2	0.7
LT	-10.8	-7.4	-4.6	-16.7	3.1	2.1	1.4	9.0
LU	-8.6	-5.8	-3.6	-6.1	1.3	1.0	0.8	0.5
LV	-11.3	-7.8	-5.1	-23.3	2.1	1.4	0.8	10.7
MT	-3.9	-2.1	-0.7	-1.2	1.1	0.8	0.6	1.1
NL	-3.4	-2.1	-1.1	-4.0	1.8	1.3	1.0	0.5
PT	-5.8	-4.2	-2.9	-2.6	3.1	2.3	1.8	1.9
SI	-6.8	-4.6	-2.9	-6.2	2.2	1.7	1.2	1.4
SK	-7.8	-5.2	-3.2	-3.9	4.3	3.2	2.4	1.3

Table 3.3: Parametric scenario: Stock prices and Government bonds yield

	Stock prices [%]				10-year gov bond spread against the german bonds [bps]			
	1%	5%	14%	2009	1%	5%	14%	2009
EA	-33	-25	-19	-38	98	72	52	98
AT	-37	-28	-21	-53	20	15	11	82
BE	-29	-22	-17	-47	1	-2	-4	55
CY	-61	-51	-42	-58				
DE	-28	-21	-14	-30				
EE	-36	-27	-19	-52				
ES	-30	-23	-17	-32	145	105	75	59
FI	-45	-35	-27	-42	3	0	-1	60
FR	-31	-23	-17	-35	40	28	18	40
GR	-54	-44	-36	-47	283	212	158	193
IE	-33	-25	-17	-56	194	146	108	189
IT	-37	-29	-21	-40	114	82	57	96
LT	-33	-24	-16	-61	96	73	55	899
LU	-47	-38	-30	-54	35	27	21	71
LV	-29	-21	-14	-57	133	95	65	95
MT	-35	-28	-22	-35	12	7	3	77
NL	-30	-22	-16	-42	32	25	19	43
PT	-40	-32	-25	-31	241	180	134	93
SI	-45	-36	-28	-47	166	123	89	103
SK	-38	-30	-23	-24	52	37	24	118

Table 3.4: Parametric scenario: Residential and commercial real estate prices

	Residential property prices [%]				Commercial property prices [%]			
	1%	5%	14%	2009	1%	5%	14%	2009
EA	-13	-8	-5	-2	-19	-13	-7	
AT	-18	-13	-8	13	-27	-19	-12	
BE	-10	-7	-4	6	-15	-10	-6	
CY	-17	-13	-10	-13	-26	-20	-14	
DE	-9	-6	-3	2	-14	-9	-5	
EE	-15	-11	-7	-30	-23	-16	-11	
ES	-20	-14	-10	-15	-30	-22	-14	
FI	-10	-7	-4	10	-15	-10	-6	
FR	-13	-9	-6	3	-20	-14	-9	
GR	-11	-8	-5	-4	-16	-11	-7	
IE	-20	-15	-9	-38	-31	-22	-14	
IT	-18	-12	-8	0	-27	-19	-12	
LT	-18	-14	-10	-33	-27	-20	-15	
LU	-7	-4	-1	7	-11	-6	-2	
LV	-21	-17	-13	-43	-32	-25	-19	
MT	-14	-9	-5	-5	-21	-14	-8	
NL	-17	-13	-9	-8	-26	-20	-14	
PT	-18	-13	-8	-7	-26	-19	-13	
SI	-17	-13	-10	-6	-26	-20	-15	
SK	-16	-11	-8	-21	-23	-17	-11	

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Appendix A

Appendix to Chapter 1

A.1 Kinked multiplier preferences vs Ghirardato, Maccheroni and Marinacci (2004)'s biseparable preferences

The main theoretical contribution of the paper consists in the introduction of the *kinked multiplier preferences* a formalization of the agents' preferences that accounts for both ambiguity aversion and loving and that makes the prevalence of each of the side to the current state of the cycle. These ne preferences are defined as follows:

$$V(c_t) = \begin{cases} \min_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta^+ \varepsilon(m_{t+1}) \right\} & \text{if} \quad V_t \leq \mathbb{E}_{t-1}\{V_t\} \\ \\ \max_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta^- \varepsilon(m_{t+1}) \right\} & \text{if} \quad V_t > \mathbb{E}_{t-1}\{V_t\} \end{cases} \tag{A.1}$$

The kinked multiplier preferences can be seen as a dynamic and state-contingent generalization of the biseparable preferences axiomatized in Ghirardato and Marinacci (2001) and Ghirardato, Maccheroni and Marinacci (2004). Both papers show that ambiguity attitudes can be formalized within a general decision model by constructing a biseparable preference, which can combine both ambiguity aversion and ambiguity seeking. Given again the specification, in equation A.1, we can represent our kinked multiplier preferences as follows:

$$\begin{aligned} V_t(c_t) = \mathbb{I}_{\theta_t \geq 0} \min_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta\varepsilon(m_{t+1}) \right\} + \\ \mathbb{I}_{\theta < 0} \max_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta\varepsilon(m_{t+1}) \right\} \end{aligned} \tag{A.2}$$

As noted in Ghirardato, Maccheroni and Marinacci (2004) the indicator function shall depend only upon expected utility mapping. We design the following expected utility mapping so that $\theta_t < 0$ whenever $V_t > \mathbb{E}_{t-1}\{V_t\}$ (which since now we often refer as the gain domain) and viceversa (in the loss domain). We can therefore re-write our preferences as:

$$V_t(c_t) = \mathbb{I}_{V_t \leq \mathbb{E}_{t-1}\{V_t\}} \min_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta^{+} \varepsilon(m_{t+1}) \right\} +$$
$$\mathbb{I}_{V_t > \mathbb{E}_{t-1}\{V_t\}} \max_{\{m_{t+1}, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t M_t u(c_t) + \beta\theta^{-} \varepsilon(m_{t+1}) \right\} \tag{A.3}$$

A.2 Microfoundation of the collateral constraint

In this section we provide micro-foundations for a delegated monitoring problem in which the collateral constraint emerges as resulting from an incentive-compatible debt contract enforced through a bank. The micro-foundations follows Bianchi and Mendoza (2018). Debt contract are signed by a bank that must enforce debtor incentives. Between periods borrowers can divert revenues for an amount $\tilde{d}$. At the end of the period the diversion is no longer possible and payment is enforced. Banks can monitor financial diversion due to special relationship lending abilities[1]. If the bank detects the diversion asset can be seized up to a percentage ϕ. As common in dynamic economies we assume that the contract is done under no memory, so that in the next periods borrowers can re-enter debt agreement even if they defaulted in the previous period. This assumption allows us to preserve the Markov structure of the contracting/intermediation problem.

We shall show that the collateral constraint can emerge as resulting from an incentive compatibility constraint imposed by the bank through the debt design. Specifically the collateral constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents banks from redeploying more than a fraction ϕ of the value of the assets owned by a defaulting borrower. Define V^R and V^D respectively the value of repayment and default and define as V the continuation value.

If the borrower defaults the diverted resources enter his budget constraint and the recursive problem reads as follows (for notational convenience we skip the beliefs constraints for the purpose of this derivation):

$$\begin{aligned} V^D(b, x, S) = \max_{c, x', b'} & \{u(c) + \beta\mathbb{E}_S + \\ & + \lambda \left[y + q(S)(x + \alpha y) + \tilde{d} + b - q(S)x' - c - \frac{b'}{R} \right] + \\ & + \mu \left[\phi q(S)x' + \frac{b'}{R} \right] \end{aligned} \tag{A.4}$$

[1] We assume zero monitoring costs for simplicity. Extending it to the case with positive monitoring costs is rather straightforward.

On the other side if the borrower repays his value function reads as:

$$\begin{aligned} V^D(b,x,S) = \max_{c,x',b'} \{u(c) + \beta \mathbb{E}_S + \\ + \lambda \left[y + q(S)(x+\alpha y) + b - q(S)x' - c - \frac{b'}{R} \right] + \\ + \mu \left[\phi q(S)x' + \frac{b'}{R} \right] \end{aligned} \tag{A.5}$$

The comparison of the two easily shows that the households repay if and only if $\tilde{d}' < \phi q(S)x'$.

A.3 GMM Estimation of the Ambiguity Parameter

In this section we detail the derivations needed to achieve the moment condition that is the object of our estimation. Further below we also provide a description of the dataset used in the estimation.

A.3.1 General Approach

We use a GMM estimation procedure based on the moment condition obtained from the combined Euler equation for debt and risky assets and is a variant of the techniques developed for asset pricing models with recursive preferences, pioneered by Epstein and Zin (1989) and Kreps and Porteus (1978). Hence, the starting point is to reformulate our value function, capturing multiplier preferences, in terms of an ambiguity term. The latter is achieved by mapping the multiplier preferences to a special case of the recursive preferences. This can be done by assuming a logarithmic continuation value, a logarithmic utility function and an ambiguity adjustment factor, Q which accounts for waves of optimism and pessimism. Indeed we depart from the well-known equivalence between multiplier and recursive preferences by embedding state-contingent ambiguity attitudes. We start by reporting the value function derived after substituting the solution of the inner problem, presented in Section 1.3.3:

$$V_t = u(c_t) - \beta\theta_t log \left[\mathbb{E}_t \left\{ \exp\left(-\frac{V_{t+1}}{\theta_t} \right) \right\} \right] \tag{A.6}$$

The above equation embeds a logarithmic ambiguity-adjusted component $Q_t(V_{t+1})$, which maps future continuation values into current realizations. Indeed we can re-write (A.6) as follows:

$$\begin{aligned} V_t =& u(c_t) + \beta h^{-1} \mathbb{E}_t \{h(V_{t+1})\} \\ & u(c_t) + \beta Q_t(V_{t+1}) \end{aligned} \tag{A.7}$$

where $h(V_{t+1}) = \exp\left(-\frac{V_{t+1}}{\theta_t}\right)$, as implied by the equivalence between specifications under recursive and multiplier preferences (see Hansen et al. (2007)). It then follows that the ambiguity adjustment component reads as follows:

$$Q_t(V_{t+1}) = h^{-1}\mathbb{E}_t\left\{h(V_{t+1})\right\} = -\theta_t \log\left[\mathbb{E}_t\left\{\exp\left(-\frac{V_{t+1}}{\theta_t}\right)\right\}\right] \tag{A.8}$$

A.3.2 Pricing Kernel-SDF

The next step to obtain our moment condition is to derive an expression for the stochastic discount factor as function of $Q_t(V_{t+1})$. To this purpose, we shall derive the marginal utility of consumption and the derivative of the current value function with respect to the next period one, which we define as MV_{t+1} and reads as follows:

$$\begin{aligned} MV_{t+1} &= \frac{\partial V_t}{\partial Q_t(V_{t+1})}\frac{\partial Q_t(V_{t+1})}{\partial V_{t+1}} = \beta\frac{\exp(-\frac{V_{t+1}}{\theta_t})}{\mathbb{E}_t\left\{\exp(-\frac{V_{t+1}}{\theta_t})\right\}} \\ &= \beta\exp\Big(-\frac{1}{\theta_t}(V_{t+1}-Q_t(V_{t+1}))\Big) \end{aligned} \tag{A.9}$$

Given a logarithmic utility function $u(c_t) = log(c_t)$, the marginal utility of consumption is $MC_t = c_t^{-1}$. Using the above expressions we can derive the SDF as function of Q_t:

$$\Lambda_{t,t+1} = \frac{MV_{t+1}MC_{t+1}}{MC_t} = \beta\frac{c_{t+1}}{c_t}^{-1}\underbrace{\exp\left(-\frac{1}{\theta_t}(V_{t+1}-Q_t(V_{t+1}))\right)}_{m_{t+1}} \tag{A.10}$$

where $m_{t+1} = \exp\left(-\frac{1}{\theta_t}(V_{t+1}-Q_t(V_{t+1}))\right)$ is the optimal likelihood ratio. Equation (A.10) shows that the SDF has a two-factor structure. The first factor is the standard consumption growth, while the second is the ambiguity factor. The latter depends upon the distance between the future value function and its certainty equivalent, namely the future insurance premium. Under no uncertainty this premium vanishes[2].

A.3.3 Estimation of the Continuation Value Ratio

Since estimation requires strictly stationary variables, we shall re-scale the value function (A.7) by consumption (see Hansen, Heaton and Li (2008) (HHL henceforth). Subtracting the log of consumption, $\tilde{c}_t = log(c_t)$, on both sides:

$$\tilde{v}_t = \beta Q_t(\tilde{v}_{t+1} + \Delta\tilde{c}_{t+1}) \tag{A.11}$$

where we define $\tilde{v}_t = V_t - \tilde{c}_t$ as the *continuation value ratio*, scaled by the log of consumption. Next substituting (A.8) into (A.11) we obtain:

$$\tilde{v}_t = -\beta\theta_t\log(\mathbb{E}_t\left\{\exp\left[\sigma_t(\tilde{v}_{t+1} + \Delta\tilde{c}_{t+1})\right]\right\}) \tag{A.12}$$

[2]Indeed the continuation value would be perfectly predictable $\left(\exp\left(-\frac{V_{t+1}}{\theta}\right) = \mathbb{E}_t\exp\left(-\frac{V_{t+1}}{\theta}\right), m^*_{t+1} = 1\right)$ with zero adjustment ($Q_t(V_{t+1}) = V_{t+1}$).

where $\sigma_t = -1/\theta_t$, and it is negative when $\theta_t > 0$ and positive when $\theta_t < 0$. An expression for equation (A.12) can be derived analytically along the lines of HHL. Indeed, since $\tilde{v}_t$ is a function of states governing the dynamic behaviour of consumption growth, g^c_{t+1}, we can guess it as a function of a Markov process, ξ_t:

$$g^c_{t+1} = \tilde{c}_{t+1} - \tilde{c}_t = \mu_c + H\xi_t + \mathbf{A}\epsilon_{\mathbf{t+1}} \tag{A.13}$$

$$\xi_{t+1} = F\xi_t + \mathbf{B}\epsilon_{\mathbf{t+1}} \tag{A.14}$$

where ϵ_{t+1} is a $(2x1)$ i.i.d. vector with zero mean and covariance matrix I. A and B are $(2x1)$ vectors. The exogenous states, ϵ_{t+1}, income shocks in our case, have an impact on consumption directly and through the states, ξ_t. Its estimated value, $\hat{\xi}_t$, is obtained through Kalman filtering consumption data. Then, given (A.13), we guess the continuation value ratio as depending only upon the estimated states, $\hat{\xi}_t$:

$$\tilde{v}_t = \mu_v + U_v\hat{\xi}_t \tag{A.15}$$

where $U_v\hat{\xi}_t$ is the discounted sum of expected future growth rates of consumption. After some derivations we can write U_v and μ_v as follows:

$$U_v \equiv \beta(I - \beta F)^{-1}H \tag{A.16}$$

$$\mu_v \equiv \frac{\beta}{1-\beta}\left(\mu_c + \frac{\sigma_t}{2}|A + U_v B|^2\right)$$

where the term $A + U_v B$ captures the dependence between the the continuation value and the exogenous shocks.

A.3.4 SDF and the Euler Equation

Next, given the estimated $\tilde{v}_t$ from (A.15). Substituting (A.12) into (A.10) delivers:

$$\Lambda_{t,t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-1}\left(\frac{\frac{\exp(V_{t+1})}{c_{t+1}}\frac{c_{t+1}}{c_t}}{\exp\left(Q_t\left(\tilde{v}_{t+1} + \Delta\tilde{c}_{t+1}\right)\right)}\right)^{\sigma} \tag{A.17}$$

Note that equation (A.17) is equivalent to the SDF obtained under Epstein and Zin (1989) preferences and given the assumption of unitary EIS. At last, upon using (A.11) into (A.17) and substituting the resulting SDF into the combined Euler for debt and risky asset (1.11) and (1.12), we obtain:

$$\mathbb{E}_t\left\{\underbrace{\beta\left(\frac{c_{t+1}}{c_t}\right)^{-1}\left(\frac{\frac{\exp(V_{t+1})}{c_{t+1}}\frac{c_{t+1}}{c_t}}{\sqrt[\beta]{\left(\frac{\exp(V_t)}{c_t}\right)}}\right)^{\sigma_t}}_{\Lambda_{t,t+1}}\left(R^s_{t+1} - \phi R_{t+1}\right) + \phi - 1\right\} = 0 \tag{A.18}$$

where $R^s_{t+1} = \frac{d_{t+1}+q_{t+1}}{q_t}$ and for the estimation we shall write the debt rate as time-varying.

A.4 Analytical Derivations

This appendix derives analytical expressions for asset prices and returns.

A.4.1 Asset Price

From the borrowers' optimality condition on risky assets:

$$\begin{aligned} q_t =& \beta \mathbb{E}_t \left\{ \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1}(q_{t+1} + d_{t+1}) \right\} + \phi \mu_t^{'} q_t \\ =& \beta \mathbb{E}_t \{ \Lambda_{t,t+1}(d_{t+1} + q_{t+1}) \} + \phi \mu_t^{'} q_t \end{aligned} \tag{A.19}$$

using the definitions for $\Lambda_{t,t+1} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)} m_{t+1}$ and $\mu_t^{'} = \frac{\mu_t}{u_c(c_t)}$. Then denoting $K_{t,t+1} = \frac{\Lambda_{t,t+1}}{1-\phi\mu_t^{'}}$, we derive the following expression for the asset price:

$$q_t = \mathbb{E}_t \{ K_{t,t+1}(d_{t+1} + q_{t+1}) \} \tag{A.20}$$

Proceeding by forward recursion:

$$\begin{aligned} q_t =& \mathbb{E}_t \{ K_{t,t+1}(d_{t+1} + K_{t+1,t+2}(d_{t+2} + q_{t+2})) \} \\ =& \mathbb{E}_t \{ K_{t,t+1}(d_{t+1} + K_{t+1,t+2} d_{t+2}) \} + \mathbb{E}_t \{ K_{t,t+1} K_{t+1,t+2} K_{t+2,t+3} (d_{t+3} + q_{t+3}) \} \\ =& \mathbb{E}_t \{ K_{t,t+1}(d_{t+1} + K_{t+1,t+2} d_{t+2} + K_{t+1,t+2} K_{t+2,t+3} d_{t+3}) \} + \\ & + \mathbb{E}_t \{ K_{t,t+1} K_{t+1,t+2} K_{t+2,t+3} K_{t+3,t+4} (d_{t+4} + q_{t+4}) \} \\ =& \mathbb{E}_t \{ K_{t,t+1}(d_{t+1} + K_{t+1,t+2} d_{t+2} + \\ & + K_{t+1,t+2} K_{t+2,t+3} d_{t+3} + K_{t+1,t+2} K_{t+2,t+3} K_{t+3,t+4} d_{t+4}) \} + \\ & + \mathbb{E}_t \{ K_{t,t+1} K_{t+1,t+2} K_{t+2,t+3} K_{t+3,t+4} q_{t+4} \} \end{aligned} \tag{A.21}$$

At the final recursion step, the solution for the asset price:

$$q_t = \mathbb{E}_t \left\{ \sum_{i=1}^{T} d_{t+i} \prod_{j=1}^{i} K_{t+j-1,t+j} \right\} + \mathbb{E}_t \left\{ \prod_{i=0}^{T} K_{t+i,t+i+1} q_{t+T} \right\} \tag{A.22}$$

Taking the limit for $T \to \infty$ of the above condition delivers equation (1.21).

A.4.2 The Risk Premium

Expanding the borrower's FOC for the risky asset and plugging in it the derivation for $\mathbb{E}_t\{\Lambda_{t,t+1}\}$ and the definition $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$ we get:

$$\begin{aligned} 1 &= \mathbb{E}_t\{\Lambda_{t,t+1}\frac{q_{t+1}+d_{t+1}}{q_t}\} + \phi\mu'_t \\ &= \mathbb{E}_t\{\Lambda_{t,t+1}\}\mathbb{E}_t\Big\{\frac{q_{t+1}+d_{t+1}}{q_t}\Big\} + \text{Cov}\Big(\Lambda_{t,t+1}, \frac{q_{t+1}+d_{t+1}}{q_t}\Big) + \phi\mu'_t \\ &= \Big(\frac{1-\mu'_t}{R}\Big)\mathbb{E}_t\{R^s_{t+1}\} + \text{Cov}(\Lambda_{t,t+1}, R^s_{t+1}) + \phi\mu'_t \end{aligned} \tag{A.23}$$

The return on risky assets is obtained:

$$\mathbb{E}_t\{R^s_{t+1}\} = \frac{R(1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu'_t)}{1-\mu'_t} \tag{A.24}$$

Dividing by the risk-free return rate, the premium between the return on the risky asset and the risk-free rate can be derived:

$$\Psi_t = \frac{1 - cov(\Lambda_{t,t+1}, R^s_{t+1}) - \phi\mu'_t}{1-\mu'_t}. \tag{A.25}$$

A.4.3 The Sharpe Ratio and the Hansen and Jagannathan (1991) Bounds

Writing down the two borrowers' optimal conditions for the risk-free and risky assets, respectively:

$$1 = \mathbb{E}_t\{\Lambda_{t,t+1}R\} + \mu'_t \tag{A.26}$$

$$1 = \mathbb{E}_t\{\Lambda_{t,t+1}R^s_{t+1}\} + \phi\mu'_t \tag{A.27}$$

where $\mu'_t = \frac{\mu_t}{u_c(c_t)}$, $\Lambda_{t,t+1} = \beta\frac{u_c(c_{t+1})}{u_c(c_t)}m_{t+1}$ and $R^s_{t+1} = \frac{q_{t+1}+d_{t+1}}{q_t}$. In order to derive the excess return between the risky asset and the risk-free asset, we subtract (A.26) from (A.27), obtaining:

$$0 = \mathbb{E}_t\{\Lambda_{t,t+1}(R^s_{t+1} - R)\} + \mu'_t(\phi - 1). \tag{A.28}$$

Then, we define the excess return as $z_{t+1} = R^s_{t+1} - R$. Assuming a linear general form for the stochastic discount factor $\Lambda_{t.,t+1}$:

$$\Lambda^*_{t.,t+1} = \bar{\Lambda}^* + \tilde{\beta^m}(z_{t+1} - E_t z_{t+1}) \tag{A.29}$$

it must satisfy the following condition:

$$0 = \mathbb{E}_t\{\Lambda^*_{t,t+1}z_{t+1}\} + \mu'_t(\phi - 1)\}, \tag{A.30}$$

which, once expanded, gives:

$$
\begin{aligned}
0 &= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + cov(\Lambda^*_{t,t+1}, z_{t+1}) + \mu^{'}_t(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \mathbb{E}_t\{(z_{t+1} - \bar{z})(\Lambda^*_{t,t+1} - \bar{\Lambda}^*)\} + \mu^{'}_t(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \mathbb{E}_t\{(z_{t+1} - \bar{z})(z_{t+1} - \bar{z})\tilde{\beta^m}\} + \mu^{'}_t(\phi - 1) \\
&= \mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} + \sigma^2_z\tilde{\beta^m} + \mu^{'}_t(\phi - 1).
\end{aligned} \tag{A.31}
$$

Hence:

$$
\tilde{\beta^m} = -(\sigma^2_z)^{-1}\mathbb{E}_t\{\Lambda^*_{t,t+1}\}\mathbb{E}_t\{z_{t+1}\} - (\sigma^2_z)^{-1}\mu^{'}_t(\phi - 1) \tag{A.32}
$$

The variance of the stochastic discount factor is then obtained:

$$
\begin{aligned}
Var(\Lambda^*_{t,t+1}) &= Var((z_{t+1} - \mathbb{E}_t\{z_{t+1}\})'\tilde{\beta^m}) \\
&= \tilde{\beta^{'m}}\sigma^2_z\tilde{\beta^m} \\
&= (-(\sigma^2_z)^{-1}\bar{\Lambda}^*_t\mathbb{E}\{z_{t+1}\} - (\sigma^2_z)^{-1}\mu^{'}_t(\phi - 1))^{'}\sigma^2_z \\
&(-(\sigma^2_z)^{-1}\bar{\Lambda}^*_t\mathbb{E}\{z_{t+1}\} - (\sigma^2_z)^{-1}\mu^{'}_t(\phi - 1)) \\
&= (\sigma^2_z)^{-1}(\bar{\Lambda}^*)^2(\mathbb{E}_t\{z_{t+1}\})^2 + \\
&+ 2\mu^{'}_t(\phi - 1)((\sigma^2_z)^{-1}\bar{\Lambda}^*_t\mathbb{E}\{z_{t+1}\} + ((\sigma^2_z)^{-1}(\mu^{'}_t)^2(\phi - 1)^2.
\end{aligned} \tag{A.33}
$$

Hence:

$$
\frac{\sigma^2_{\Lambda^*_t}}{\bar{\Lambda}^{*2}} = \frac{(\mathbb{E}_t\{z_{t+1}\})^2}{\sigma^2_z} + 2\mu^{'}_t\frac{(\phi - 1)\mathbb{E}_t\{z_{t+1}\}}{\sigma^2_z} + \frac{\mu^{'2}_t}{\Lambda^{*2}}\frac{(\phi - 1)^2}{\sigma^2_z}. \tag{A.34}
$$

The Sharpe Ratio (SR hereafter) on stock asset returns over bonds results to be:

$$
SR = \frac{(\mathbb{E}_t\{z_{t+1}\})^2}{\sigma^2_z} = \frac{\sigma^2_{\Lambda^*_t}}{\bar{\Lambda}^{*2}} - 2\mu^{'}_t\frac{(\phi - 1)\mathbb{E}_t\{z_{t+1}\}}{\sigma^2_z} - \frac{\mu^{'2}_t}{\Lambda^{*2}}\frac{(\phi - 1)^2}{\sigma^2_z} \tag{A.35}
$$

Thus, the SR depends on the variance of the adjusted for distorted beliefs stochastic discount factor and on $\mu^{'}_t$.

A.5 Numerical Method

Our numerical method extends the algorithm of Jeanne and Korinek (2010) to persistent shocks and state-contingent ambiguity attitudes. The method, following the endogenous grid points approach of Carroll (2006), performs backwards time iteration on the agent's optimality conditions. We derive the set of policy functions $\{c(b, s),\ b'(b, s),\ q(b, s),\ \mu(b, s),\ V(b, s)\}$ that solve competitive equilibrium characterized by the system:

$$c(b,s)^{-\gamma} = \beta R\, \mathbb{E}\left\{m(b',s')c(b',s'^{-\gamma})\right\} + \mu(b,s) \tag{A.36}$$

$$q(b,s) = \beta\, \frac{\mathbb{E}\left\{m(b',s')c(b',s'^{-\gamma}[q(b',s') + \alpha y']\right\}}{c(b,s)^{-\gamma} - \phi\mu(b,s)} \tag{A.37}$$

$$\mu(b,s)\left[\frac{b'(b,s)}{R} + \phi q(b,s)\right] = 0 \tag{A.38}$$

$$c(b,s) + \frac{b'(b,s)}{R} = y + b \tag{A.39}$$

$$V(b,s) = \frac{c(b,s)^{1-\gamma} - 1}{1-\gamma} + \frac{\beta}{\sigma}\ln \mathbb{E}\left\{\exp\{\sigma V(b',y')\}\right\} \tag{A.40}$$

where $m(b,s)$ is the expectation distortion increment. The solution method works over the following steps:

1. We set a grid $G_b = \{b_1, b_2, \ldots, b_H\}$ for the next-period bond holding b'; and a grid $G_s = \{s_1, s_2, \ldots, s_N\}$ for the shock state space $s = \{y, \sigma\}$. The income process y, is discretized with Tauchen and Hussey (1991) method, while the grid for the *inverse* of the penalty parameter σ (recall that θ is the inverse of σ) follows a simple two-state rule:[3]

$$\sigma = \begin{cases} \sigma^{+} & \text{if} \quad V < \mathbb{E}\{V\} \\ \sigma^{-} & \text{if} \quad V \geq \mathbb{E}\{V\} \end{cases} \tag{A.41}$$

2. In iteration step k, we start with a set of policy functions $c_k(b,s)$, $q_k(b,s)$, $\mu_k(b,s)$ and $V_k(b,s)$. For each $b' \in G_b$ and $s' \in G_s$:

   a) we derive the expectation distortion increment:

   $$m_k(b',s') = \frac{\exp\{\sigma V_k(b',s')\}}{\mathbb{E}\left[\exp\{\sigma V_k(b',s')\}\right]} \tag{A.42}$$

   and then, the distorted expectations in the Euler equation for bonds and for the risky assets (equations (1) and (2)).

   b) we solve the system of optimality conditions under the assumption that the collateral constraint is slack:

   $$\mu^u(b',s) = 0 \tag{A.43}$$

   As a result, $c^u(b',s)$, $q^u(b',s)$, $\mu^u(b',s)$, $V^u(b',s)$ and $b^u(b',s)$ are the policy functions for the unconstrained region;

   c) in the same way, we solve the system for the constrained region of the state space, where the following condition holds:

   $$q^c(b',s) = -\frac{b'/R}{\phi} \tag{A.44}$$

[3] We use 800 grids point for bonds and 45 grid points for the exogenous shocks; we implement linea interpolation in order to approximate the policy functions outside the grids

$c^c(b', s)$, $q^c(b', s)$, $\mu^c(b', s)$, $V^c(b', s)$ and $b^c(b', s)$ are the respective policy functions.

d) we derive the next period bond holding threshold $\bar{b}'$ such that the borrowing constraint is marginally binding. For each $s \in G_s$ it satisfies the following condition:

$$\bar{b}'^c(\bar{b}', s) + \frac{\bar{b}'(s)}{R} = 0 \tag{A.45}$$

When this point is out of the grid we use linear interpolation. Given this value, we can derive for each policy function the frontier between the binding and non-binding region: $x^u(\bar{b}'^c(\bar{b}', s)$ for $x = \{c, b, q, \mu, V\}$.

3. In order to construct the step k+1 policy function, $x_{k+1}(b, s)$, we interpolate on the pairs $(x^c(b'^c(b', s))$ in the constraint region, and on the pairs $(x^u(b'^u(b', s))$ in the unconstrained region. As a result we find: $c_{k+1}(b, s)$, $q_{k+1}(b, s)$, $\mu_{k+1}(b, s)$ and $V_{k+1}(b, s)$

4. We evaluate convergence. If

$$\sup ||x_{k+1} - x_k|| < \epsilon \qquad \text{for} \qquad x = c, q, \mu, V \tag{A.46}$$

we find the competitive equilibrium. Otherwise, we set $x_k(b, s) = (1 - \delta)x_{k+1}(b, s) + \delta x_k(b, s)$ and continue the iterations from point 2. We use a value of δ close to 1.

A.6 Data Description for Empirical Moments

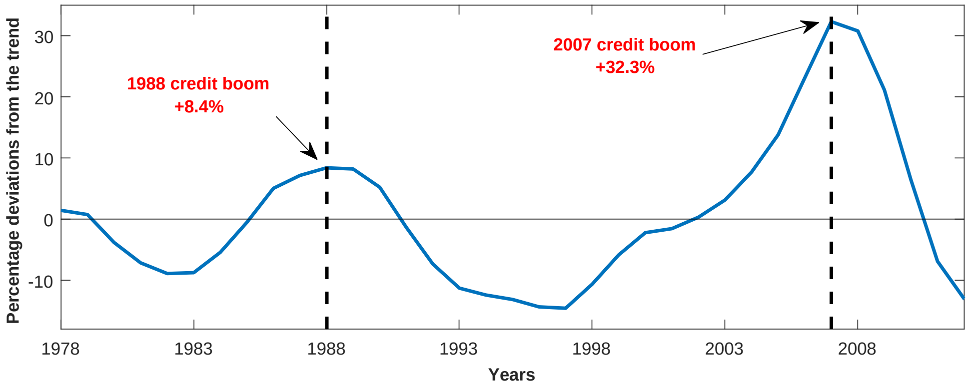
In this section we describe the data employed for the computation of the empirical moments used for model matching. We compute several moments for asset prices, returns and debt data. Data are from the US. The used sample spans 1980:Q1 to 2016:Q4, since this corresponds to the period of rapid debt growth The dataset is composed as follows: debt is given by private non-financial sector, by all sectors (BIS: http://www.bis.org/publ/qtrpdf/r_qt1403g.pdf); consumption is given by Personal Consumption Expenditure (NIPA Tables[4]), GDP (NIPA Tables); the risk-free rate is the 3month T-bill rate (CRSP Indices database[5]); risky returns are proxied by the $S\&P500$ equity return with dividends (Shiller Database[6]). All variables are deflated by CPI index. Note that HP-filtered series are computed as deviations from a long-term trend. Therefore, we work with a much larger smoothing parameter ($\lambda = 400,000$) than the one employed in the business cycle literature, to pick up the higher expected duration of the credit cycle (see http://www.bis.org/publ/bcbs187.pdf).

[4]See https://www.bea.gov/iTable/index_nipa.cfm.

[5]See http://www.crsp.com/products/research-products/crsp-historical-indexes.

[6]See http://www.econ.yale.edu/~shiller/data.htm

Figure A.1: US leverage cycle and credit booms



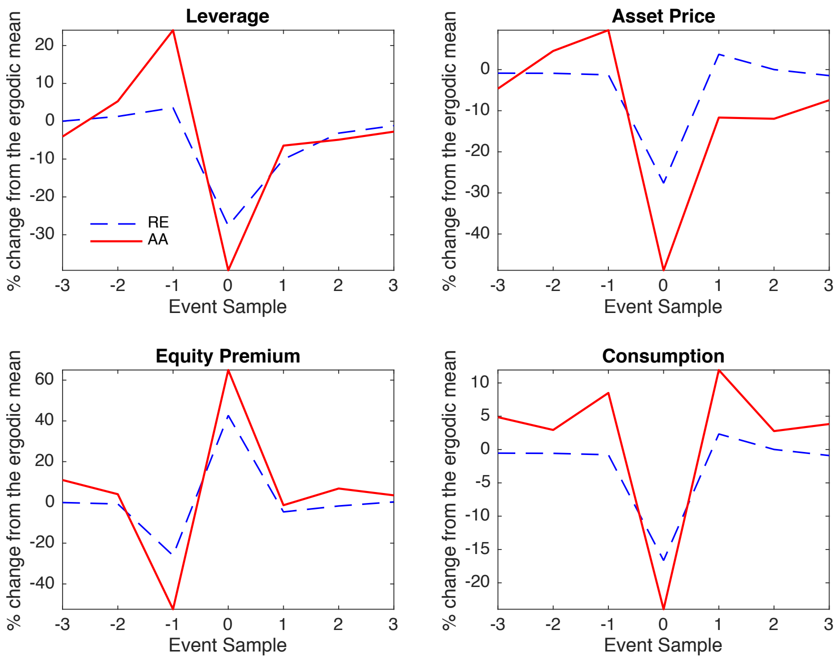
Note: In the chart we plot the deviation of the aggregate credit with respect to the long terms trend. The series is detrended applying the HP-filtered with a smoothing parameter λ equal to 400.000. therefore, we set a larger value for this parameter with respect to the one employed in the business cycle literature, in order to pick up the higher expected duration of the credit cycle. Further details on the time series used can be found in the . The two credit boom, indicated with the dotted vertical lines, are identified following Mendoza and Torrones (2012).

A.7 Intermediation Sector and Intermediation Shocks

Lack of transparency and ambiguity play an important role in crises developments as we showed so far, but by no means instability stemming from the intermediation sector, hence originating in the credit supply, has a major role too. This is particularly true within the context of the 2007-2008 financial crisis. While including all possible sources of intermediation disincentives is beyond the scope of this paper, we nevertheless wish to assess the role of the intermediation channel. This is important as one should test whether the beliefs-related channels described so far persist even when the supply side of credit is inserted in the model. In fact, we find that not only the role of ambiguity attitudes is preserved, but in most cases is amplified and the interaction with the intermediation channels is compelling.

We introduce intermediation by assigning the role of debt monitoring to a bank. This is actually realistic since atomistic lenders do not monitor or screen debtors individually, but largely assign this function to an intermediary. In this context the collateral constraint results from the bank design of a debt contract that is incentive compatible, meaning that it reduces the incentives of the borrower to divert resources and default. We formalize this type of contracts and show how the collateral constraint emerges from such incentive compatibility constraint in Appendix A.2 Within this context an intermediation shock, which suddenly tightens the supply of credit, affects the parameter governing the loan-to-value ratio, ϕ, which itself governs the strength of the incentive problems. Intuitively the shock can be interpreted in two ways, both affecting the contractual agreement in a similar vein. It could capture financial innovation in the form of derivatives and/or asset back securities issuance, which being pervasive prior to the crisis, allowed banks to off-load credit risk and reduced the tightness of the debt contract. A sudden freeze of the asset backed market liquidity due for instance to the sub-prime shock would have then induced a sudden fall in ϕ. A second interpretation, linked to the first, is that higher availability of liquidity[7] prior to the crisis had lessened banks' monitoring incentives, something which resulted in higher loan-to-value ratios, ϕ. After the crisis occurs, the squeeze in liquidity, hence banks' funding, could suddenly tightens the loan-to-value ratio. Both interpretations, which are realistic particularly in the context of the recent financial crisis, have the effect of producing a sudden tightening of credit supply. Within this context we subject our model to an intermediation shock to ϕ and assess its role as well as its interaction with ambiguity attitudes. We do so by analysing again policy functions, crisis events and second moments of the model.

Figure A.2: Crises Event Study with income and intermediation shock



Before proceeding to the assessment of the quantitative results, a few words are needed regarding the calibration of the shock. We define a high and a low level of the loan-to-value ratio, respectively $\phi_l = 0.22$ and $\phi_h = 0.28$, calibrated in order to match the empirical volatility of debt. The shock then follows a two-state regime-switching Markov process, with a transition matrix calibrated to replicate the empirical probability and duration of the crises events, as in Bianchi and Mendoza Bianchi and Mendoza (2018). More in detail, the probability to remain in a high state, π_{hh} is set equal to 0.955 in order to match a frequency of crises close to 4%, while the transition probability from a low to high state π_{lh} is equal to one, implying a one year duration of the crises. The remaining transition probabilities are set as complements of the previous ones, i.e $\pi_{hl} = 1 - \pi_{hh}$ and $\pi_{ll} = 1 - \pi_{lh}$.

We start from the crisis event study. Figure A.2 compares the crisis event in the model with ambiguity attitudes and with rational expectations. The crisis event is defined as before,

[7]This again could be due either to the possibility of raising additional bank liabilities through asset backed securities or through the ample availability of liquidity in interbank and repos markets prior to the 2007-2008 crisis.

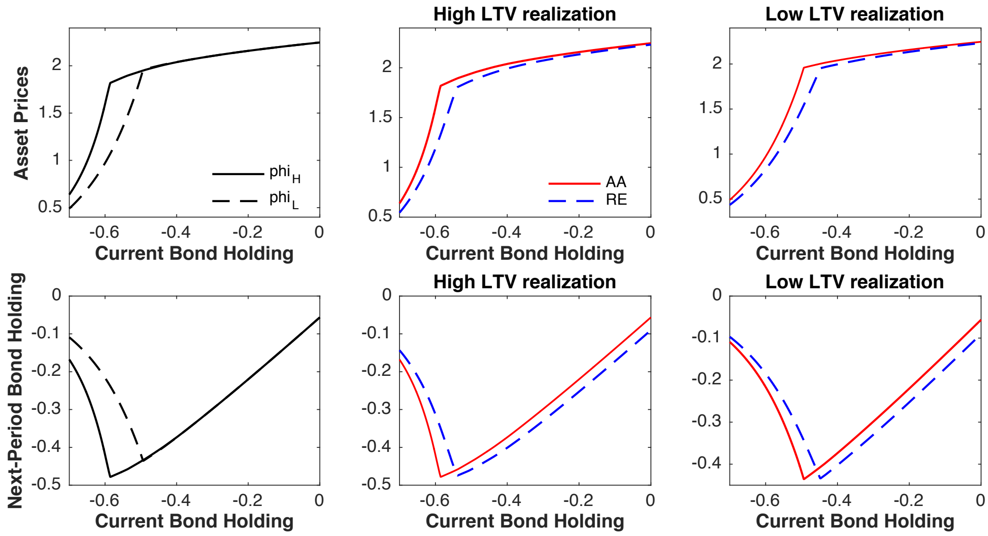
but now it is triggered by a combination of income and intermediation shocks. Specifically, we simulate the model in response to both shocks, we then observe that the crisis originates exactly when both shocks turn negative. The Figure shows strongly that the role of ambiguity attitudes remains. It is still true that beliefs formation by affecting the value of collateral through endogenous skewed beliefs induce sharper crises than under the case with no ambiguity. Given the calibration of the intermediation states values, the difference between the crises event study of the baseline model is negligible.

To examine more in details the intermediation channel we examine the policy functions for debt and asset prices. Figure **??** below shows the policy functions conditional to positive realizations of the income shock for asset prices and debt by comparing various scenarios. In the first column we compare the model with ambiguity attitudes for two values of ϕ. This case allows us to isolate only the contribution of credit supply. As before the kink represents the turn in which the constraint shifts from binding to non-binding. The comparison shows that a low ϕ, namely tight credit due to high monitoring standards or low availability of liquidity, has two effects. On the one side, it enlarges the constrained region. On the other side, it reduces leverage, and this effect can be beneficial in the medium to long run. The second and the third columns compare the models with and without ambiguity attitudes, respectively for low levels of ϕ (second column) and high levels of ϕ (third column). Two interesting observations emerge. First, as before under the model with ambiguity attitudes asset prices are higher and debt displays the previously underlined nonlinear dynamics over constrained and unconstrained regions. This as before is due to the nature of the positive skewed beliefs that emerge under positive income shocks. Second, the comparison between a high and a low level of ϕ shows that the qualitative pattern of the policy functions remains unaltered, albeit the constrained region is expanded under the low loan to value ratio. In other words, the forces operating through the ambiguity channel remain active even when introducing supply side elements. The dominant effect of the latter is more evident in terms of changes in the size of the constrained region. To fully complete the assessment of the policy functions, Figure A.3b shows the results for the policy functions conditional on negative income realizations. The message is largely symmetric to the one described above.

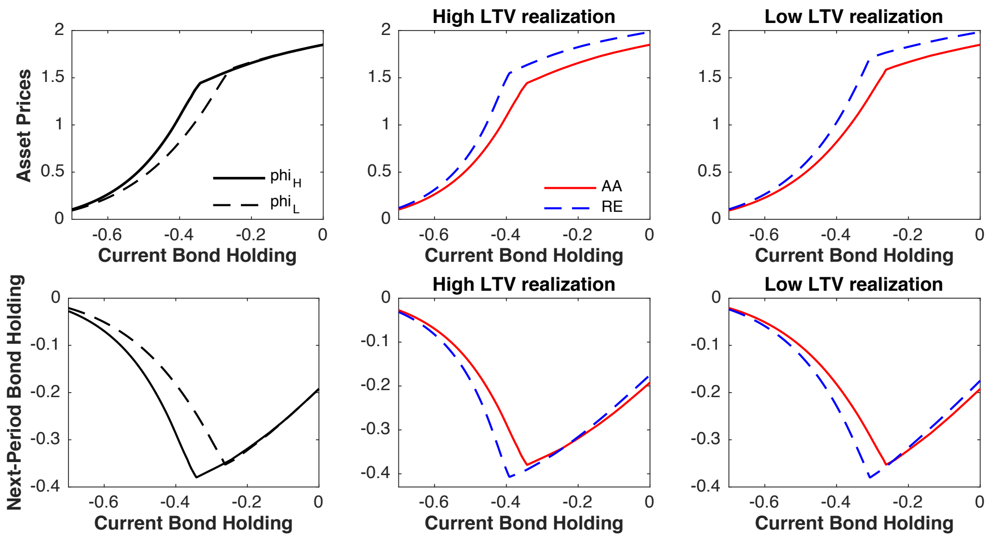
At last, we ask whether the introduction of the intermediation shock can improve upon the moment matching and if so along which dimension. Table**??** below shows again the comparison of a selected numbers of second moments between the data, the model with and without ambiguity attitudes. This time the comparison is done by simulating the model also in response to the intermediation shock. The addition of the intermediation shock preserves most of the previous moments and improves in terms of data matching along other dimensions. The Table highlights primarily moments that change with the introduction of the intermediation shock. The most noteworthy result is that the introduction of credit supply fluctuations increases debt pro-cyclicality, which as discussed before, is an important stylized fact. The reason is intuitive. The double occurrence of the negative income and credit supply shock tightens leverage much more sharply. Equally the double-coincidence of the positive income and credit supply realizations heightens the build-up of leverage. Those movements on the tails help to increase average

Figure A.3: Policy Functions for the model with intermediation

(a) Positive intermediary shock realization



(b) Negative intermediary shock realization



pro-cyclicality. The volatility of debt is also somewhat higher, mostly so in the model with ambiguity attitudes, and is closer to the data value. This again might be due to the contribution of the tails. On the other side, it shall be mentioned that the introduction of the intermediation shock worsens the volatility of risky returns, which now goes above the one detected in the data. This effect is possibly due to the fact that our model does not account for loss absorption capacity of the intermediation sector in terms of equity capital and/or liquidity buffers. Those elements would indeed limit the extent of fire sales in risky assets when credit supply tightens, hence they would reduce fluctuations in asset prices.

Table A.1: Empirical and model-based moments

Moments	Mnemonics	Empirical	AA	AA + shock[1]	RE + shock
Debt volatility	σ^b	12.52	12.37	11.55	9.78
Debt persistence	ρ^b	0.846	0.539	0.432	0.385
Debt cyclicality	$Corr(\Delta b_t, \Delta c_t)$	0.668	0.378	0.792	0.795
Equity return	$E_t(R_t^s)$	9.38	8.19	8.67	7.88
Equity return volatility	$\sigma^{R_t^s}$	16.21	17.46	23.45	19.40
Equity return cyclicality	$Corr(\Delta R_t^s, \Delta c_t)$	0.474	0.989	0.983	0.992

[1] *Column 4 shows the theoretical moments of the* AA *model without the intermediation shock. Instead columns 5 and 6 show the theoretical moments for the* AA *and* RE *models with the intermediation shocks. For the the* AA *and* RE *specifications a different moment matching exercises are run, then the two models could differs in the parameter values.*

To sum up the main contribution of the intermediation channel in our model is that of modifying the size of the constrained versus the unconstrained region and facilitate the matching of the probability of financial crises.

A.8 Three Period Model (work in progress)

In this section we outline an extended version of the three period model with occasionally binding collateral constraints and with ambiguity attitudes. The goal is to show the combined effect of those two elements on debt growth. The economy we consider is populated by a continuum of agents, who live for three periods: $t = 0, 1, 2$. Preferences are given by the following specification:

$$U = u(c_0) + \mathbb{E}_S\left[\beta u(c_1) + \beta^2 u(c_2)\right] \tag{A.47}$$

where $u(c) = \frac{1}{2}[\bar{c} - c]^2$. In period 0 we can assume a linear utility function $u(c_0) = c_0$ in order to simplify the analysis. We also assume that $\beta R = 1$. The endowment structure is characterized as follows. Agents receive endowment income in period 1 and 2, but none in period 0. In period 1 the endowment is stochastic depending on the realization of the state $s \in S$. We assume that $S = \{s_1, s_2, ...s_N\}$ is a monotone increasing sequence. The realization of the endowment are affected monotonically from the realization of s, so that for example $y^{s_n} > y^{s_{n-1}}$. The probability that a state s occurs is given by π_s. Similarly to the main text we assume that the dividend is lead by the same source of volatility. This allows us to simplify the state space. Therefore, in each period a fraction $(1-\alpha)y_t$ is the labor income, and the fraction $d_t = \alpha y_t$ is the dividends' income. The budget constraints for each period reads as follows:

$$c_0 + q_0 x_0 + \frac{b_0}{R} = 0 \tag{A.48}$$

$$c_1^s + q_1^s x_1^s + \frac{b_1^s}{R} = (1-\alpha)y_1^s + x_0(q_1^s + \alpha y_1^s) + b_0 \tag{A.49}$$

$$c_2^s = (1-\alpha)y_2 + x_1 \alpha y_2 + b_1^s \tag{A.50}$$

Note that the sup-index s in period 1 indicates that uncertainty materializes in this period. We have assumed that $b_{-1} = b_2 = 0$, $x_{-1} = x_2 = 0$, $q_2 = 0$ and $d_{-1} = 0$. In period 1 the collateral constraint limits the amount of debt:

$$-\frac{b_1^s}{R} \leq \phi q_1^s x_1^s \tag{A.51}$$

The agents expectation formation process is derived as in the main text. Since uncertainty refers to period 1 income, agents form expectation in period 0. Their optimal likelihood ratio in period 0 is given by:

$$m_1^s = \frac{\exp\{\sigma_0 V_1^s\}}{\mathbb{E}_0\left\{\exp\{\sigma_0 V_1^s\}\right\}} \tag{A.52}$$

where the value function recursion is defined as following[8]: $V_1^s = u(c_1^s) + \beta u(c_2^s)$. The relation that links the level of m_1^s to the state of the economy is:

$$\text{if} \qquad V_1^s < \mathbb{E}_0\left\{V_1^s\right\} \qquad \text{then} \qquad m_1^s > 1 \tag{A.53}$$

Given the above optimization problems the *decentralized equilibrium* is characterized as follows. The bonds' Euler equations between periods 0 and 1 and between periods 1 and 2, read as follows:

$$1 = \beta R \mathbb{E}_0\left\{m_1^s u_c(c_1^s)\right\} \tag{A.54}$$

$$u_c(c_1^s) = \beta R u_c(c_2^s) + \mu_1^s \tag{A.55}$$

The Euler conditions on the risky asset between periods 0 and 1 and between periods 1 and 2 read as follows:

$$q_0 = \beta \mathbb{E}_0\left\{m_1^s u_c(c_1^s)[q_1^s + \alpha y_1^s]\right\} \tag{A.56}$$

$$q_1^s = \beta \frac{u_c(c_2^s)\alpha y_2}{u_c(c_1^s) - \phi \mu_1^s} \tag{A.57}$$

[8] This simplified representation is obtained under the assumption that there is no uncertainty in period 2.

The complementarity slackness condition is:

$$\mu_1^s \left[\frac{b_1^s}{R} + \phi q_1^s \right] = 0 \tag{A.58}$$

Finally, the decentralized equilibrium is closed with a condition on expectations, equation (1.3), and the following market clearing conditions:

$$c_0 + q_0 + \frac{b_0}{R} = 0 \tag{A.59}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.60}$$

$$c_2^s = y_2 + b_1^s \tag{A.61}$$

where we have imposed the stock market clearing condition $x_t = 1$.

A.8.1 Time 1 Continuation Equilibrium

We now proceed to the model solution by backward induction. We start from period the last period and since there is no uncertainty between time 1 and time 2 we can solve for the two periods simultaneously. We start from characterizing the continuation value under the *unconstrained region.* The system of equilibrium conditions for the unconstrained region (the sup-index U will be used since now on to indicate the solution for this region) is (we can use $\beta = R^{-1}$ and $\mu_1 = 0$):

$$u_c(c_1^s) = u_c(c_2^s) \qquad c_1^s = c_2^s = c^{U,s} \tag{A.62}$$

$$q_1^s = \beta \frac{u_c(c_2^s)}{u_c(c_1^s)} \alpha y_2 \tag{A.63}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.64}$$

$$c_2^s = y_2 + b_1^s \tag{A.65}$$

Given the above the consumption function depends on lifetime wealth and reads as follows:

$$c^{U,s} = \frac{1}{1+\beta} \left(y_1^s + b_0 + \frac{y_2}{R} \right) \tag{A.66}$$

Using the budget constraint and the consumption function one can derive the optimal level of debt:

$$b_1^U(s) = \frac{\beta}{1+\beta} \left(y_1(s) + b_0 - \frac{y_2}{R} \right) \tag{A.67}$$

Finally, the equilibrium asset price condition, which depends on the value of the dividend in the last period, reads as follows:

$$q_1 = \beta \alpha y_2 \tag{A.68}$$

In the *constrained region* ($\mu_t > 0$, the sup-index C is used since now onward to indicate equilibrium values for this region)), the system of equilibrium conditions reads as follows:

$$\mu_1^s = u_c(c_1^s) - u_c(c_2^s) \qquad c_1^s < c_2^s \tag{A.69}$$

$$q_1^s = \beta \frac{u_c(c_2^s)}{u_c(c_1^s) - \phi \mu_1^s} \alpha y_2 \tag{A.70}$$

$$c_1^s + \frac{b_1^s}{R} = y_1^s + b_0 \tag{A.71}$$

$$c_2^s = y_2 + b_1^s \tag{A.72}$$

$$\frac{b_1^s}{R} = -\phi q_1^s \tag{A.73}$$

A.8.2 Time Zero Equilibrium

To characterize the time 0 equilibrium we first partition the state space into two blocks, S^C and S^U, where the constraint is binding and slack respectively. Assuming that the $u(c_0) = c_0$ we have:

$$1 = \sum_{s \in S^U} \pi_s m_1^{U,s} u_c^{U,s}(b_0; y_1, y_2)) + \sum_{s \in S^C} \pi_s m_1^{C,s} u_c^{C,s}(b_0; y_1, y_2)) \tag{A.74}$$

$$q_0 = \beta \left\{ \begin{array}{c} \sum_{s \in S^U} \pi_s m_1^{U,s} u_c^{U,s}(b_0; y_1, y_2))[q_1^U + y_1^s] \\ + \sum_{s \in S^C} \pi_s m_1^{C,s} u_c^{C,s}(b_0; y_1, y_2))[q_1^{C,s}(b_0; y_1, y_2) + y_1^s] \end{array} \right\} \tag{A.75}$$

$$c_0 = -\frac{b_0}{R} - q_0 \tag{A.76}$$

where $c_1^{i,s}, b_1^{i,s}, q_1^{i,s}$ are the solutions of the time 1 continuation equilibrium.

A.8.3 The Expectation Distortion under a Binomial State Space

Our goal is to assess the role of ambiguity attitudes on debt growth. To this purpose we shall derive a closed form solution for policy functions. To do that we assume a simple binomial structure for the state space. Hence we assume that the state space is comprised of two states, which we label high, with sup-index h, occurring with probability π, and low, with sup-index l, occurring with probability $(1 - \pi)$. The exogenous state space therefore reads as follows $S = \{h, l\}$. We assume that the in state h the income realization is high enough that the collateral constraint is slack. Similarly we assume that in state l, the income realization is low enough that the collateral constraint binds. Given this structure for the objective probability, the expectation distortions are given by:

$$m_1^s = \frac{\exp\{\sigma_0 V_1^s\}}{\pi \exp\{\sigma_0 V_1^h\} + (1-\pi)\exp\{\sigma_0 V_1^l\}} \tag{A.77}$$

where the value function has the following form, $V_1^s = u(c_1^s) + \beta u(c_2^s)$. Given the assumptions on the state space, it follows that:

$$V_1^h > \mathbb{E}_0\{V_1^s\} \quad \text{and} \quad V_1^l < \mathbb{E}_0\{V_1^s\} \tag{A.78}$$

Equation (A.77) and jointly imply that, if $\theta_0 > 0$, hence $\sigma_0 = -\frac{1}{\theta_0} < 0$, the following holds:

$$\exp \sigma_0 V_1^h < \mathbb{E}_0 \{\exp \sigma_0 V_1^s|\} \quad \Rightarrow \quad m_1^h < 1 \tag{A.79}$$

$$\exp \sigma_0 V_1^l > \mathbb{E}_0 \{\exp \sigma_0 V_1^s\} \quad \Rightarrow \quad m_1^l > 1 \tag{A.80}$$

Intuitively the above implies that agents assign an higher subjective probability (with respect to the objective) to the bad history and a lower probability to the good history. We can call this behaviour *pessimism*. Similarly if $\theta_0 < 0$, then $\sigma_0 = -\frac{1}{\theta_0} > 0$, we have that:

$$\exp \sigma^0 V_1^h > \mathbb{E}_0 \{\exp \sigma_0 V_1^s\} \quad \Rightarrow \quad m_1^h > 1 \tag{A.81}$$

$$\exp \sigma_0 V_1^l < \mathbb{E}_0 \{\exp \sigma_0 V_1^s\} \quad \Rightarrow \quad m_1^l < 1 \tag{A.82}$$

Note that in this second case agents assign an higher subjective probability to the good history and a lower probability to the bad history, depicting borrowers' *optimistic* behaviour. We shall now solve the equilibrium and derive the implied debt policy functions under the above beliefs' structure. We start by characterizing the equilibrium at time zero, given by the optimal decisions (b_0, c_0, q_0). We also compare the two solutions to the case with rational expectations. The debt policy function is best characterized by the following relation:

$$b_0 = -R[c_0 + q_0] \tag{A.83}$$

Next to characterize the time 0 policy function for consumption we rely on the Euler equation between period 0 and period 1:

$$u_c(c_0) = \pi m_1^h u_c(c_1^h) + (1-\pi) m_1^l u_c(c_1^l) \tag{A.84}$$

We can reformulate the above equation in terms of the subjective weights of the ambiguity averse agent:

$$u_c(c_0) = \psi^h u_c(c_1^h) + (1-\pi)\psi^l u_c(c_1^l) \tag{A.85}$$

where $\psi^h = \pi m_1^h$ and $\psi^l = (1-\pi)m_1^l$. Given the model structure (incomplete financial markets, hence lack of insurance to equalize consumption), the events structure and the condition on the collateral constraint, we can conclude that:

$$c_1^h > c_1^l \quad \Rightarrow \quad u_c(c_1^h) < u_c(c_1^l) \tag{A.86}$$

Next, recall that in the *optimism* case beliefs imply:

$$\psi^h = \pi m_1^h > \pi \tag{A.87}$$

$$\psi^l = (1-\pi) m_1^l < (1-\pi) \tag{A.88}$$

This implies that agents assign a higher weight, with respect to the RE case, to the component $u_c(c_1^h)$. Hence, the marginal utility of consumption in $t = 0$ is lower (than under rational expectations) and the consumption is higher:

$$c_0^o > c_0^{RE} \tag{A.89}$$

where c_0^o indicates consumption under optimism behaviour, while c_0^{RE} indicates consumption under no ambiguity. Intuitively agents assign higher weight to good future states, hence they prefer to postpone consumption and to invest in the risky asset. This in turn will raise asset price, since the demand of asset has increased. As investment takes place through leverage, they will also leverage more. In the *pessimism* case the borrower assigns the following weights:

$$\psi^h = \pi m_1^h < \pi \tag{A.90}$$

$$\psi^l = (1-\pi)m_1^l > (1-\pi) \tag{A.91}$$

This implies:

$$c_0^u < c_0^{RE} \tag{A.92}$$

where c_0^u indicates consumption under pessimistic behaviour. In this case the agent expects more likely the bad state to take place in the future. The agent will then anticipate consumption and invest less in the risky asset. They will in turn leverage less. We can generalize this relation with the following condition:

$$c_0^o > c_0^{RE} > c_0^u \tag{A.93}$$

Appendix B

Appendix to Chapter 2

B.1 Model Dynamics

This section describes the model dynamics looking at the IRFs associated to the main exogenous shocks of the model: demand, leverage, monetary policy and macroprudential policy. First, we analysis the dynamics of the closed economy and, then, we look at the interactions among the two symmetric countries. In the latter exercise, we show how higher degrees of trade and financial integration increase the synchronization between countries.

B.1.1 Closed Economy

Demand Shock

Figure B.1a displays the impulse responses to a 1% positive demand shock for two different policy regimes: the only monetary policy regime (black solid line) and the monetary & macroprudential policy regime (blue dashed line). After the demand shock, the aggregate credit (leverage) surges, triggering a steep rise in the credit spread. As a consequence, the response of the output gap turns to be negative after few periods in the model without macroprudential policy. Instead, when the macroprudential policy is active, she reacts to the rise of the credit spread in order to tame the credit growth. In doing so, the economic expansion is made safer and more stable. Indeed, the output gap response remain positive over the entire period.

Leverage Shock

Figure B.1b displays the impulse responses to a 1% positive leverage shock under the same two policy regimes. Few words are needed regarding the interpretation of the variable L_t and the identification of the associated shock. The steady state value of the credit must be interpreted as the *maximum safe level* of private credit, and therefore, quantities of credit above this limit produces strong demand costs. Indeed, as we can see in the charts, the economy experiences crises after the shock materialization characterized by significant collapse in output and inflation. In this situation, macroprudential policy reduces the severity and the duration of the crises, restricting the credit growth and controlling the credit spread.

Policy Interventions

Figure B.1c shows the effect of monetary policy restriction, defined as by a 1% positive monetary policy shock ($\varepsilon_t^M P$) and its interaction with the macroprudential policy. The IRFs reflect qualitatively those of the Curdia and Woodford (2016) model (see Fig. 3, pag 46). In this case the macroprudential policy, supporting the credit growth, makes the monetary restrictions less harsh. Comparing the reactions of the macroprudential authorities to the demand shock and to the monetary restriction allows us to understand the countercyclical nature of this policy. After the demand shock, the macroprudential instrument rises producing a restriction of the credit. During a monetary contraction, instead, the macroprudential authority tries to reduce the impact of the higher policy rate on the credit growth. Finally, figure B.1d shows as the macroprudential authority can neutralize the effects of the credit spreads contracting the credit growth.

B.1.2 The open economy

This sections analysis the transmission of the shocks studied above from the domestic to the foreign country. The IRFs are calculated for different degrees of financial and trade integration in order to compare the relative role of the two international channels in increasing the business and financial synchronization of the two countries. Looking at the IRFs of the demand and leverage shocks we can appreciate the difference between the two international transmission channels give by the trade and the financial flows. The synchronization in the countries' responses to the demand shock (Figure B.2) is generate by a high degree in the trade integration $\tilde{\alpha}_Y$, that however is not enough to generate synchronization of the financial (leverage) shock (Figure B.3). The latter is mainly transmitted through the financial flows, as the bottom panels of Figure B.3 show. Indeed, only for a high degree of financial integration $\tilde{\alpha}_L \geq 0.5$ the synchronization of the two countries start to be strong.

Finally, Figure B.4 and Figure B.4 show the transmission of the monetary and macroprudential policies from the domestic to the foreign countries. As we can notice, the transmission of the macroprudential policy is stronger and passes mainly trough the financial channel. Indeed, Figure B.4 shows that until $\tilde{\alpha}_L = 0$ the effect of the domestic macroprudential policy on the foreign variables is treasurable. Instead, for the highest level of synchronization, domestic macroprudential is able to spread strongly the effects on the other country, in particular in terms of output and credit spread reduction.

B.2 Within-Country analysis

B.2.1 The role of international integration: decomposing total gains

This section complements the open economy within-country analysis of section 2.4.3. Figure B.6 and Figure B.6 display how the degree of trade and financial integration affects the loss functions in the cooperative and Nash equilibrium, as well as the gains from cooperation, differentiating between the two authorities. Two main comments are worth mentioning. First, higher degrees of trade openness implies lower gains from cooperation for both the monetary and the macroprudential analysis (see Figure B.6) . Second, the financial integration, instead, increases the value of the cooperation from both the authority-specific and country point of views. However, the combination of high degree of financial and trade openness can reduce the gains from cooperation for the monetary authority (see Figure B.7).

Figure B.1: Closed Economy

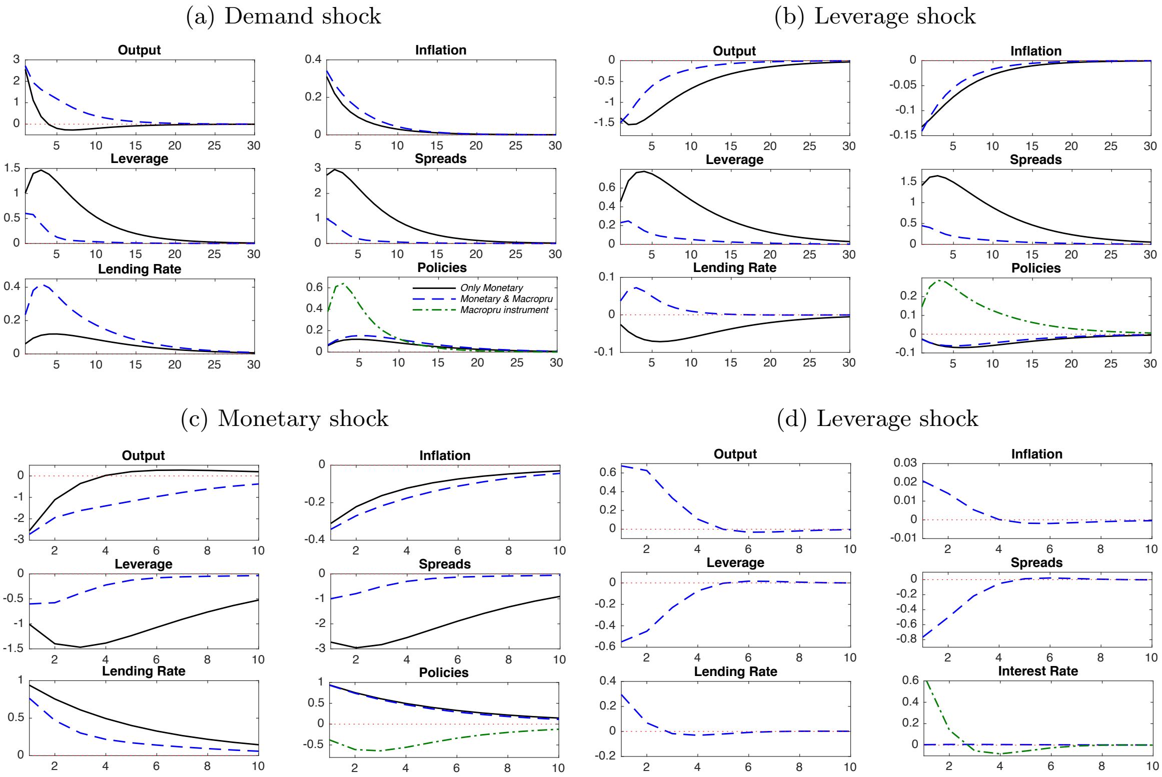
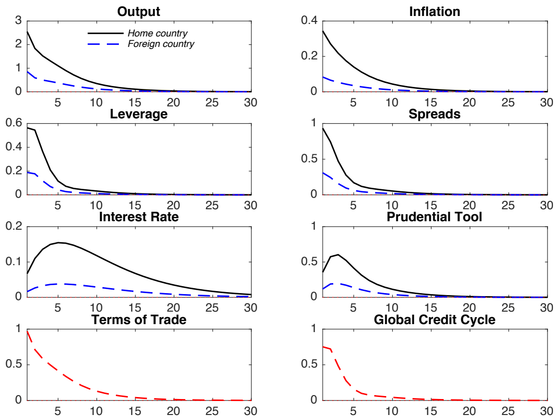
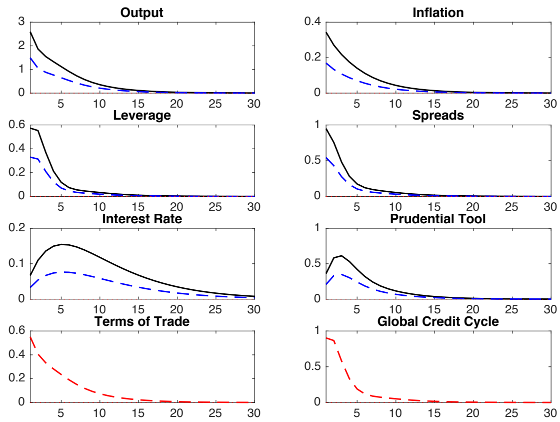


Figure B.2: Open Economy: Demand Shock

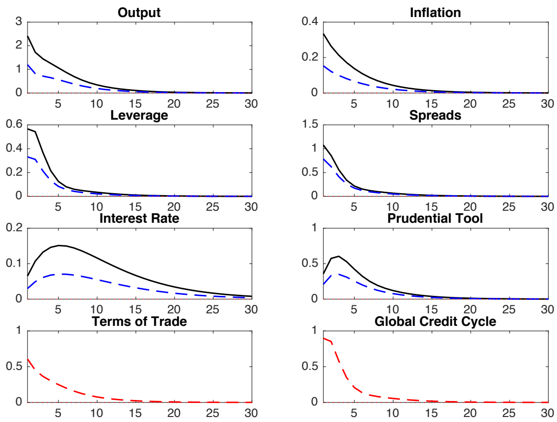
(a) $\alpha_Y = 0.5$, $\alpha_L = 0$



(b) $\alpha_Y = 1$, $\alpha_L = 0$



(c) $\alpha_Y = 1$, $\alpha_L = 0.5$



(d) $\alpha_Y = 1$, $\alpha_L = 1$

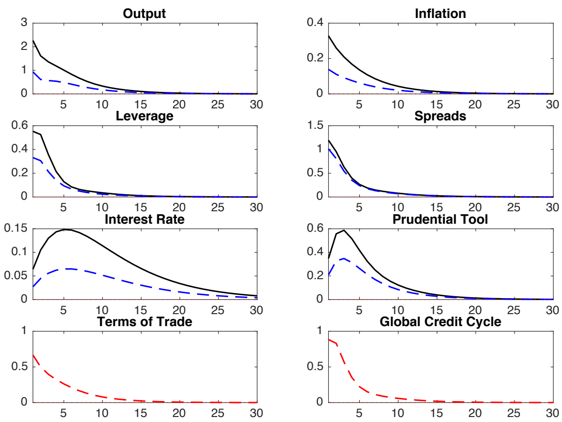
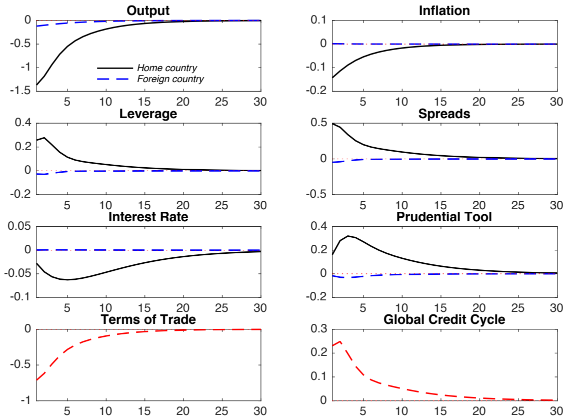
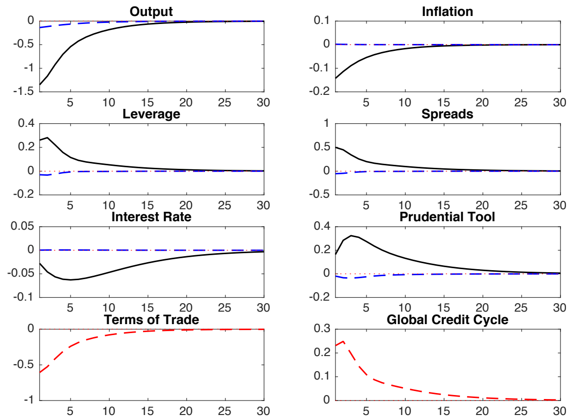


Figure B.3: Open Economy: Leverage Shock

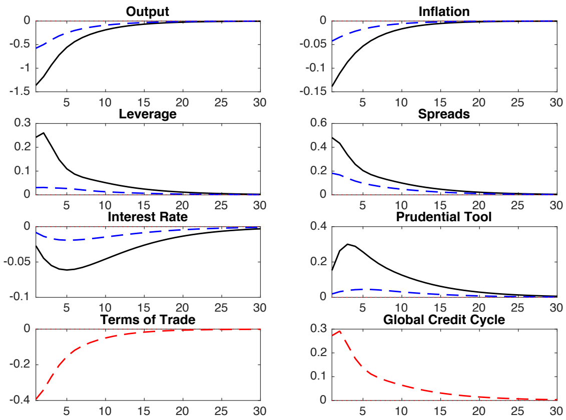
(a) $\alpha_Y = 0.5$, $\alpha_L = 0$



(b) $\alpha_Y = 1$, $\alpha_L = 0$



(c) $\alpha_Y = 1$, $\alpha_L = 0.5$



(d) $\alpha_Y = 1$, $\alpha_L = 1$

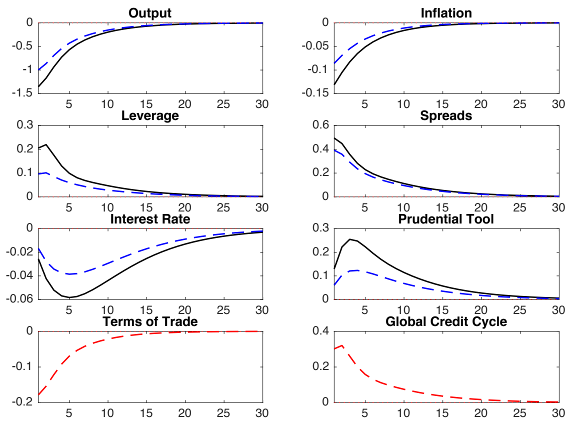
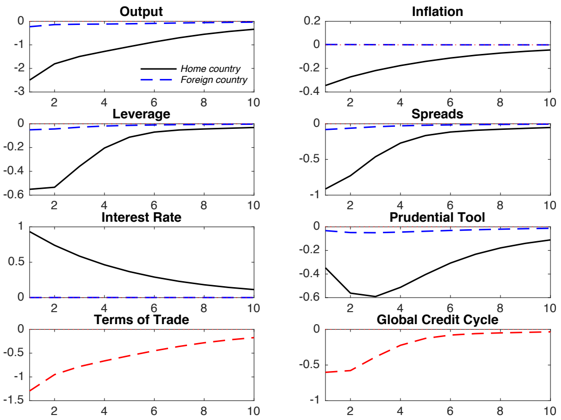
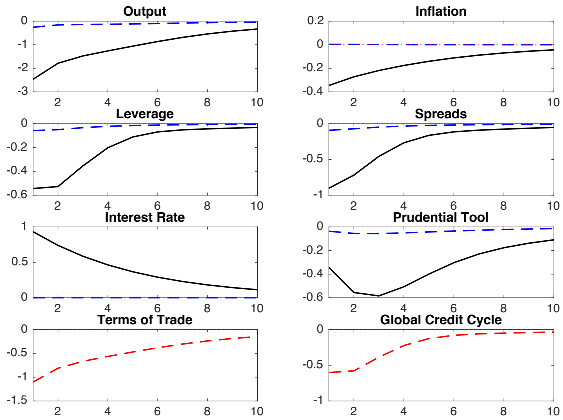


Figure B.4: Open Economy: Monetary Shock

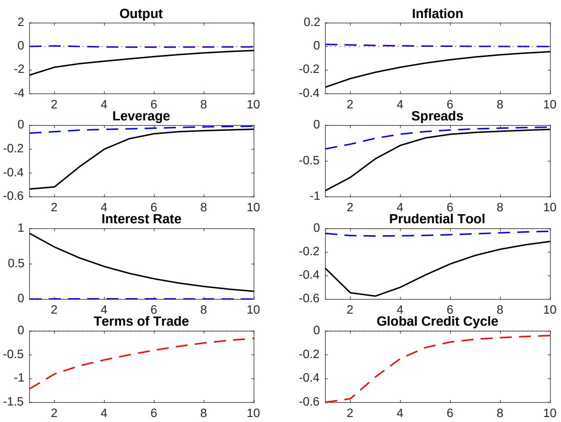
(a) $\alpha_Y = 0.5$, $\alpha_L = 0$



(b) $\alpha_Y = 1$, $\alpha_L = 0$



(c) $\alpha_Y = 1$, $\alpha_L = 0.5$



(d) $\alpha_Y = 1$, $\alpha_L = 1$

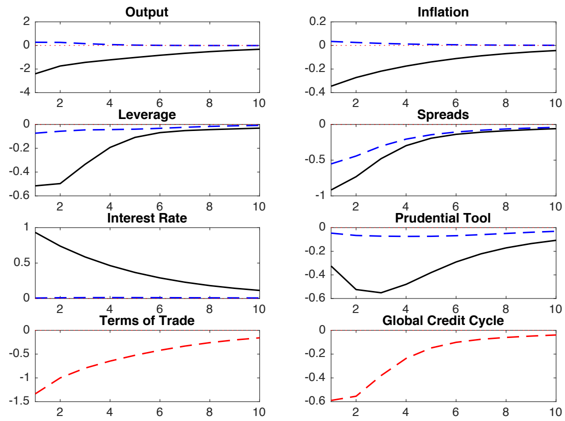
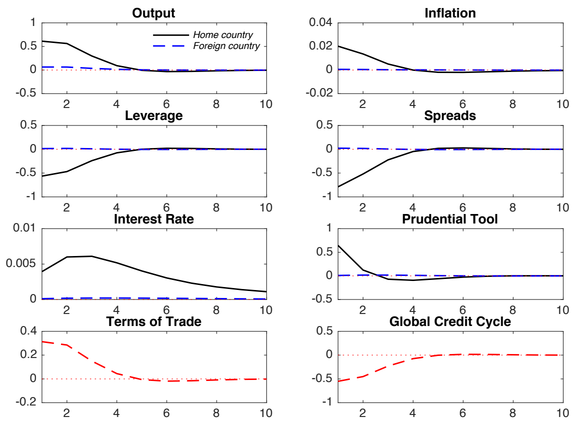
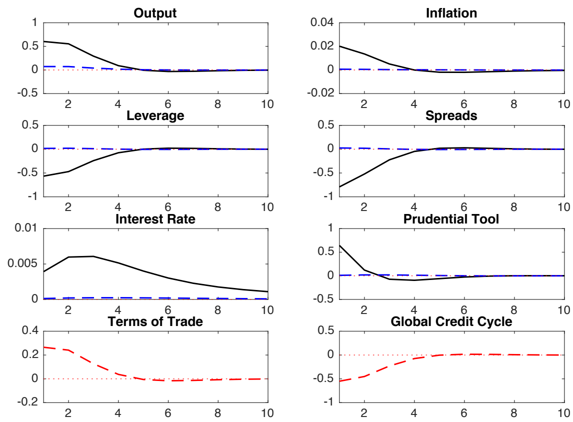


Figure B.5: Open Economy: Macroprudential Shock

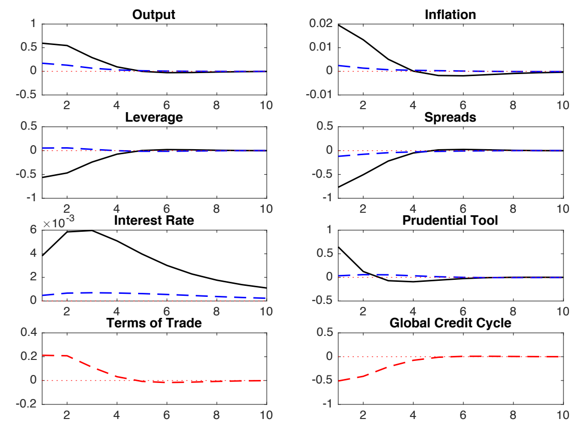
(a) $\alpha_Y = 0.5$, $\alpha_L = 0$



(b) $\alpha_Y = 1$, $\alpha_L = 0$



(c) $\alpha_Y = 1$, $\alpha_L = 0.5$



(d) $\alpha_Y = 1$, $\alpha_L = 1$

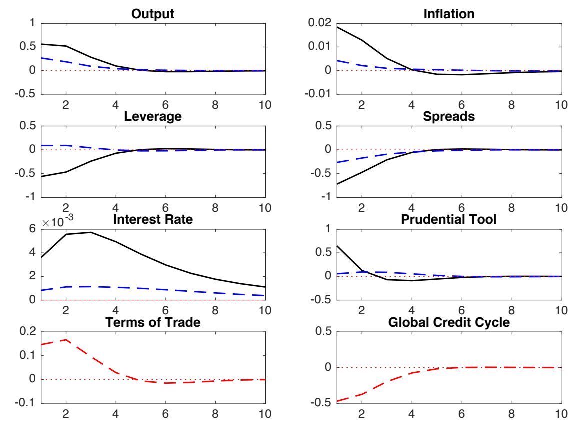
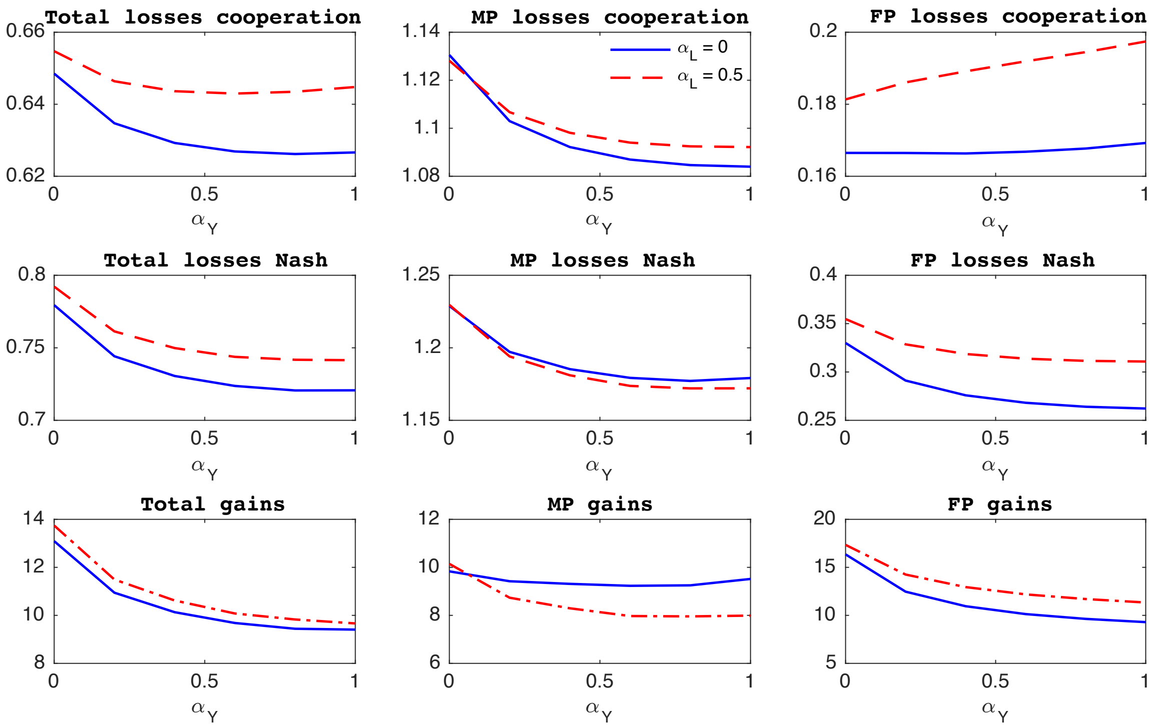
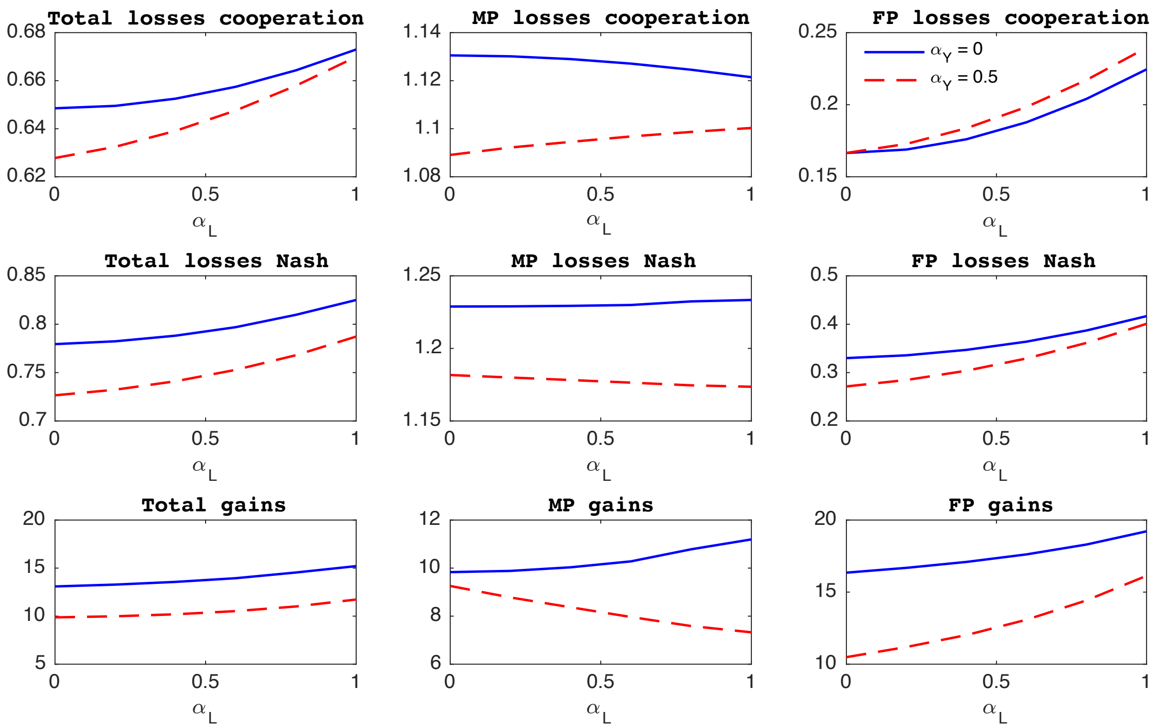


Figure B.6: Gains decomposition and trade openness



Note: the upper panels display how the total loss functions and the authorities' loss functions in the cooperative equilibrium are affected by the degree of trade openness (α_Y). The middle panels replicate the exercise for the Nash equilibrium. The lower panels show how the percentage difference between the two equilibria changes with respect to the degree of openness.

Figure B.7: Gains decomposition and financial openness



Note: the upper panels display how the total loss functions and the authorities' loss functions in the cooperative equilibrium are affected by the degree of trade openness (α_L). The middle panels replicate the exercise for the Nash equilibrium. The lower panels show how the percentage difference between the two equilibria changes with respect to the degree of openness.

Appendix C

Appendix to Chapter 3

C.1 A technical note on the MacroFin Copula

The expression for each measure is provided first in a general formula and then the particular one under the Gaussian methodology, which gives us a closed form formula.

C.1.1 Expected Shortfall (ES)

The $VaR_x(\alpha)$ gives information about how large is the minimum return for the variable x with $(1-\alpha)100\%$ confidence level. It is obtained by solving the implicit equation

$$P_{t-1}\left[r_x \leq VaR_x(\alpha)\right] = \alpha. \tag{C.1}$$

Equation (C.1) under Gaussian assumptions is

$$VaR_x(\alpha) = \mu_x - \sigma_x \Phi^{-1}(\alpha) \tag{C.2}$$

where Φ^{-1} is the inverse standardized cumulative Gaussian distribution function. For the smoothed empirical distribution would be

$$VaR_x(\alpha) = F_x^{-1}(\alpha),$$

The Value-at-Risk only looks at a certain quantile, consequently it is not a sub-additive measure. The properties of this risk measure can be enhanced if we look further than the quantile of interest for the VaR. The Expected Shortfall tells us how large are the average return in the financial market if these losses are higher than $-VaR_x(\alpha)$, i.e.,

$$\begin{aligned} ES_x(\alpha) &= \mathbb{E}_x\left[r_x | r_x < VaR_x(\alpha)\right] \\ &= \frac{1}{\alpha}\int_0^{\alpha} VaR_x(s) \quad ds \end{aligned} \tag{C.3}$$

where for the Gaussian case we have a closed form without computing numerically the integral

$$ES_x(\alpha) \quad = \quad \mu_x - \sigma_x \alpha^{-1} \phi(\Phi^{-1}(\alpha). \tag{C.4}$$

where ϕ is the probability standardized Gaussian distribution function.

Expected Shortfall (ES) under Gaussian framework Equation (C.4) can be rewritten in a Gaussian framework using VaR definition provided in (C.2), i.e.

$$\begin{aligned} ES_x(\alpha) &= \frac{1}{\alpha} \int_0^\alpha \mu_+ - \sigma_x \Phi^{-1}(s) \quad ds \\ &= \mu_x + \frac{\sigma_x}{\alpha} \int_0^\alpha \Phi^{-1}(s) \quad ds. \end{aligned}$$

Consequently, the problem is reduced to the integration of the inverse cumulative Gaussian distribution function from 0 to α. Define a change of variable $s = \Phi(r)$, then $ds = \phi(r)dr$ so $\int_0^\alpha \Phi^{-1}(s) \quad ds = \int_{-\infty}^{\Phi^{-1}(\alpha)} r\phi(r) \quad dr$ where ϕ is the probability Gaussian distribution function. Subsequently,

$$\begin{aligned} \int_{-\infty}^{\Phi^{-1}(\alpha)} r\phi(r) \quad dr &= \int_{-\infty}^{\Phi^{-1}(\alpha)} \frac{r}{\sqrt{2\pi}} \exp(-r^2/2) \quad dr \\ &= \frac{1}{\sqrt{2\pi}} \left[-\exp(-r^2/2) \right]_{-\infty}^{\Phi^{-1}(\alpha)} \\ &= -\phi(\Phi^{-1}(\alpha)). \end{aligned}$$

As a result the ES is

$$ES_x(\alpha) \quad = \quad \mu_x - \frac{\sigma_x}{\alpha} \phi(\Phi^{-1}(\alpha)).$$

C.1.2 Conditional Mean Response (CMR)

The Conditional Mean Response of a variable y is the mean loss of the variable y when variable x is below its $VaR_x(\alpha)$, i.e.

$$\begin{aligned} CMR_y(\alpha) &= E\left(r_y | r_x < VaR_{x,t}(\alpha)\right) \\ &= \int_0^1 P(F_y(r_y) = s | r_x < VaR_x(\alpha)) F_y^{-1}(s) \quad ds, \end{aligned} \tag{C.5}$$

where F_y is the cumulative distribution function of variable y and F_y^{-1} is its inverse. For the Gaussian case, the CMR expression is

$$MES_y(\alpha) \quad = \quad \mu_y - \frac{\sigma_y \rho_{y,x} \phi\left(\Phi^{-1}(\alpha)\right)}{\alpha}. \tag{C.6}$$

Conditional Mean Response (CMR) in a Gaussian framework $r = (r_x, r_y)'$ can be expressed as

$$\begin{pmatrix} r_x \\ r_y \end{pmatrix} = \underbrace{\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}}_{\mu} + \underbrace{\begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}}_{D^{1/2}} \underbrace{\begin{pmatrix} 1 & 0 \\ \rho_{x,y} & \sqrt{1-\rho_{x,y}^2} \end{pmatrix}}_{L_t} \begin{pmatrix} \Phi^{-1}(U_x) \\ \Phi^{-1}(U_y) \end{pmatrix} \tag{C.7}$$

where L_t matrix represents Choleski decomposition and $\rho_{x,y}$ is the correlation parameter between x and y. U_x and U_y are uniform independent distributed variables while Φ^{-1} is the inverse cumulative Gaussian distribution function.

The vector r is normally distributed with mean μ and covariance matrix $D^{1/2}L_tL_t'D^{1/2}$. Given a value for the triggering variable r_x, the distribution of the stationary transformation of y becomes $r_y|r_x \; N\left(\mu_y + \frac{\sigma_y \rho_{x,y}}{\sigma_x}(r_x - \mu_x), \sqrt{1-\rho_{x,y}^2}\sigma_y\right)$, where N refers to the Gaussian distribution where the first input is the mean ($\mu_{y|x}$) and the second one is the standard deviation ($\sigma_{y|x}$).

If the realization of r_x is expressed in terms of quantiles, i.e. $r_x = \Phi^{-1}(q)\sigma_x + \mu_x$, the mean value of r_y given that r_x is in its q quantile is $\mu_y + \sigma_y \rho_{x,y}\Phi^{-1}(q)$, i.e. $E_{t-1}(r_y|r_x = VaR_x(q))$. Then, the mean value of r_y given that r_x is at most in its α quantile would be

$$E(r_y|r_x < VaR_x(\alpha)) = \mu_y + \sigma_y \rho_{x,y} \frac{\int_0^\alpha \Phi^{-1}(q)dq}{\alpha}.$$

Because of the solution of previous integral, the CMR expression is

$$CMR_y(\alpha) = \mu_y - \frac{\sigma_y \rho_{x,y} \phi\left(\Phi^{-1}(\alpha)\right)}{\alpha}.$$

C.1.3 Conditional Expected Shortfall ($CoES$)

The Conditional Expected Shortfall of variable y given that variable x is below its quantile α is expressed as

$$\begin{aligned} CoES_y(\alpha,\beta) &= E\left(r_y|r_y < CoVaR_x(\alpha,\beta)\right) \\ &= \frac{1}{\beta}\int_0^{s^*} P(F_y(r_y) = s|r_x < VaR_x(\alpha))F_y^{-1}(s) \quad ds, \end{aligned} \tag{C.8}$$

where s^* is such that $P(F_y(r_y) < s^*|r_x < VaR_x(\alpha)) = \beta$. In a Gaussian framework this expression can be rewritten as

$$CoES_y(\alpha,\beta) = \mu_y - \sigma_y\left(\sqrt{1-\rho_{x,y}^2}\frac{\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \rho_{x,y}\frac{\phi(\Phi(\alpha))}{\alpha}\right), \tag{C.9}$$

Conditional Expected Shortfall (CoES) in a Gaussian framework From Equation (C.6) and taking under consideration the representation of r in Equation (C.7), Equation (**??**)

can be rewritten as

$$\begin{aligned}
CMR_y(\alpha) &= \mu_y - \sigma_y \left\{ \frac{\rho_{x,y}\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha} - \sqrt{1-\rho_{x,y}^2}\left(\int_0^\beta \Phi^{-1}(q)dq + \int_\beta^1 \Phi^{-1}(q)dq\right)\right\} \\
&= \underbrace{\mu_y - \sigma_y \frac{\rho_{x,y}\phi\left(\Phi^{-1}(\alpha)\right)}{\alpha}}_{\mu_{y|x}} + \\
&\quad \overbrace{\sigma_y\sqrt{1-\rho_{x,y}^2}}^{\sigma_{y|x}} \underbrace{\left(\frac{1}{\beta}\int_0^\beta \Phi^{-1}(q)dq\right)}_{E(A)} \overbrace{\beta}^{P(A)} + \\
&\quad \overbrace{\sigma_y\sqrt{1-\rho_{x,y}^2}}^{\sigma_{y|x,t}} \underbrace{\left(\frac{1}{1-\beta}\int_\beta^1 \Phi^{-1}(q)dq\right)}_{E_{t-1}(A^C)} \overbrace{(1-\beta)}^{P(A^C)}
\end{aligned}$$

where

$$\begin{aligned}
E(A) &= E\left(\frac{(r_y-\mu_{y|x})}{\sigma_{y|x}} | r_y < CoVaR_{y|x}(\alpha,\beta), r_x < VaR_x(\alpha)\right), \\
P(A) &= P(r_y < CoVaR_{y|x} | r_x < VaR_x(\alpha)), \\
E(A^C) &= E\left(\frac{(r_y-\mu_{y|x})}{\sigma_{y|x}} | r_y > CoVaR_{y|x}(\alpha,\beta), r_x < VaR_x(\alpha)\right) \text{ and} \\
P(A^C) &= P(r_y > CoVaR_{y|x} | r_x < VaR_x(\alpha)).
\end{aligned}$$

From the solution of these integrals,

$$\begin{aligned}
E(A) &= \frac{-1}{\beta}\phi\left(\Phi^{-1}(\beta)\right) \\
E(A^C) &= \frac{1}{1-\beta}\phi\left(\Phi^{-1}(\beta)\right).
\end{aligned}$$

Consequently

$$CoES_{y|x}(\alpha,\beta) = \mu_y - \sigma_y\left(\frac{\sqrt{1-\rho_{x,y}^2}\phi\left(\Phi^{-1}(\beta)\right)}{\beta} + \frac{\rho_{x,y}\phi(\Phi(\alpha))}{\alpha}\right),$$

C.2 Data Description and Source

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