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A pricing technique to calculate the Solvency Capital Requirement for non-life Premium Risk

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Contents

INTRODUCTION


ONONDAGA
MUTUAL FIRE INSURANCE CO.
THIS POLICY WITNESSETH,
That Whereas,
Onondaga County Mutual Insurance Company.

Figure 1: 1844-06-20 - Onondaga County Mutual Insurance Company – Policies.

Onondaga County New York Mutual Fire Insurance Co. 1844 policy, issued to Truman Woodford & Philip Alexander on June 1844. This policy is for coverage of Dwelling House & Barn. Handwriten name of town appears to be Tully located in the County of Onondaga.

"Each human activity is the result of a long chain of events, of the fate, opportunity, chances and the continuous modest work of millions of men: there is no human adventure which not involves risk"[1].

Early methods of transferring or distributing risk were practiced by Chinese and Babylonian traders as long ago as the 3rd and 2nd millennium BC, respectively. Chinese merchants travelling treacherous river rapids would redistribute their wares across many vessels to limit the loss due to any single vessel's capsizing. The Babylonians

[1] http://www.golinucci.it/

developed a system which was recorded in the famous Code of Hammurabi, c. 1750 BC, and practiced by early Mediterranean sailing merchants. If a merchant received a loan to fund his shipment, he would pay the lender an additional sum in exchange for the lender's guarantee to cancel the loan should the shipment be stolen or lost at sea.

Achaemenian monarchs of Ancient Persia were the first to insure their people and made it official by registering the insuring process in governmental notary offices. The insurance tradition was performed each year in Norouz (beginning of the Iranian New Year); the heads of different ethnic groups as well as others willing to take part, presented gifts to the monarch. The most important gift was presented during a special ceremony. When a gift was worth more than 10 000 Derrik (Achaemenian gold coin) the issue was registered in a special office. This was advantageous to those who presented such special gifts. For others, the presents were fairly assessed by the confidants of the court. Then the assessment was registered in special offices. The purpose of registering was that whenever the person who presented the gift registered by the court was in trouble, the monarch and the court would help him. Jahez, a historian and writer, writes in one of his books on ancient Iran: "*Whenever the owner of the present is in trouble or wants to construct a building, set up a feast, have his children married, etc. the one in charge of this in the court would check the registration. If the registered amount exceeded 10 000 Derrik, he or she would receive an amount of twice as much*".

A thousand years later, the inhabitants of Rhodes invented the concept of the 'general average'. Merchants whose goods were being shipped together would pay a proportionally divided premium which would be used to reimburse any merchant whose goods were deliberately jettisoned in order to lighten the ship and save it from total loss.

The Greeks and Romans introduced the origins of health and life insurance c. 600 AD when they organized guilds called "benevolent societies" which cared for the families and paid funeral expenses of members upon death. Guilds in the Middle Ages served a similar purpose. The Talmud deals with several aspects of insuring goods. Before insurance was established in the late 17th century, "friendly societies" existed in

England, in which people donated amounts of money to a general sum that could be used for emergencies.

Separate insurance contracts (i.e., insurance policies not bundled with loans or other kinds of contracts) were invented in Genoa in the 14th century, as were insurance pools backed by pledges of landed estates. These new insurance contracts allowed insurance to be separated from investment, a separation of roles that first proved useful in marine insurance. Insurance became far more sophisticated in post-Renaissance Europe, and specialized varieties developed.

Some forms of insurance had developed in London by the early decades of the seventeenth century. For example, the will of the English colonist Robert Hayman mentions two "policies of insurance" taken out with the diocesan Chancellor of London, Arthur Duck. Of the value of £100 each, one relates to the safe arrival of Hayman's ship in Guyana and the other is in regard to "one hundred pounds assured by the said Doctor Arthur Ducke on my life". Hayman's will was signed and sealed on 17 November 1628 but not proved until 1633. Toward the end of the seventeenth century, London's growing importance as a center for trade increased demand for marine insurance. In the late 1680s, Edward Lloyd opened a coffee house that became a popular haunt of ship owners, merchants, and ships' captains, and thereby a reliable source of the latest shipping news. It became the meeting place for parties wishing to insure cargoes and ships, and those willing to underwrite such ventures. Today, Lloyd's of London remains the leading market for marine and other specialist types of insurance, but it works rather differently than the more familiar kinds of insurance.

Insurance as we know it today can be traced to the Great Fire of London, which in 1666 devoured more than 13 000 houses. The devastating effects of the fire converted the development of insurance "from a matter of convenience into one of urgency, a change of opinion reflected in Sir Christopher Wren's inclusion of a site for "the Insurance Office" in his new plan for London in 1667." A number of attempted fire insurance schemes came to nothing, but in 1681 Nicholas Barbon and eleven associates, established England's first fire insurance company, the "Insurance Office for Houses", at the back of the Royal Exchange. Initially, 5 000 homes were insured by Barbon's Insurance Office.

The first insurance company in the United States underwrote fire insurance and was formed in Charles Town (modern-day Charleston), South Carolina, in 1732. Benjamin Franklin helped to popularize and make standard the practice of insurance, particularly against fire in the form of perpetual insurance. In 1752, he founded the Philadelphia Contributionship for the Insurance of Houses from Loss by Fire. Franklin's company was the first to make contributions toward fire prevention. Not only did his company warn against certain fire hazards, it refused to insure certain buildings where the risk of fire was too great, such as all wooden houses.

It’s clear that one of the key elements of insurance is security, but what would happen if the insurance company is unable to pay the cost of the claim?

In this context, the solvency of insurance company is fundamental.

The pioneering works done by Cornelis Campagne in the Netherlands at the end of the 1940s and by Teivo Pentikäinen in Finland in the beginning of the 1950s are important, as they introduced the solvency research for insurance undertakings. Before the term solvency was introduced, a concept like statutory reserves was often used, which have been formed in the course of years and which serve as an extra guarantee for fulfilling the obligations undertaken. Initially, Campagne called this type of reserve for life insurance for a stabilization reserve. In Finland a special equalization reserve was introduced in 1953 to take account of the stochastic fluctuations in the annual claims amount in non-life insurance. During the 1950s Campagne enlarged the solvency assessment to non-life insurance. As Campagne’s work became leading for the approach of assessing an extra minimum reserve for both life and non-life companies he was asked to present a report on solvency (“Minimum Standards of Solvency for Insurance Firms”) in 1957 to the OEEC[2] Insurance Committee. As a chairman of a working group within the Insurance Committee his work was developed and a final report was presented in 1961.

In life insurance the approach adopted was the same as in the 1940s. As the risk on investments is the most important factor for life insurance companies and as the technical provisions are the most important invested amount, Campagne considered a

[2] Organization for European Economic Co-operation;

minimum solvency margin as given by a percentage of the technical provisions. Campagne asked “how great has the extra reserve to be, so that with a probability smaller than 0.01 respectively 0.001 this can be expressed to be insufficient for the financing of investment losses and deviations of foundations; in which case furthermore distinctions have to be made between cases in which the stabilization reserve has to be sufficient for one year or more years.”. Campagne concluded that an extra reserve of 6% of the technical provision would be adequate with a probability of 99%.

With a probability of 95% the percentage of the extra reserve became 4% and this was the extra reserve proposed by Campagne. It was implemented in the first life directive within the European Union in 1979.

In non-life insurance the model was simple but elegant. Let the net retained premium be 100%. From this we deduct a constant fraction equal to the average expense ratio[3] (fixed to 42%). The remaining part is what remains for claims payment. With data from different European countries he estimated the Value-at-Risk[4] of the loss ratio at 0.9997% as 83%. Thus the combined ratio[5] will be 42% + 83% =125%. In other words the company will need an extra 25% of the premium during 1 year to meet the requirements. After further works during the 1960s and political negotiations this framework became the base for the first non-life directive in Europe in 1973. Research on solvency assessment was initiated as many countries in Europe had got the non-life and life directives during the 1970s implicating minimum solvency margins. Work was done in e.g. United Kingdom, the Netherlands, but also in Finland.

The research and works done were all stepwise towards a risk based capital (RBC) approach.

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[3] Expenses divided by premiums;

[4] Value at Risk (VaR) is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) in the given probability level.

[5] The combined ratio is comprised of the claims ratio and the expense ratio. The claims ratio is claims owed as a percentage of revenue earned from premiums. The expense ratio is operating costs as a percentage of revenue earned from premiums. The combined ratio is calculated by taking the sum of incurred losses and expenses and then dividing them by earned premium.

In Europe the Life insurance directives (EEC 1979) and the non-life insurance directives (EEC 1973) can be considered the starting point of a formal set of solvency requirements that insurance companies were required to fulfill in a free market. The approach adopted those days were simple and straight forward to operate. Solvency assessment was based on simple factors and formulae that were applied on accounting results after adjustment for reinsurance. The findings of Müller report[6] and the work done by a few other committees paved the way for the introduction of Solvency I in the EU in the year 2002. It introduced some additional parameters in solvency evaluation. Solvency I provided a simple, but robust mechanism to regulate insurer solvency. It has improvements over the early day regulations, but still maintained its simplicity. A positive consequence of this was that it made the administration and compliance management easy and inexpensive. In spite of its relative simplicity, Solvency I did significantly increase the protection of the policyholders. However significant changes had taken place in the insurance industry, creating the need to adapt the rules appropriately, in addition the working document for Solvency I had already indicated the need for a better system which recognizes the various risks that an insurance company is exposed to in a more holistic manner. In some sense, Solvency I had already paved the way for the development of a more sophisticated approach. In the beginning of 2000, the Commission Services together with Member States initiated a fundamental and wide-ranging review of the overall financial position of an insurance undertaking: "The Solvency II Project". One of the objectives for the project is to establish a solvency system that is better matched to the true risks of an insurance company.

The insurance regulator in Switzerland (Federal Office of Private Insurance – "FOPI") was assigned the goal to ensure that the receivables of policyholders are protected. Historically (as in many other countries) this goal has been achieved with a combination of measures.

[6] H. Müller et al. (1997) Solvency of Insurance Undertakings, Conference of the Insurance Supervisory Services of the Member States of the European Union.

These include prudent reserving and pricing requirements as well as prescriptions over what assets are allowed to be held by insurance companies. On top of this, there is a requirement to meet a minimum solvency margin based on a simple standard formula.
In Switzerland the financial stability of several insurers has been shaken in the past few years. Events which have had significant adverse effects include the crash in the equity markets in 2001 and 2002, the steady fall in bond yields as well as the impact of increased longevity. These events have significantly reduced market values of equity investments, and at the same time have increased the value of some embedded options and guarantees which have been sold by insurers in the past, leading to required reserve increases. For some insurers, the effects of the fall in the equity markets have been compounded by deteriorating technical results and large catastrophe claims.
This has led to a number of changes in the way insurance companies are being regulated, monitored and valued around the world. This includes changes to accounting rules, increased requirements for corporate governance within insurance companies, and enhanced solvency regulations and standards.
Herbert Lüthy, director of the FOPI, embarked on an analysis project for the reorientation of insurance supervision in autumn 2002 with the support of a task force. At the same time, a draft Insurance Supervison Act (ISA) was elaborated, submitted to the Federal Council and subsequently tabled in Parliament. In reference to solvency, the bill states that the solvency requirement should take account of the risks to which an insurance company is exposed.
In spring 2003 the director of the FOPI initiated the Swiss Solvency Test (SST) project with the aim of defining basic principles of a future system for determining solvency. This was done in cooperation with the insurance industry, consulting companies and academia.
In Europe, the birth of Solvency II and Swiss Solvency Test has pushed insurance Companies to build their own models in order to represent consistently their risk profiles.

If the focus is on the non-life Premium Risk model, the perception is a lack of connection with actuarial best practices of pricing, for that reason the scope of this paper is to build an internal model that is able to connect strongly these two worlds.

The Premium Risk results from fluctuation in timing of frequency and severity, of insured events, which ensure that the premiums income will be not enough to pay future claims.

Since the level of the premiums income is based on actuarial techniques of pricing, which define the risk level of each profile in portfolio, it is clear that a coherent model, used to define the Premium Risk SCR, should be based on these best practices.

The model developed considers all these aspects and helps the Company Board to take decisions, answering to the following fundamental question: "What will happen in terms of expected profitability, loss ratio, SCR, size of portfolio if ....?"

In Chapter One we introduce the linear models, while in Chapter Two the generalized linear models are presented.

The journey of Solvency II is explained in Chapter Three, where the focus is mainly on the Non-Life Premium risk, and finally, in Chapter Four, a Frequency-Severity internal model is developed and the results are compared with Solvency II standard formula.

CHAPTER ONE

LINEAR MODELS

1.1 GENERAL LINEAR MODELS

In a general linear model (GLM), the observed value of the dependent variable y for observation number i $(i = 1,2,\cdots,n)$ is modeled as a linear function of $(p-1)$ so called independent variables $x_1, x_1, \cdots, x_{p-1}$ as

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_{(p-1)} x_{i(p-1)} + \varepsilon_i \qquad 1.1$$

Or in matrix terms

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \qquad 1.2$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{pmatrix}$$

is a vector of observations on the dependent variable,

$$\boldsymbol{X} = \begin{pmatrix} 1 & x_{11} & \dots & x_{1\,(p-1)} \\ 1 & x_{21} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \dots & x_{n\,(p-1)} \end{pmatrix}$$

is a matrix of dimension $n \cdot p$, that contains the values of the independent variables and a column of 1s corresponding to the intercept,

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_p \end{pmatrix}$$

is a vector containing p parameters that must be estimated, and

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{pmatrix}$$

is a vector of residuals.

The hypothesis underlying the model are:

- ε_i are independent
- ε_i are homoscedasticity
- $\varepsilon_i \sim N(0, \sigma^2)$.

1.2 ESTIMATION

In general linear models, estimation of parameters is usually done with the method of least squares.

The parameters are estimated with those values for which the sum of squared residuals is minimal

$$\beta : min \sum \varepsilon_i^2 \qquad 1.3$$

In matrix term the sum of squared residuals is

$$\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}' = (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \qquad 1.4$$

Minimizing 1.4 with respect to the parameters in $\boldsymbol{\beta}$ gives the normal equations

$$\boldsymbol{X}'\boldsymbol{X}\boldsymbol{\beta} = \boldsymbol{X}'\boldsymbol{y} \qquad 1.5$$

If the matrix $\boldsymbol{X}'\boldsymbol{X}$ is nonsingular, the $det(\boldsymbol{X}'\boldsymbol{X}) \neq 0$, the estimators of the parameters will be

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y} \qquad 1.6$$

If $det(\boldsymbol{X}'\boldsymbol{X}) = 0$ we can still find a solution, but it may not be unique.

1.3 THE MODEL FITTING

After the estimation of the parameters we must understand how the model fits the observed data. In order to reach this scope the variation in the data is subdivided in two parts: systematic variation and unexplained variation.

The fitted value of the response variable is:

$$\hat{y}_i = \sum_{j=0}^{p-1} \hat{\beta}_j \ x_{ij} \qquad 1.7$$

or in matrix terms

$$\hat{\boldsymbol{y}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}} \qquad 1.8$$

The difference between the observed value and the predicted value is the observed residual

$$\hat{\varepsilon}_i = y_i - \hat{y}_i \qquad 1.9$$

If the model had been perfect we would get residuals equal to zero.

We now introduce some parameters to understand better the fitting of our model.

First of all the total variation in the data can be measured as the total sum of squares

$$SS_T = \sum_i (y_i - \bar{y})^2 \qquad 1.10$$

Where $\bar{y}$ is the observed mean of the data.

This measure can be divided as:

$$\begin{aligned}\sum_i (y_i - \bar{y})^2 &= \sum_i (y_i + \hat{y}_i - \hat{y}_i - \bar{y})^2 \\ &= \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2 + 2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y})\end{aligned} \qquad 1.11$$

The last term can be shown to be zero, thus the total sum of squares $\mathrm{SS_T}$ is just composed by two elements:

$$SS_\varepsilon = \sum_i (y_i - \hat{y}_i)^2 \qquad 1.12$$

$$SS_{model} = \sum_i (\hat{y}_i - \bar{y})^2 \qquad 1.13$$

If the data fits well, the so called residual sum of squares - SS_ε -, will be small.

We can write everything in matrix terms as follow:

$$SS_T = \sum_i (y_i - \bar{y})^2 = \boldsymbol{y}'\boldsymbol{y} - n\bar{y} \; with \; n-1 \; degrees \; of \; freedom \; (df) \qquad 1.14$$

$$SS_\varepsilon = \sum_i (y_i - \hat{y}_i)^2 = \boldsymbol{y}'\boldsymbol{y} - \widehat{\boldsymbol{\beta}}\boldsymbol{X}' \; with \; n-p \; df \qquad 1.15$$

$$SS_{model} = \sum_i (\hat{y}_i - \bar{y})^2 = \widehat{\boldsymbol{\beta}}\boldsymbol{X}'\boldsymbol{y} - n\bar{y}^{\mathbf{2}} \; with \; p - 1 \; df \tag{1.16}$$

A descriptive measure of the fit of the model to data can be calculated as:

$$R^2 = {SS_{model}}/{SS_T} \tag{1.17}$$

R^2 is the coefficient of determination and has values comprises between 0 and 1.

For models with predicted values $\hat{y}_i$ always equal to the observed ones y_i, R^2 would be 1.

This is a useful coefficient, but it could be dangerous because it doesn't take into account the number of parameters used by the model.

To solve this problem we can use the adjusted R^2.This index decreases when irrelevant variables are added to the model, and it's defined as:

$$R^2_{adj} = 1 - \frac{n-1}{n-p}(1 - R^2) = 1 - {MS_\varepsilon}\Big/{{SS_T}/{(n-1)}} \tag{1.18}$$

Where

$$MS_\varepsilon = {SS_T}/{df}$$

This can be interpreted as the complementary of the variation estimated by the model and the variation estimated without any model.

In general linear models, parameter estimators are linear functions of the observed data. Thus, the estimator of any parameters, can be written as:

$$\beta_j = \sum_i \omega_{ij} y_i \qquad 1.19$$

Where ω_{ij} are known weights. Assuming that all y_i has the same variance σ^2 this makes it possible to obtain the variance of any parameter estimator as

$$Var(\hat{\beta}_j) = \sum_i \omega_{ij}^2 \sigma^2 \qquad 1.20$$

The variance σ^2 can be estimated from the data as :

$$\hat{\sigma}^2 = \frac{\sum_i \hat{\varepsilon}_i^2}{n-p} = MS_\varepsilon \qquad 1.21$$

The variance of a parameter estimator $\hat{\beta}_j$ can now be estimated as

$$\widehat{Var}(\hat{\beta}_j) = \sum_i \omega_{ij}^2 \hat{\sigma}^2 \qquad 1.22$$

Now we are able to calculate confidence intervals and to build some hypothesis tests about single parameters.

$$\begin{cases} H_0 \colon \beta_j = 0 \\ H_1 \colon \beta_j \neq 0 \end{cases} \qquad 1.23$$

Such test is made comparing:

$$t = \frac{\hat{\beta}_j}{\widehat{Var}(\hat{\beta}_j)} \qquad 1.24$$

With the appropriate percentage point of the t distribution with $n - p$ degrees of freedom.

Similarly,

$$\hat{\beta}_j \pm t_{\left(1-\frac{\alpha}{2}, n-p\right)} \sqrt{\widehat{Var}(\hat{\beta}_j)} \qquad 1.25$$

Would provide a $(1 - \alpha) \cdot 100\%$ confidence interval for the parameters β_j.

CHAPTER TWO

GENERALIZED LINEAR MODELS

2.1 GENERALIZED LINEAR MODEL

General linear models are limited in many ways, formally they rest on the assumptions of:

- normality
- linearity
- homoscedasticity.

Generalized linear models provide a unified approach to modeling different types of response variables:

- continuous variables
- binary variables
- proportion variables
- ....

In addition they are a generalization of general linear models. We can relax the assumption that the Y-components are independently normally distributed with constant variance and any distribution that belongs to the exponential family is admitted.

Instead of modeling directly $\mu = E(y)$ as a function of the linear predictors $\boldsymbol{X\beta}$, we model some function $g(\mu)$, thus the model becomes

$$g(\mu) = \eta = \boldsymbol{X\beta} \qquad 2.1$$

Where $g(\cdot)$ is called link function.

The specifications of a generalized linear model involve:

- specification of the distribution
- specification of the link function
- specification of the linear predictor $\boldsymbol{X\beta}$.

2.2 THE EXPONENTIAL FAMILY

The exponential family is a general class of distributions that includes many well-known distributions as special cases. It can be written in the form

$$f(y;\vartheta,\phi) = exp\left[\frac{y\vartheta - b(\vartheta)}{a(\phi)} + c(y,\phi)\right] \qquad 2.2$$

Where $a(\cdot), b(\cdot)\ and\ c(\cdot)$ are some functions specified in advance.

2.2.1 THE FUNCTION $\boldsymbol{b}(\cdot)$

Function $b(\cdot)$ is of special importance in generalized linear models because describes the relationship between the mean value and the variance in the distribution.

We consider Maximum Likelihood estimation of the parameters of the model and we denote the log likelihood function with $l(\vartheta;\phi,y) = logf(y;\vartheta,\phi)$. According to the likelihood theory it holds that:

$$E\left(\frac{\partial l}{\partial \vartheta}\right) = 0 \qquad 2.3$$

and that:

$$E\left(\frac{\partial^2 l}{\partial \vartheta^2}\right) + E\left[\left(E\left(\frac{\partial l}{\partial \vartheta}\right)\right)^2\right] = 0 \qquad 2.4$$

From 2.2 we obtain that $l(\vartheta; \phi, y) = \frac{y\vartheta - b(\vartheta)}{a(\phi)} + c(y, \phi)$, therefore:

$$\frac{\partial l}{\partial \vartheta} = \frac{y - b'(\vartheta)}{a(\phi)} \qquad 2.5$$

And

$$\frac{\partial^2 l}{\partial \vartheta^2} = \frac{-b''(\vartheta)}{a(\phi)} \qquad 2.6$$

Where $b'\ and\ b''$ denote the first and second derivative, respectively, of b with respect to ϑ. From 2.3 and 2.5 we get

$$E\left(\frac{\partial l}{\partial \vartheta}\right) = E\left(\frac{y - b'(\vartheta)}{a(\phi)}\right) = 0 \qquad 2.7$$

so that

$$E(y) = \mu = b'(\vartheta) \qquad 2.8$$

Thus the mean value of the distribution is equal to the first derivative of b with respect to ϑ.

From 2.4 and 2.6 we get

$$\frac{-b''(\vartheta)}{a(\phi)} + \frac{Var(y)}{a(\phi)^2} = \frac{-b''(\vartheta)}{a(\phi)} + E\left[\left(\frac{y - b'(\vartheta)}{a(\phi)}\right)^2\right] = \qquad 2.9$$

$$= \frac{-b''(\vartheta)}{a(\phi)} + \frac{1}{a(\phi)^2} E[(y-\mu)^2]$$

So that

$$Var(y) = a(\phi) \cdot b''(\vartheta) \qquad 2.10$$

We see that the variance of y is a product of two terms: the second derivative of b and the function $a(\phi)$ which is independent of ϑ. The parameter ϕ is called the dispersion parameter and $b''(\vartheta)$ is called the variance function.

2.2.2 THE POISSON DISTRIBUTION

The Poisson distribution can be written as a special case of an exponential family distribution. It has probability function

$$\begin{aligned} f(y;\mu) = \frac{\mu^y e^{-\mu}}{y!} &= e^{-\mu} \cdot e^{ylog(\mu)} \cdot e^{-log(y!)} \\ &= exp\{ylog(\mu) - \mu - log(y!)\} \end{aligned} \qquad 2.11$$

We can compare 2.11 with 2.2. We note that $\vartheta = log(\mu)$ which means that $\mu = exp(\vartheta)$. We insert this into 2.11 and get

$$f(y;\mu) = exp\{y\vartheta - exp(\vartheta) - log(y!)\} \qquad 2.12$$

Thus 2.11 is a special case of 2.2 with $\vartheta = log(\mu)$, $b(\vartheta) = exp(\vartheta)$, $c(y,\phi) = -log(y!)$ and $a(\phi) = 1$.

2.2.3 THE BINOMIAL DISTRIBUTION

The binomial distribution can be written as

$$f(y;p) = \binom{n}{y} p^y (1-p)^{n-y}$$
$$= exp\left\{ylog\left(\frac{p}{1-p}\right) + nlog(1-p) + log\binom{n}{y}\right\} \qquad 2.13$$

We use with $\vartheta = log\left(\frac{p}{1-p}\right)$, $p = \frac{\exp(\vartheta)}{1+\exp(\vartheta)}$ this can be inserted in 2.13 to give

$$f(y;p) = exp\left\{y\vartheta + nlog\left(\frac{1}{1+exp(\vartheta)}\right) + log\binom{n}{y}\right\} \qquad 2.14$$

It follows that the binomial distribution is an exponential family distribution with $\vartheta = log\left(\frac{p}{1-p}\right)$, $b(\vartheta) = nlog\left(\frac{1}{1+exp(\vartheta)}\right)$, $c(y,\phi) = log\binom{n}{y}$ and $a(\phi) = 1$.

2.2.4 THE NORMAL DISTRIBUTION

The normal distribution can be written as

$$f(y;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{-(y-\mu)^2}{2\sigma^2}\right\}$$
$$= exp\left\{\frac{y\mu - \frac{\mu^2}{2}}{\sigma^2} - \frac{y^2}{2\sigma^2} - \frac{1}{2}log(2\pi\sigma^2)\right\} \qquad 2.15$$

This is an exponential family distribution with $\vartheta = \mu, \phi = \sigma^2, a(\phi) = \phi, b(\vartheta) = \frac{\vartheta^2}{2}$ and $c(y,\phi) = \frac{-\left[\frac{y^2}{\phi} + log(2\pi\phi)\right]}{2}$.

2.3 THE LINK FUNCTION

The link function $g(\cdot)$ is a function relating the expected value of the response variable Y to the predictors $X_1, X_2, \dots, X_p$. It has the general form $g(\mu) = \eta = \boldsymbol{X\beta}$. The function $g(\cdot)$ must be monotone and differentiable. For a monotone function we can define the inverse function $g^{-1}(\cdot)$ by the relation $g^{-1}(g(\mu)) = \mu$. The choice of the link function depends on the type of data, for example, for continuous normal-theory data an identity link may be appropriate. For counts data the link function should restrict μ to be positive, while for data in the form of proportions should use a link that restricts μ to the interval [0,1].

There are some link functions that are "natural" for certain distributions. These are called *canonical links*. The canonical link is that function which transform the mean to a canonical location parameter of the exponential dispersion family member. This means that the canonical link is that function $g(\cdot)$ for which $g(\mu) = \vartheta$. It holds that:

Poisson: $\vartheta = log(\mu)$ so the canonical link is log.

Binomial: $\vartheta = log\left(\frac{p}{1-p}\right)$ which is the logit link.

Normal: $\vartheta = \mu$ so the canonical link is the identity link.

2.4 THE LINEAR PREDICTOR

The linear predictor $\boldsymbol{X\beta}$ plays the same role in generalized linear models as in general linear models. In regression settings, $\boldsymbol{X}$ contains values of independent variables. In ANOVA settings, $\boldsymbol{X}$ contains dummy variables corresponding to qualitative predictors. In general, the model states that some function of the mean of y is a linear function of the predictors: $\eta = \boldsymbol{X\beta}$. $\boldsymbol{X}$ is called a design matrix.

2.5 MAXIMUM LIKELIHOOD ESTIMATION

Estimation of the parameters of generalized linear models is often done using the Maximum Likelihood method. The estimates are those values that maximize the log likelihood, which for a single observation can be written

$$l = log[L(\vartheta, \phi, y)] = \frac{y\vartheta - b(\vartheta)}{a(\phi)} + c(y, \phi) \qquad 2.16$$

The parameters of the model is a $p\,x\,1$ vector of regression coefficients $\boldsymbol{\beta}$, which are functions of ϑ. Differentiation of l with the respect of the elements of $\boldsymbol{\beta}$, using the chain rule, yields

$$\frac{\partial l}{\partial \beta_j} = \frac{\partial l}{\partial \vartheta}\frac{\partial \vartheta}{\partial \mu}\frac{\partial \mu}{\partial \eta}\frac{\partial \eta}{\partial \beta_j} \qquad 2.17$$

We have shown earlier that $b'(\vartheta) = \mu$ and $b''(\vartheta) = V$ the variance function. Thus $\frac{\partial \mu}{\partial \vartheta} = V$. From the expression for the linear predictor $\eta = \sum_j x_j \beta_j$ we obtain $\frac{\partial \eta}{\partial \beta_j} = x_j$. Putting things together

$$\frac{\partial l}{\partial \beta_j} = \frac{y - b'(\vartheta)}{a(\phi)}\frac{1}{V}\frac{\partial \mu}{\partial \eta}x_j \qquad 2.18$$

If we define

$$W^{-1} = V\left(\frac{\partial \mu}{\partial \eta}\right)^2 \qquad 2.19$$

we finally obtain

$$\frac{\partial l}{\partial \beta_j} = \frac{W}{a(\phi)}(y-\mu)\frac{\partial \mu}{\partial \eta}x_j \qquad 2.20$$

So far, we have written the likelihood for one single observation. By summing over the observations, the likelihood equation for one parameter β_j is given by

$$\sum_{1} \frac{W_i}{a(\phi)}(y_i-\mu_i)\frac{\partial \mu_i}{\partial \eta_i}x_{ij} = 0 \qquad 2.21$$

We can solve 2.21 with respect to β_j since the μ_i are functions of the parameters β_j. Asymptotic variances and covariances of the parameter estimates are obtain through the inverse of the Fisher information matrix. Thus

$$\begin{bmatrix} Var(\hat\beta_0) & Cov(\hat\beta_0,\hat\beta_1) & \cdots & Cov(\hat\beta_0,\hat\beta_{p-1}) \\ Cov(\hat\beta_1,\hat\beta_0) & Var(\hat\beta_1) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ Cov(\hat\beta_{p-1},\hat\beta_0) & \cdots & \cdots & Var(\hat\beta_{p-1}) \end{bmatrix}$$
$$= -E\begin{bmatrix} \frac{\partial^2 l}{\partial \beta_0^2} & \frac{\partial l}{\partial \beta_0}\frac{\partial l}{\partial \beta_1} & \cdots & \frac{\partial l}{\partial \beta_0}\frac{\partial l}{\partial \beta_{p-1}} \\ \frac{\partial l}{\partial \beta_1}\frac{\partial l}{\partial \beta_0} & \frac{\partial^2 l}{\partial \beta_1^2} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \frac{\partial l}{\partial \beta_{p-1}}\frac{\partial l}{\partial \beta_0} & \cdots & \cdots & \frac{\partial^2 l}{\partial \beta_{p-1}^2} \end{bmatrix}^{-1} \qquad 2.22$$

Maximization of the log likelihood 2.16, which is equivalent to solve the likelihood equations 2.21 is done using numerical procedures.

2.6 THE FITTING OF THE MODEL

The fit of a generalized linear model to data may be assessed through the deviance which is used even to compare nested models.

Different models can have different degrees of complexity. The null model has only one parameter that represents a common mean value μ for all observations. In contrast, the full model has n parameters, one for each observation. For the saturated model, each observation fits the model perfectly, i.e. $y = \hat{y}$. The full model is used as a benchmark for assessing the fit of any model to the data and this is done by calculating the deviance. The deviance is defined as follows:

Let $l[\hat{\mu}, \vartheta, \phi]$ be the log likelihood of the current model at the Maximum Likelihood estimate, and let $l[y, \phi, y]$ be the log likelihood of the full model. The deviance D is defined as

$$D = 2\big[l[y, \phi, y] - l[\hat{\mu}, \vartheta, \phi]\big] \qquad 2.23$$

It can be noted that for a Normal distribution the deviance is just the residual sum of the squares. We can defined even the scaled deviance

$$D^* = \frac{D}{\phi} \qquad 2.24$$

which is used for inference. For distribution like Poisson and Binomial, deviance and scaled deviance are identical.

If the model is true the deviance will asymptotically tends towards a χ^2 distribution as n increases. This can be used as an over-all test of the adequacy of the model. A second and perhaps a more important use of the deviance is in comparing competing models. Suppose that a certain model gives a deviance D_1 on df_1 degrees of freedom, and that a simpler model produces a deviance D_2 on df_2. The simpler model would have a larger deviance and less df. To compare the two models we can calculate the difference in deviance $(D_2 - D_1)$ and relate this to a χ^2 distribution with $(df_2 - df_1)$ degrees of freedom. This would give a larger sample-test of the significance of the parameters that are included in model 1 but not in model 2. This requires that the parameters included in model 2 is a subset of the parameters of model 1. An alternative to the deviance for testing and comparing models is the Pearson χ^2, which can be defined as

$$\chi^2 = \sum_i \frac{(y_i - \hat{\mu})^2}{\hat{V}(\hat{\mu})} \qquad 2.25$$

Here $\hat{V}(\hat{\mu})$ is the estimated variance function. For the Normal distribution this is again the residual sum of the squares of the model, so in this case, the deviance and Pearson χ^2 coincide. In other cases, the deviance and Pearson's χ^2. have different asymptotic properties and many produce different results.

Another indicator is the *Akaike's information criterion*, where the main idea is to penalize the likelihood functions such that simpler models are being preferred. A general expression of this idea is to measure the fit of a model to data by a measure such as

$$D_c = D - \alpha q \phi \qquad 2.26$$

Here, D is the deviance, q is the number of parameters in the model and ϕ is the dispersion parameter. If ϕ is constant, it can be shown that $\alpha \approx 4$ is roughly equivalent to test one parameter at 5% level. The model with the smallest value of D_c would then be preferred.

CHAPTER THREE

SOLVENCY II FRAMEWORK

3.1 INTRODUCTION

Solvency II is a new, risk-sensitive system for measuring the financial stability of insurance companies in the EU. It is intended to provide greater security for policyholders and stability for financial markets by providing insurance supervisors with better information and tools to assess the financial strength and the overall solvency of insurance companies. Solvency II will be based on economic principles for the measurement of assets and liabilities. It will also be a risk-based system, as risk will be measured on consistent principles and capital requirements based directly on this measurement.

A set of simple factors, as used for Solvency I, cannot cope well with the diversity of risks in typical insurance portfolios. The more advanced companies have developed sophisticated internal models to measure the effects of adverse events on their portfolios. Provided they can be validated to an adequate standard, these models will form the basis of the capital assessment under Solvency II.

Companies that do not have an internal model of the required standard will still be able to use a factor-based system (the "Standard Approach"), although it is likely to be more complex than the current system.

The aim of Solvency II is not to increase overall levels of capital but rather to ensure a high standard of risk assessment and efficient capital allocation. It should also contribute to increased transparency and help in the development of a level playing field across Europe.

Solvency II is an opportunity to improve insurance regulation by introducing:

- a risk-based system;

- an integrated approach for insurance provisions and capital requirements;
- a comprehensive framework for risk management;
- capital requirements defined by a standard approach or internal model;
- Recognition of diversification and risk mitigation.

The Solvency II has been developed in accordance with the EU's "Lamfalussy process", this means it's divided in four levels. The first level sets out key principles and relates the adoption of provisions under the existing framework procedure of "co-decision" based on the proposals that the Commission submit to the Council and the European Parliament for adoption.

The second level relates to the implementing measures, defining detailed requirements that are tested through Quantitative Impact Studies (QIS).

The third level relates to the transposition and application of legislation of the first and second level in the Member States by the CEIOPS to ensure harmonized outcome. Finally the fourth level is a review by the Commission of the proper implementation of legislation across the European Union.

3.2 THE STRUCTURE

The structure adopted in Solvency II has embraced a 3 pillar approach:

- Pillar I, which focuses on quantitative requirements: valuing assets, liabilities and capital;
- Pillar II, which focuses on supervisory activities: which provides qualitative review through the supervisory process including a focus upon the company's internal risk management processes;
- Pillar III, which addresses supervisory reporting and public disclosure of financial and other information by insurance companies.

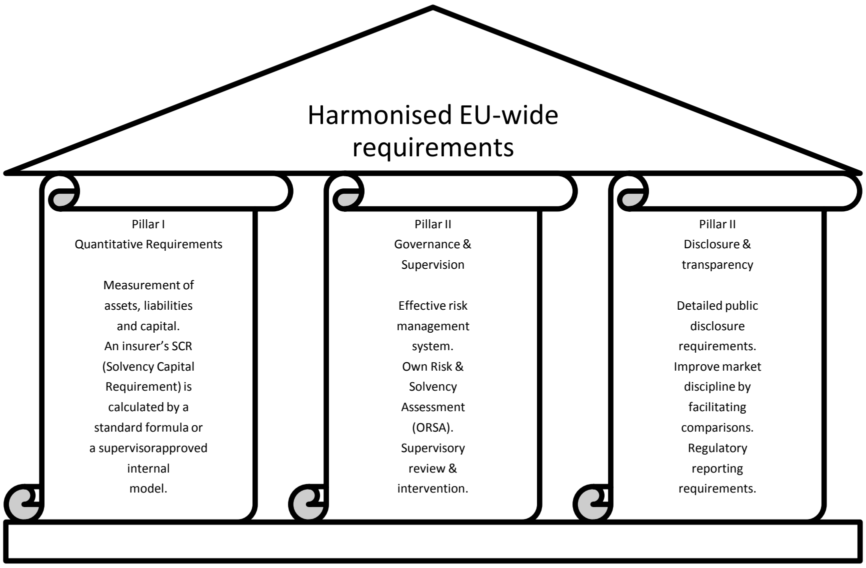

Figure 2: Solvency II structure.

Pillar I requires demonstration of adequate financial resources and there will be dual-level requirements: the Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR).

Below the MCR, which is calculated with a simple method, policyholders are exposed to unacceptable risk and breaching the MCR leads to serious supervisory action.

The Solvency Capital should be determined in order to consider all quantifiable risks faced by a company and be based on the amount of economic capital corresponding to a default probability of ruin and a specific time horizon.

With regard to the calibration parameters, the European Commission suggests a probability of ruin of 0.5%, in particular a VaR risk measure with a confidence level equal to 99.5%, and a time horizon of one year.

For what concerns the risks to be considered in determining the requirements, the European Commission refers to the classification adopted by the International Actuarial Association (IAA), includes:

- underwriting risk;
- credit risk;

- market risk;
- operational risk.

The SCR may be calculated in different ways, using:

- Standard Model: this is default formula currently being constructed and will be available for all companies to use;
- Internal Models: are firm specific calculations designed to maximise capital efficiency. They will encompass all the risks present in the standard model, however will be structured to capitalise on the entity"s" unique composition and inherent risk diversification. As these internal models are produced by the firms themselves, they require regulatory sign-off before they can be used. This ensure they capture all the risks within the standard model to an adequate degree;
- Partial Model: due to the potentially prohibitive costs of constructing an entity specific internal model (particularly for smaller companies), the partial model is an amalgamation of the above.

A firm can choose to use the standard model, but on certain risk modules it can provide its own calculations. As in the internal model these specific areas will require local regulatory permission.

Pillar II comprises two aspects. First, an insurance firm must have in place sound and effective strategies and processes to assess and manage the risks to which it is subject and to assess and maintain its capital needs. Second, those strategies and processes should be subject to review by the supervisory authority.

The starting point should be for all firms to undertake their own individual risk and capital assessment. Where under Pillar I a firm is using either an internal model or is using a stress or scenario approach under standard formula, the firm's Pillar I calculation and its Pillar II assessment will overlap significantly. However, there are important differences between these two pillars. The Pillar II assessment should represent the firm's view of its capital needs, based on an analysis of all the risks that it is exposed to – and factoring in any reasonable mitigation management action.

Requiring all firms to conduct an individual risk and capital assessment in Pillar II will act as powerful tools to foster, encourage and reward, as well as to help embed better and more comprehensive risk management practices in firms. This in turn will lead to a much better assessment and alignment of actual capital needed by a firm to meet its risk profile. For those firms using the standard formula, the assessment should also consider the extent to which it adequately captures the capital required for the risks of that particular firm, including internal controls, risk management systems and governance of the firm. For firms using an internal model the assessment should also take account of control and model risks and demonstrate the extent to which the quantitative models appropriately reflects the actual capital required.

The supervisory review under Pillar II has two aims: to ensure that a firm is well run and meets adequate risk management standards; and to ensure that the firm is adequately capitalised. If, as a result of the supervisory review, the supervisor concludes that the firm should hold more or higher quality of capital, an "add-on" of capital could be applied and this would be the adjusted SCR. Extra capital is of course not the always the best response or even always an appropriate response to problems identified in the supervisory review. In particular, inadequate risk management standards at an insurance firm identified in the supervisory review, quite simply needs to be remedied by the firm.

The purpose of Pillar III is to enhance market discipline on regulated firms by requiring them to disclose publicly key information that is relevant to market participants. It follows from this that in choosing which information should be selected for disclosure under Pillar III, supervisors should be guided by the actual needs of market participants rather than by their own information needs. Pillar III public disclosure serves a different purpose from private regulatory reporting of information to supervisors.

In the following section we analyze the journey of the capital requirements for underwriting premium and reserve risks under the different quantitative impact studies.

3.3 THE STANDARD FORMULA UNDER QIS2

The second quantitative impact study introduced a first structure for the determination of the SCR, defining the risks to be considered and must be evaluated by appropriate methods.

The QIS 2 structure is the following:

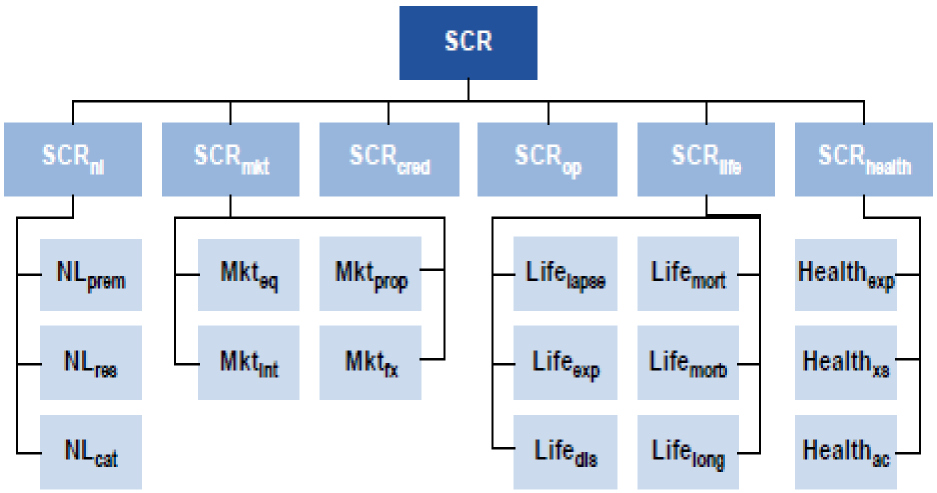

Figure 3: QIS2 SCR Structure.

3.3.1 SCR NON-LIFE UNDERWRITING RISK MODULE

Non-life underwriting risk is split into three components: reserve risk, premium risk and cat risk.

The capital charges for the sub-risks should be combined using a correlation matrix as follows:

$$SCR_{nl}^{QIS2} = \sqrt{\sum_{r\cdot c} Corr^{r\cdot c} \cdot NL_r \cdot NL_c}$$
$$= \sqrt{NL_{pr}^2 + NL_{re}^2 + NL_{CAT}^2 + 2 \cdot 0.5 \cdot NL_{pr} \cdot NL_{re}} \qquad 3.1$$

Where SCR_{nl}^{QIS2}, is the placeholder capital charge for non-life underwriting risk, NL_c and NL_r are capital charges for the individual non-life underwriting sub-risks according to the rows and columns of correlation matrix $CorrNL$ which is defined as follows

CorrNL	NL_{res}	NL_{prem}	NL_{CAT}
NL_{res}	1		
NL_{prem}	0.5	1	
NL_{CAT}	0	0	1

Table 1: QIS2 correlation matrix between reserve risk, premium risk and CAT risk.

The QIS2 LoB classification is:

	Line of Business
1	Accident and health
2	Motor, third-party liability
3	Motor, other classes
4	Marine, aviation, transport (MAT)
5	Fire and other property damage
6	Third-party liability
7	Credit and suretyship
8	Legal expenses
9	Assistance
10	Miscellaneous
11	Reinsurance

Table 2: QIS2 LoB classification.

3.3.1.1 NL PREMIUM RISK

To define the capital requirement for premium risk a factor based approach is used, and it's:

$$NL_{pr} = p(\sigma)P \qquad 3.2$$

Where P is the estimate of net earned premium of the overall business in forthcoming year, is the estimate of the standard deviation of the overall combined ratio and is a function of the standard deviation specified as follows:

$$p(x) = \frac{0.99 - \phi\left(N_{0.99} - \sqrt{log(x^2 + 1)}\right)}{0.01} \qquad 3.3$$

Where ϕ is the cumulative distribution function of the standard normal distribution and $N_{0.99}$ equal to the 99% quantile of the standard normal distribution. In addition the estimate P of the volume of net earned premium for the overall non-life business in the forthcoming year is defined as follows:

$$P = \sum_{lob} P_{lob} \qquad 3.4$$

And σ of the total business is:

$$\sigma = \sqrt{\frac{1}{P^2}\sum_{r,c} CorrLobPr^{r,c} \cdot P_r \cdot P_c \cdot \sigma_r \cdot \sigma_c} \qquad 3.5$$

CorrLob_Prem	1	2	3	4	5	6	7	8	9	10	11
1	1										
2	0.25	1									
3	0	0.5	1								
4	0	0	0.5	1							
5	0	0	0.5	0.25	1						
6	0.25	0	0	0	0	1					
7	0	0	0	0	0	0.75	1				
8	0.5	0.25	0	0	0	0.5	0.75	1			
9	0	0	0.5	0.5	0.5	0	0	0	1		
10	0	0	0	0	0	0	0	0	0	1	
11	0	0	0.5	0.5	0.5	0.5	0	0	0	0	1

Table 3: QIS2 Premium Risk LoB correlation.

And σ_r, σ_c respectively the σ of lob_r and lob_c.

To estimate σ are available two different approaches:

- "Market-wide approach";
- "Undertaking-specific approach".

The market wide approach estimates the standard deviation of the combined ratio in the individual $LoBs$ as follows:

$$\sigma_{M,lob} = sf_{lob} \cdot f_{lob} \tag{3.6}$$

Where sf_{lob} is the size factor defined as follows

$$sf_{lob} = \begin{cases} 1 & if\ P_{lob,gross} > 100mln \\ \dfrac{10}{\sqrt{P_{lob,gross} \cdot 10^{-6}}} & if\ 100mln > P_{lob,gross} \geq 20mln \\ \dfrac{10}{\sqrt{20}} & otherwise \end{cases} \tag{3.7}$$

And f_{lob} is the volatility factor specific for each Line of Business and equal to

LoB	1	2	3	4	5	6	7	8	9	10	11
f_{lob}	0.05	0.125	0.075	0.15	0.10	0.25	0.10	0.15	0.10	0.15	0.15

Table 4: QIS2 Premium Risk volatility factors.

The undertaking specific approach estimates the standard deviation of the combined ratio in the individual $LoBs$ as follows

$$\sigma_{U,lob} = \sqrt{c_{lob} \cdot \sigma^2_{CR,lob} + (1 - c_{lob}) \cdot \sigma^2_{M,lob}} \qquad 3.8$$

Where $\sigma^2_{CR,lob}$ is the estimate of the standard deviation of the combined ratio in the individual lob on the basis of historic combined ratios of the undertaking and c_{lob} is the credibility factor for lob defined as

$$c_{lob} = 0.2 \cdot max \binom{0}{J_{lob} - 10} \qquad 3.9$$

Where J_{lob} is equal to the number of historic combined ratios for each lob (to the extent available, not more than 15 years). In case of $J_{lob} > 10$ the estimate $\sigma_{CR,lob}$ of the standard deviation of the combined ratio in the individual lob on the basis of historic combined ratios of the undertaking and is defined as

$$\sigma_{CR,lob} = \sqrt{\frac{1}{(J_{lob} - 1) \cdot P_{lob}} \sum_y P_{lob,y} \cdot \left(CR_{lob,y} - \mu_{lob}\right)^2} \qquad 3.10$$

Where μ_{lob} is the company specific estimate of the expected value of the combined ratio in the individual lob and equal to

$$\mu_{lob} = \frac{\sum_y P_{lob,y} \cdot CR_{lob,y}}{\sum_y P_{lob,y}} \qquad 3.11$$

3.4 THE STANDARD FORMULA UNDER QIS3

The QIS 3 structure is the following:

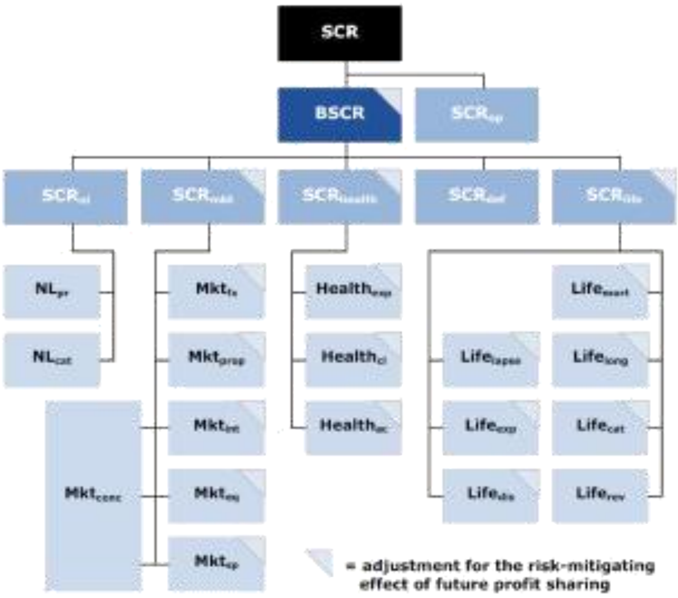

Figure 4: QIS3 SCR Structure.

QIS3 defines the same risk categories of QIS2 but reviews the structure.

3.4.1 SCR NON-LIFE UNDERWRITING RISK MODULE

The third quantitative impact study reviewed the non-life underwriting risk module.

First of all has been reviewed the aggregation structure unifying the premium and the reserve module and then aggregating the result obtained with the CAT risk capital requirement supposing incorrelation.

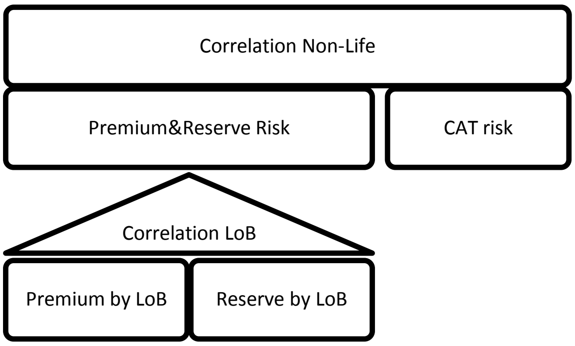

Figure 5: QIS3 aggregation between Premium Risk, Reserve Risk and CAT Risk.

Even the LoB classification has been reviewed:

	Line of Business
1	Accident and health-workers compensation
2	Accident and health-health insurance
3	Accident and health-others/default
4	Motor, third-party liability
5	Motor, other classes
6	Marine, aviation, transport (MAT)
7	Fire and other property damage
8	Third-party liability
9	Credit and suretyship
10	Legal expenses
11	Assistance
12	Miscellaneous non-life insurance
13	Non-proportional reinsurance – property
14	Non-proportional reinsurance – casualty
15	Non-proportional reinsurance – MAT

Table 5: QIS3 LoB classification.

The capital requirements for Premium & Reserve Risk are determinated multiplying a transformation for the total volume, net of reinsurance, of the reserve amount (only Best Estimate) and the premiums of the following year maintaining an annual growth at least 5%.

$$NL_{pr} = p(\sigma) \cdot V \qquad 3.12$$

Where

$$V = \sum_{lob} V_{prem,lob} + V_{res,lob} \qquad 3.13$$

The function $p(\sigma)$ is defined as follows

$$p(x) = exp\left\{-\frac{\sigma^2}{2} + N_{0.995} \cdot \sqrt{log(\sigma^2 + 1)}\right\} - 1 \qquad 3.14$$

With $N_{0.995}$ equal to the 99.5% quantile of the standard normal distribution.

The standard deviation σ of the combined ratio for the overall non-life insurance portfolio are determinated in two steps:

- in a first step, for each individual line of business standard deviations and volume measures for both premium risk and reserve risk are determined;
- in a second step, the standard deviations and volume measures for the premium risk and the reserve risk in the individual lob are aggregated to derive an overall volume measure V and an overall standard deviation σ.

The standard deviation for premium risk in the individual lob is derived as a credibility mix of an undertaking-specific estimate and a market-wide estimate as follows

$$\sigma_{prem,lob} = \sqrt{c_{lob} \cdot \sigma^2_{U,prem,lob} + (1 - c_{lob}) \cdot \sigma^2_{M,prem,lob}} \tag{3.15}$$

Where c_{lob} is the credibility factor for lob, $\sigma_{U,prem,lob}$ is the undertaking-specific estimate of the standard deviation for premium risk and $\sigma_{M,prem,lob}$ is the market-wide estimate of the standard deviation for premium risk.

The market-wide estimate of the standard deviation for premium risk in the individual lob is determined as follows:

LoB	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
f_{lob}	7.5%	3%	5%	10%	10%	12.5%	10%	10%	12.5%	5%	7.5%	12.5%	15%	15%	15%

Table 6: QIS3 Premium Risk volatility factors.

And the credibility factor is

$$c_{lob} = \begin{cases} \dfrac{n_{lob}}{n_{lob} + 4} & if\ n_{lob} \geq 7 \\ 0 & otherwise \end{cases} \tag{3.16}$$

The undertaking-specific estimate $\sigma_{U,prem,lob}$ is determined on the basis of the volatility of historic loss ratios as follows

$$\sigma_{U,lob} = \sqrt{\frac{1}{(n_{lob} - 1) \cdot V_{prem,lob}} \sum_{y} P_{lob,y} \cdot \left(LR_{lob,y} - \mu_{lob}\right)^2} \tag{3.17}$$

Where μ_{lob} is the company-specific estimate of the expected value of the Loss Ratio in the individual lob and it's defined as the premium-weighted average of historic Loss Ratios

$$\mu_{lob} = \frac{\sum_y P_{lob,y} \cdot LR_{lob,y}}{\sum_y P_{lob,y}} \tag{3.18}$$

The standard deviations for reserve risk in the individual lob are the following

LoB	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
f_{lob}	15%	7.5%	15%	12.5%	7.5%	15%	10%	15%	10%	10%	10%	15%	15%	20%	20%

Table 7: QIS3 Reserve Risk volatility factors.

The overall volume measure V is determined as follows

$$V = \sum_{lob} V_{prem,lob} + V_{res,lob} \tag{3.19}$$

The overall standard deviation σ is determined as follows

$$\sigma = \sqrt{\frac{1}{V^2} \sum_{r,c} CorrLob^{r,c} \cdot V_r \cdot V_c \cdot a_r \cdot a_c} \tag{3.20}$$

Where r, c are indices of the form $(prem, lob)$ or (res, lob), V_r and V_c are volume measures for the individual line of business, the factors a_r and a_c are defined as follows

$$a_r = \begin{cases} \sigma_{prem,lob} & if\ r = (prem, lob) \\ \sigma_{res,lob} & if\ r = (res, lob) \end{cases} \tag{3.21}$$

And $CorrLob^{r,c}$ is the correlation matrix defined as

$$CorrLob^{r,c} = \begin{bmatrix} CorrLob_{pr} & 0.5 \cdot CorrLob_{pr} \\ 0.5 \cdot CorrLob_{pr} & CorrLob_{pr} \end{bmatrix} \tag{3.22}$$

With $CorrLob_{pr}$

CorrLob_Pr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1														
2	0.5	0.5													
3	0.5	0.5	1												
4	0.25	0.25	0.25	1											
5	0.25	0.25	0.25	0.5	1										
6	0.25	0.25	0.25	0.5	0.25	1									
7	0.25	0.25	0.25	0.25	0.25	0.25	1								
8	0.5	0.25	0.25	0.5	0.25	0.25	0.25	1							
9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.5	1						
10	0.5	0.25	0.5	0.5	0.5	0.25	0.25	0.5	0.5	1					
11	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1				
12	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1			
13	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.5	0.25	1		
14	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1	
15	0.25	0.25	0.25	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.25	1

Table 8: QIS3 Premium Risk LoB correlation.

3.5 THE STANDARD FORMULA UNDER QIS4

The QIS3 structure is utilized even in QIS4. The total capital requirement is determined adding to the operational risk capital requirement, assuming full correlation with all the other risks but, differently from QIS3, are subtracted two adjustments for future discretionary benefit and for the deferred taxes:

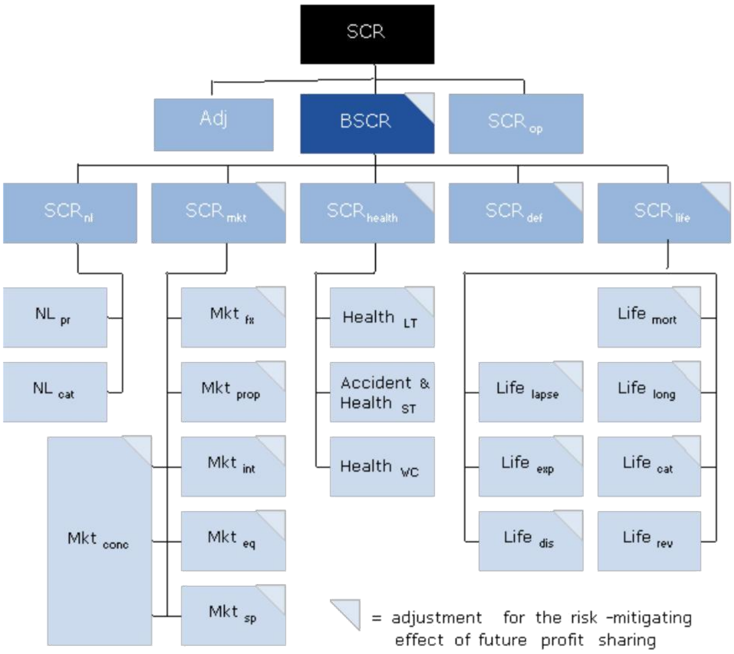

Figure 6: QIS4 SCR Structure.

3.5.1 SCR NON-LIFE UNDERWRITING RISK MODULE

The lob classification has been reviewed in QIS4:

	Line of Business
1	Motor, third-party liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance – property
11	Non-proportional reinsurance – casualty
12	Non-proportional reinsurance – MAT

Table 9: QIS4 LoB classification.

For the non-life module, the QIS 4 maintains the same standard of QIS3 with some modifications:

$$NL_{pr} = p(\sigma) \cdot V \qquad 3.23$$

$$V = \sum_{lob} V_{lob} \qquad 3.24$$

$$V_{lob} = \left(V_{prem,lob} + V_{res,lob}\right) \cdot \left(0.75 + 0.25 \cdot DIV_{pr,lob}\right) \qquad 3.25$$

$$DIV_{pr,lob} = \frac{\sum_j \left(V_{prem,j,lob} + V_{res,j,lob}\right)^2}{\left\{\sum_j \left(V_{prem,j,lob} + V_{res,j,lob}\right)\right\}^2} \qquad 3.26$$

That meant it's introduced a geographical diversification.

The new market-wide estimate of the standard deviation for premium risk in the individual lob are

LoB	1	2	3	4	5	6	7	8	9	10	11	12
$\sigma_{(prem,lob)}$	9%	9%	12.5%	10%	12.5%	15%	5%	7.5%	11%	15%	15%	15%

Table 10: QIS4 Premium Risk volatility factors.

The maximum value of for the determination of is fixed according to the line of business in the following table:

LoB	Maximum n_{lob}
2, 4, 7, 8, 10	5
3, 9, 12	10
1, 5, 6, 11	15

Table 11: QIS4 maximum number of $\boldsymbol{n_{lob}}$.

The new values of the standard deviation for reserve risk in the individual lob are:

LoB	1	2	3	4	5	6	7	8	9	10	11	12
$\sigma_{(res,lob)}$	12%	7%	10%	10%	15%	15%	10%	10%	10%	15%	20%	20%

Table 12: QIS4 Reserve Risk volatility factors.

In QIS4 after having derived $\sigma_{prem,lob}$ and $\sigma_{res,lob}$ the standard deviation for premium and reserve risk in the individual lob is defined by aggregating the standard deviations for both subrisks under the assumption of a correlation coefficient of $\alpha = 0.5$

$$\sigma_{lob} = \frac{\sqrt{\left(\sigma_{prem,lob} \cdot V_{prem,lob}\right)^2 + \left(\sigma_{res,lob} \cdot V_{res,lob}\right)^2 + 2 \cdot \alpha \cdot \left(\sigma_{prem,lob} \cdot V_{prem,lob}\right) \cdot \left(\sigma_{res,lob} \cdot V_{res,lob}\right)}}{V_{prem,lob} + V_{res,lob}} \quad 3.27$$

The credibility factor c_{lob} are defined as

	Number of historical years of data available (excluding the first 3 years after the line of business was first written)														
Maximum value of n_{lob}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	0	0	0	0	0	0	0.64	0.67	0.69	0.71	0.73	0.75	0.76	0.78	0.79
10	0	0	0	0	0.64	0.69	0.72	0.74	0.76	0.79	-	-	-	-	-
5	0	0	0.64	0.72	0.79	-	-	-	-	-	-	-	-	-	-

Table 13: QIS4 $\boldsymbol{c_{lob}}$.

Finally the new correlation matrix is:

CorrLob	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	0.5	1										
3	0.5	0.25	1									
4	0.25	0.25	0.25	1								
5	0.5	0.25	0.25	0.25	1							
6	0.25	0.25	0.25	0.25	0.5	1						
7	0.5	0.5	0.25	0.25	0.5	0.5	1					
8	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1				
9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1			
10	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.5	0.25	1		
11	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1	
12	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.25	1

Table 14: QIS4 Premium Risk LoB correlation.

3.6 THE STANDARD FORMULA UNDER QIS5

The QIS5 structure is:

Figure 7: QIS5 SCR Structure.

3.6.1 SCR NON-LIFE UNDERWRITING RISK MODULE

The capital requirement for non-life underwriting risk is derived by combining the capital requirements for the non-life sub-risks using a correlation matrix as follows

$$SCR_{nl}^{QIS5} = \sqrt{\sum_{r\cdot c} Corr^{r\cdot c} NL_r NL_c} \quad 3.28$$

Where $Corr^{r\cdot c}$ is set equal to

CorrNL	NL_{res}	NL_{prem}	NL_{CAT}
NL_{res}	1		
NL_{prem}	0	1	
NL_{CAT}	0	0.25	1

Table 15: QIS5 correlation matrix between reserve risk, premium risk and CAT risk

The lob classifications is not reviewed in QIS5

	Line of Business
1	Motor, third-party liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance – property
11	Non-proportional reinsurance – casualty
12	Non-proportional reinsurance – MAT

Table 16: QIS5 LoB classification.

The QIS5 standard formula is the same of QIS4 but introduces some modifications.

The market-wide estimates of the net standard deviation for premium risk for each line of business are:

Line of Business	$\sigma_{M,prem,lob}$
1	10% NP_{lob}
2	7% NP_{lob}
3	17% NP_{lob}
4	10% NP_{lob}
5	15% NP_{lob}
6	21.5% NP_{lob}
7	6.5% NP_{lob}
8	5% NP_{lob}
9	13% NP_{lob}
10	17.5%
11	17%
12	16%

Table 17: QIS5 Premium Risk volatility factors.

Where the adjustment factor for non-proportional reinsurance of a line of business allows undertakings to take into account the risk-mitigating effect of particular per risk excess of loss reinsurance.

The volatility factors for the reserve risk are:

LoB	1	2	3	4	5	6	7	8	9	10	11	12
$\sigma_{(res,lob)}$	9.5%	10%	14%	11%	11%	19%	9%	11%	15%	20%	20%	20%

Table 18: QIS5 Reserve Risk volatility factors.

3.7 THE STANDARD FORMULA UNDER LONG TERM GUARANTEE ASSESMENT

The LTG structure is

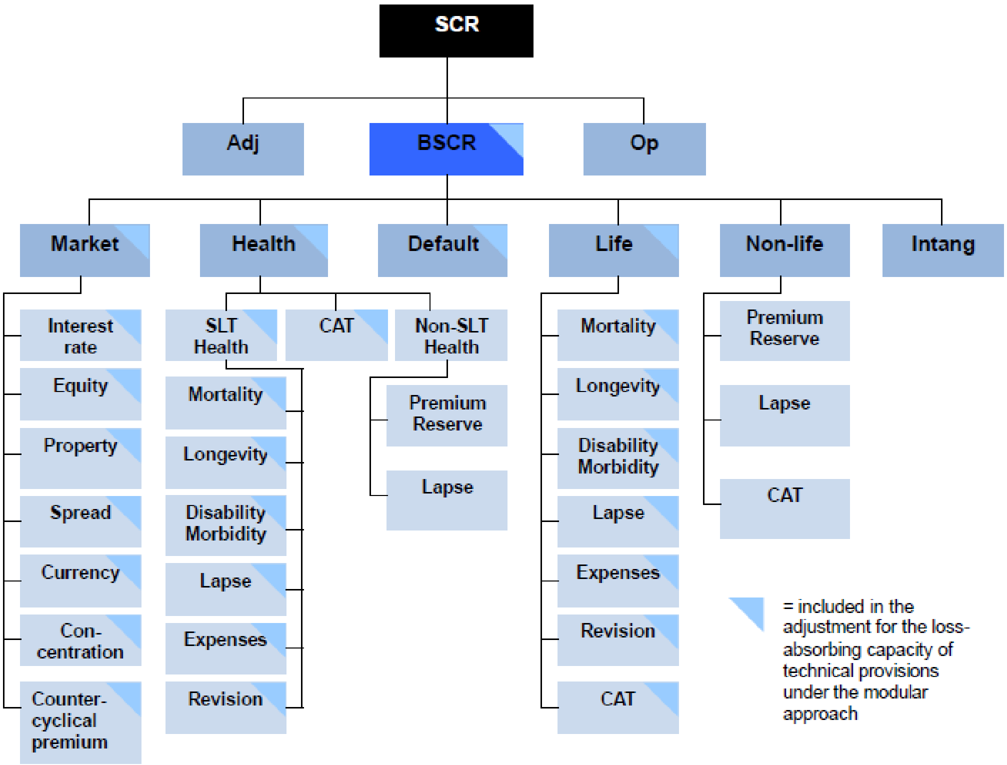

Figure 8: LTG SCR Structure.

3.7.1 SCR NON-LIFE UNDERWRITING RISK MODULE

The capital requirement for non-life underwriting risk is derived by combining the capital requirements for the non-life sub-risks using a correlation matrix as follows

$$SCR_{nl}^{LTG} = \sqrt{\sum_{r\cdot c} Corr^{r\cdot c} NL_r NL_c} \tag{3.29}$$

Where $Corr^{r\cdot c}$ is set equal to

CorrNL	NLpr	NL_{lapse}	NL_{CAT}
NLpr	1		
NL_{lapse}	0	1	
NL_{CAT}	0.25	0	1

Table 19: LTGA correlation matrix between reserve&premium risk, lapse risk and CAT risk

The capital requirement for the combined premium and reserve risk is determined as follows

$$NL_{pr} = 3 \cdot \sigma \cdot V \tag{3.30}$$

V is the volume measure and σ is the standard deviation of the overall non-life insurance portfolio and are determined in two steps:

- for each lob the standard deviations and volume measures for both premium and reserve risk are determined;
- the standard deviations and volume measures for the premium risk and the reserve risk in the individual lob are aggregated to derive an overall volume measure V and a combined standard deviation σ.

The premium and reserve risk sub-module is based on the following segmentation

	Line of Business
1	Motor, third-party liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance – casualty
11	Non-proportional reinsurance – property
12	Non-proportional reinsurance – MAT

Table 20: LTG LoB classification.

The volume measure for premium risk in the individual segment is

$$V_{prem,lob} = max(P_{lob}; P_{last,lob}) + FP_{existing,lob} + FP_{future,lob} \quad 3.31$$

Where P_{lob} is the estimate of the premiums to be earned during the following 12 months, $P_{last,lob}$ is the amount of premiums earned during the last 12 months, $FP_{existing,lob}$ is the expected present value of premiums to be earned after the following 12 months for existing contracts and $FP_{future,lob}$ is the expected present value of premiums to be earned for contract where the initial recognition date falls in the following 12 months but excluding the premiums to be earned during the 12 months after the initial recognition date.

The standard deviations for premium risk gross of reinsurance are

LoB	1	2	3	4	5	6	7	8	9	10	11	12
σ(prem,lob)	10% NP_{lob}	8% NP_{lob}	15% NP_{lob}	8% NP_{lob}	14% NP_{lob}	12% NP_{lob}	7% NP_{lob}	9% NP_{lob}	13% NP_{lob}	17%	17%	17%

Table 21: LTGA Premium Risk volatility factors.

While the standard deviations for reserve risk are

LoB	1	2	3	4	5	6	7	8	9	10	11	12
σ(res,lob)	9%	8%	11%	10%	11%	19%	12%	20%	20%	20%	20%	20%

Table 22: LTGA Reserve Risk volatility factors.

The new correlation matrix is

CorrLob	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	0.5	1										
3	0.5	0.25	1									
4	0.25	0.25	0.25	1								
5	0.5	0.25	0.25	0.25	1							
6	0.25	0.25	0.25	0.25	0.5	1						
7	0.5	0.5	0.25	0.25	0.5	0.5	1					
8	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1				
9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1			
10	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	1		
11	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.25	0.5	0.25	1	
12	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.5	0.25	0.25	0.25	1

Table 23: LTGA Premium Risk LoB correlation.

CHAPTER FOUR

CASE STUDY

4.1 INTERNAL MODEL

The main idea behind this study is to create a strong connection between the best pricing techniques, for example the GLM introduced in the previous chapter, and the calculation of Premium Risk SCR (SCR_{PR}). We will focus on the non-life Premium Risk connected with an Insurance Company "ALFA" that operates only on the MTPL market.

To calculate the SCR_{PR} we will use a simulation method based on the Risk Theory Collective Approach. Following this approach, the aggregate claim amount is analyzed considering the entire portfolio, which must be composed of risk with a high homogeneity level. The aggregate claim amount is given by a compound process :

$$\tilde{X}_{coll} = \sum_{i=1}^{\tilde{N}} \tilde{Z}_i \qquad 4.1$$

Where $\tilde{N}$ is the random variable of the number of claims occurred and $\tilde{Z}_i$ is the random claim size of the i$^{\text{th}}$ claim occurred. $\tilde{N}$ and $\tilde{Z}_i$ must be independent each other and $\tilde{Z}_i$ must be independent and identically distributed (iid). Usually, in the modeling of $\tilde{N}$ is used a Poisson distribution, but, unfortunately, is not always possible. This is due to external factors, like climatic, pandemic, political and seasonal conditions, that can cause deviations of the number of claims from the expected value, which are not included in the pure random fluctuations of the variable. In general we can affirm that such factors act on the parameter of the simple Poisson distribution, causing its alteration.

There are different kind of fluctuations:

- trends: when a slow moving change of the claim probabilities is occurring. They can produce an either increase or decrease of the expected value since a systematic change in the line environment conditions;
- short-period fluctuations: when fluctuations are affecting only in the short-term (usually less than a year) the assumed probability distribution, without any time dependency;
- long-period cycles: when changes are not mutually independent and they produce their effect on a long term and a cycle period of several years may be assumed. They are usually correlated to general economic conditions.

In the case under examination, we will consider a sort of trend factor.

The first peculiarity of the Italian market, is the presence of the CARD system. From this convention born four types of claims:

- NC claim;
- CD claim;
- CG claim;
- FG claim.

Due to this Italian characteristic the 4.1 becomes

$$\tilde{X}_{coll} = \sum_{i=1}^{\tilde{N}_{NC}} \tilde{Z}_i^{NC} + \sum_{j=1}^{\tilde{N}_{CD}} \tilde{Z}_j^{CD} + \sum_{t=1}^{\tilde{N}_{CG}} \tilde{Z}_t^{CG} - \sum_{p=1}^{\tilde{N}_{FG}} \tilde{Z}_p^{FG} \qquad 4.2$$

We can now point out better the different elements of 4.2 starting from the NC claims.

To this type belong the claims that do not have the right characteristics to be part of the CARD system, for example incidents with more than two cars involved or mortal claims.

In that kind of claim it's easy to find big amount of payment, for this peculiarity we will model the attritional claims and the large ones, so the NC claim amount is

$$\tilde{X}_{NC} = \sum_{s=1}^{\tilde{N}_{NC-att}} \tilde{S}_s^{NC-att} + \sum_{k=1}^{\tilde{N}_{NC-large}} \tilde{Y}_k^{NC-att} \qquad 4.3$$

Being H the threshold between attritional and large claims we have

$$\tilde{N}_{NC-large} = \tilde{N}_{NC} \cdot P\left(\tilde{Z}_i^{NC} \geq H\right) \qquad 4.4$$

CG claims are referred to damages suffered by our customer that have no cause the claims. The Company is going to pay the whole damage and will receive a fixed amount (depending on the province) from the "guilty-car Company".

In addition, if on the car of our costumer there would be a third person, the Company must pay even this damage and is going to receive another Forfait (CTT claims).

Last peculiarity of CG claims arises when the two cars involved on the incident are both insured with the same Company. In this special case we have a DN claim and the Company simply pays the damage.

We can now write the CG claims amount as:

$$\tilde{X}_{CG}^{j} = \sum_{m=1}^{\tilde{N}_{CG-STD}^{j}} {}_{j}\tilde{Y}_m^{CG-STD} + \sum_{n=1}^{\tilde{N}_{DN}^{j}} {}_{j}\tilde{Y}_n^{DN} + \sum_{o=1}^{\tilde{N}_{CG-CTT}^{j}} {}_{j}\tilde{Y}_o^{CTT} \ with\ j = 1,2,3 \qquad 4.5$$

Where

$$\widetilde{N}_{CG}^{j} = \widetilde{N}_{CG} \cdot P(province \in j) \qquad 4.6$$

$$\widetilde{N}_{DN}^{j} = \widetilde{N}_{CG}^{j} \cdot P(CG \in DN) \qquad 4.7$$

$$\widetilde{N}_{CG-CTT}^{j} = \widetilde{N}_{CG}^{j} \cdot P(CG \in CTT) \qquad 4.8$$

$$\widetilde{N}_{CG-STD}^{j} = \widetilde{N}_{CG}^{j} \cdot \{1 - P(CG \in CTT) - P(CG \in DN)\} \qquad 4.9$$

and j is the province group, which is a basic element of the modeling because the claim size distribution and the forfait are connected to this variable.

When we have a CG claim (except the DN claim) we receive a forfait (FG). For this reason we now focus on the FG Claims Amount:

$$\widetilde{X}_{FG}^{j} = \sum_{m=1}^{\widetilde{N}_{FG-STD}^{j}} {}_{j}F_{m}^{FG-STD} + \sum_{o=1}^{\widetilde{N}_{FG-CTT}^{j}} {}_{j}\widetilde{Y}_{o}^{FG-CTT} \; with\; j = 1,2,3 \qquad 4.10$$

Where

$$\widetilde{N}_{FG-STD}^{j} = \widetilde{N}_{CG-STD}^{j} \qquad 4.11$$

$$\widetilde{N}_{FG-CTT}^{j} = \widetilde{N}_{CG-CTT}^{j} \qquad 4.12$$

The CD claims are the opposite of the CG claims, are referred to damages caused by a costumer insured with the Company. For this reason the amount paid would be a fixed forfait which depends on the province. If on the car not guilty there would be a third person, the company is going to pay a special forfait. For each province group the CD claim cost would be

$$\tilde{X}_{CD}^{j} = \sum_{e=1}^{\tilde{N}_{CD-std}^{j}} {}_{j}F_{e}^{CD-std} \qquad 4.13$$

While the whole CD-CTT claims amount (not depending on the province)

$$\tilde{X}_{CD-CTT} = \sum_{v=1}^{\tilde{N}^{CD-CTT}} \tilde{Y}_{v}^{CD-CTT} \qquad 4.14$$

Where

$$\tilde{N}^{CD-CTT} = \tilde{N}_{CD} \cdot P(CD \in CTT) \qquad 4.15$$

$$\tilde{N}_{j}^{CD-std} = \tilde{N}_{CD} \cdot \{1 - P(CD \in CTT)\} \cdot P(province \in j) \qquad 4.16$$

We can easily rewrite the total CD claims amount as follow

$$\tilde{X}_{CD} = \tilde{N}^{CD-std} \cdot \sum_{j=1}^{3} \left\{ F_{j}^{CD-std} \cdot P(province = j) \right\} + \sum_{v=1}^{\tilde{N}^{CD-CTT}} \tilde{Y}_{v}^{CD-CTT} \qquad 4.17$$

Now the 4.2 can be written as

$$
\begin{aligned}
\tilde{X}_{coll} = & \sum_{s=1}^{\tilde{N}_{NC-att}} \tilde{S}_s^{NC-att} + \sum_{k=1}^{\tilde{N}_{NC-large}} \tilde{Y}_k^{NC-large} + \\
& \sum_{j=1}^{3} \left\{ \sum_{m=1}^{\tilde{N}_j^{CG-std}} {}_j\tilde{Y}_m^{CG-std} + \sum_{n=1}^{\tilde{N}_j^{DN}} {}_j\tilde{Y}_n^{DN} + \sum_{o=1}^{\tilde{N}_j^{CG-CTT}} {}_j\tilde{Y}_o^{CG-CTT} \right\} - \\
& - \sum_{j=1}^{3} \left\{ \sum_{m=1}^{\tilde{N}_j^{FG-std}} {}_jF_i^{FG-std} + \sum_{o=1}^{\tilde{N}_j^{FG-CTT}} {}_j\tilde{Y}_i^{FG-CTT} \right\} + \\
& + \tilde{N}^{CD-std} \cdot \sum_{j=1}^{3} \left\{ F_j^{CD-std} \cdot P(province = j) \right\} + \sum_{v=1}^{\tilde{N}^{CD-CTT}} \tilde{Y}_v^{CD-CTT}
\end{aligned}
\tag{4.18}
$$

We have just built the structure of the total claims amount that the Company must face, now we have to estimate the different parameters.

The idea is to use the GLM of the pricing to estimate the different frequency parameters, $\lambda_{NC}, \lambda_{CG}, \lambda_{CD}$. In order to reach a high level of connection with the Pricing area and the risk profile of the Company, we will manage directly with the portfolio of the MTPL policies.

In the following we will focus on the λ_{NC}, but the procedure is the same for the other types of claim. Imagine that we are on the first of January 2014, the starting points of the estimation of the future λ_{NC} are the policies that had at least one day of coverage during the previous year, and the GLM model that explains the expected number of NC claims.

First of all the GLM structure is

$$
\lambda_{NC-year\ exposure}^{custumer\ h} = base_{NC} \prod_{d=1}^{n} C_d^h \tag{4.19}
$$

This means that the expected number of claims of the customer h^{th} in one year is equal to the product of the n coefficients (and the base) of the GLM model connected with his characteristics.

Being the GLM models calculated on databases that usually have more than one year inside, we must recalculate the base of the model in order to have

$$
\begin{aligned}
&frequency_{2013}^{obs-NC} \cdot IBNyR_{2013}^{NC} = frequency_{2013}^{estimated-GLM\,NC} = \\
&\qquad = \frac{\sum_{h=1}^{r} \lambda_{NC-year\,exposure}^{custumer\,h} \cdot exposure_{2013}^{h}}{\sum_{h=1}^{r} exposure_{2013}^{h}} = \\
&\qquad = \frac{base_{NC} \cdot \sum_{h=1}^{r} \prod_{d=1}^{n} C_d^h \cdot exposure_{2013}^{h}}{\sum_{h=1}^{r} exposure_{2013}^{h}}
\end{aligned}
\qquad 4.20
$$

From than we have

$$
base_{NC} = \frac{frequency_{2013}^{obs-NC} \cdot IBNyR_{2013}^{NC} \cdot \sum_{h=1}^{r} exposure_{2013}^{h}}{\sum_{h=1}^{r} \prod_{d=1}^{n} C_d^h \cdot exposure_{2013}^{h}} \qquad 4.21
$$

We obtain:

	Frequency Ultimate	Model Base
NC	1,047%	0,669%
CD	4,450%	3,701%
CG	5,109%	3,798%

Table 24: Ultimate frequency & model bases.

Once calculated the base of the frequency model we can start to project the portfolio in order to estimate the $\lambda_{NC}^{Solvency}$.

Before going on the explanation of the model, it's important to introduce the main characteristics of the portfolio, in terms of exposure, frequency and average premiums. The followings maps show these important features.

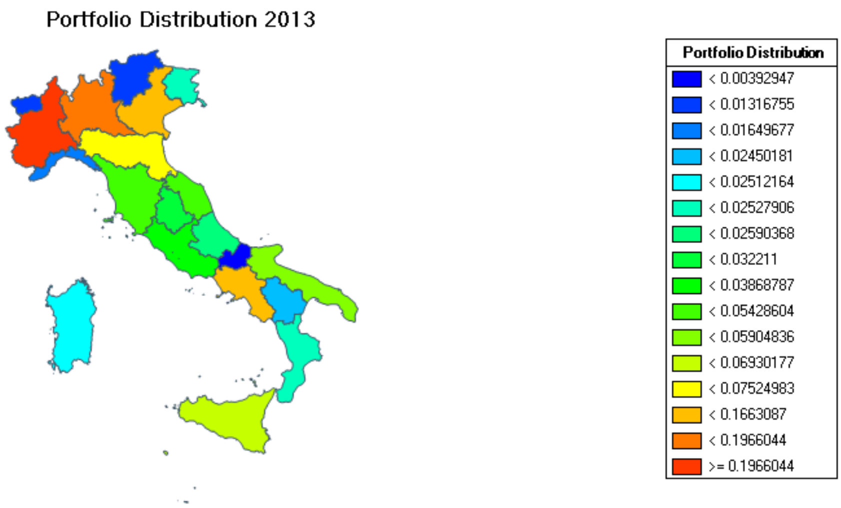

Figure 9: 2013 portfolio distribution by region.

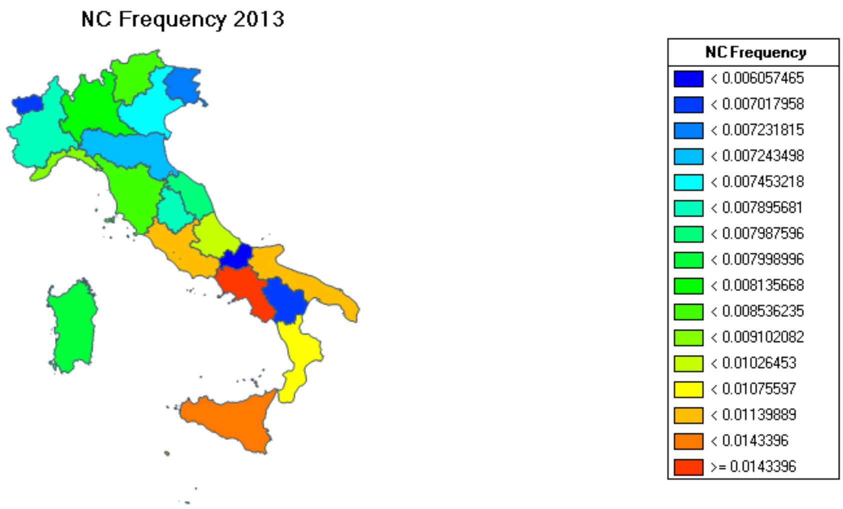

Figure 10: 2013 NC frequency by region.

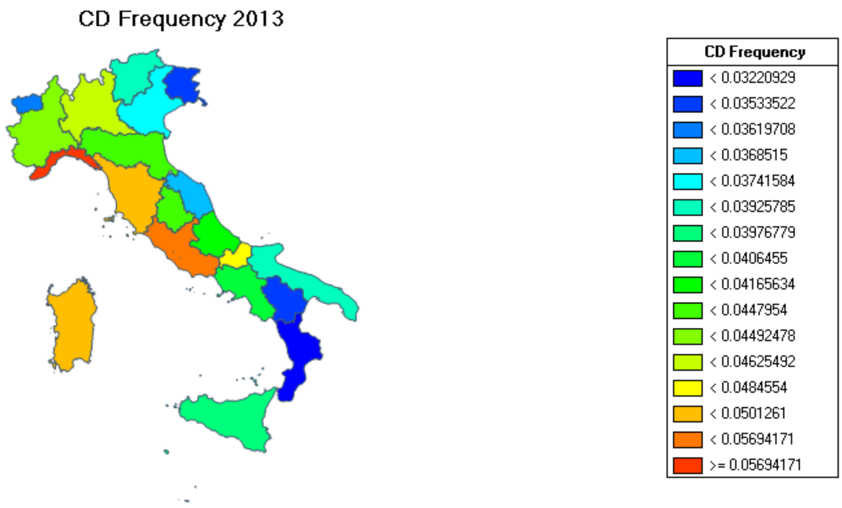

Figure 11: 2013 CD frequency by region.

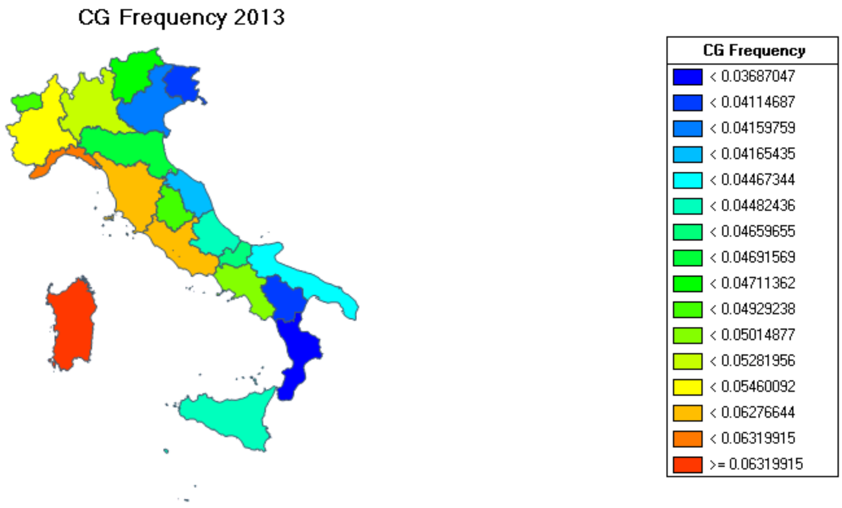

Figure 12: 2013 CG frequency by region.

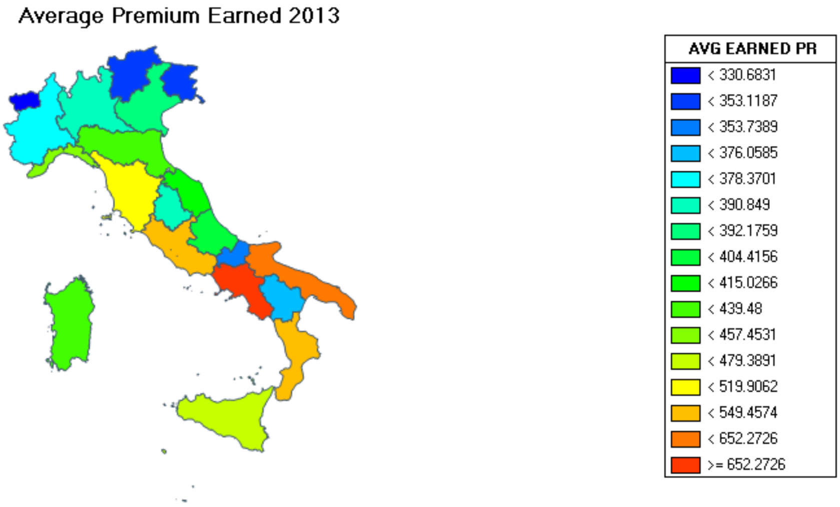

Figure 13: 2013 average premium earned by region.

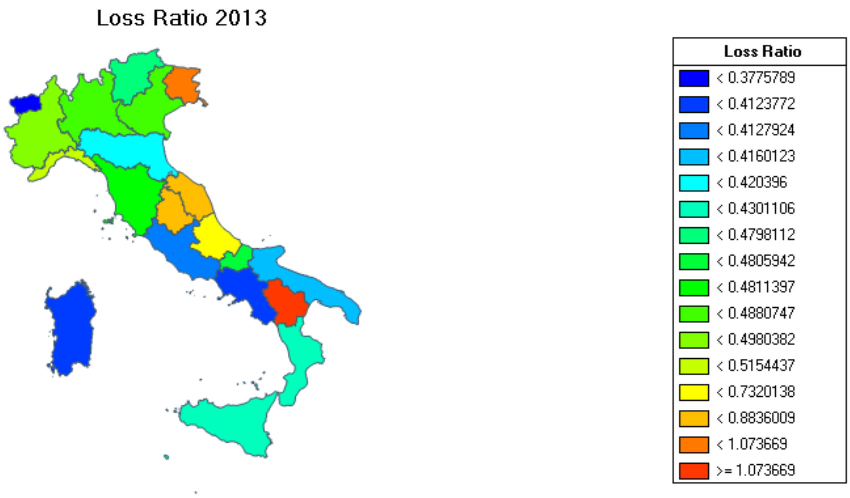

Figure 14: 2013 Loss Ratio by Region.

Region	Portfolio Distribution	CD Frequency	CG Frequency	NC Frequency	Loss Ratio	Average Premium Earned
Emilia Romagna	6.9%	4.5%	4.7%	0.7%	41.6%	€ 415.03
Friuli Venezia Giulia	2.5%	3.2%	3.7%	0.7%	88.4%	€ 330.68
Liguria	1.3%	5.7%	6.3%	0.9%	49.8%	€ 439.48
Lombardia	16.6%	4.5%	5.0%	0.8%	48.1%	€ 378.37
Piemonte	19.7%	4.5%	5.3%	0.8%	48.8%	€ 376.06
Trentino Alto Adige	1.0%	3.8%	4.7%	0.8%	43.0%	€ 350.21
Valle D Aosta	0.4%	3.5%	4.7%	0.6%	28.4%	€ 288.38
Veneto	8.3%	3.7%	4.1%	0.7%	48.2%	€ 390.85
Lazio	3.2%	5.0%	5.5%	1.1%	41.2%	€ 519.91
Marche	3.9%	3.6%	4.2%	0.8%	73.2%	€ 404.42
Toscana	5.3%	4.8%	5.6%	0.8%	48.1%	€ 479.39
Umbria	2.6%	4.2%	4.8%	0.7%	88.2%	€ 387.78
Abruzzo	2.5%	4.1%	4.5%	0.9%	51.5%	€ 392.18
Basilicata	1.6%	3.4%	4.0%	0.6%	107.4%	€ 353.74
Calabria	2.5%	3.1%	3.6%	1.0%	42.1%	€ 521.66
Campania	7.5%	4.0%	4.9%	1.4%	37.8%	€ 652.27
Molise	0.3%	4.6%	4.5%	0.5%	48.0%	€ 353.12
Puglia	5.4%	3.7%	4.2%	1.1%	41.3%	€ 549.46
Sardegna	2.5%	4.9%	6.3%	0.8%	38.6%	€ 421.32
Sicilia	5.9%	3.9%	4.5%	1.1%	42.0%	€ 457.45

Table 25: Summary of data by region.

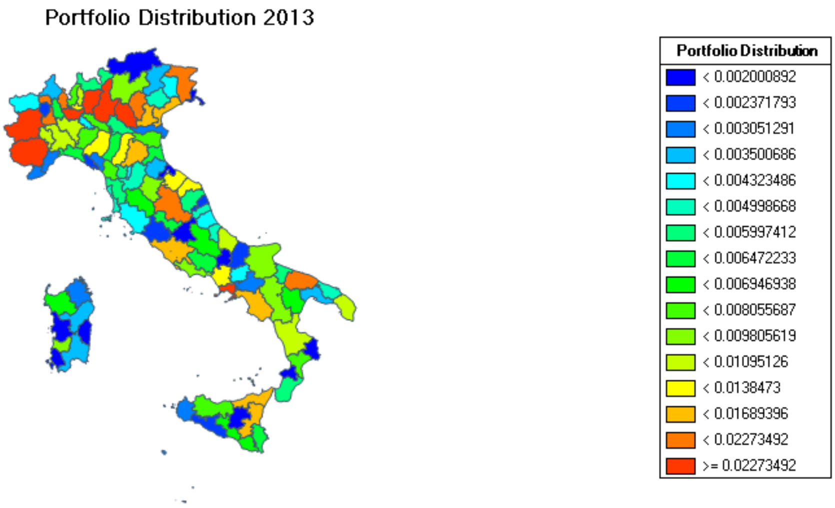

Figure 15: 2013 portfolio distribution by province.

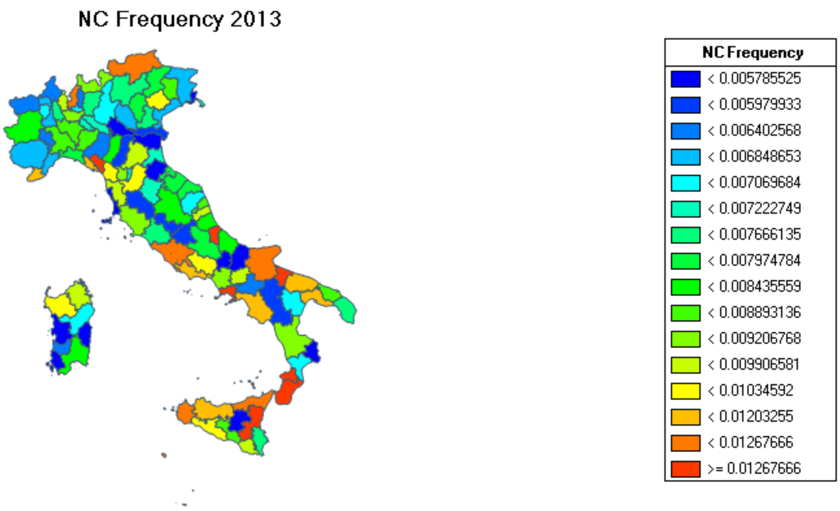

Figure 16: 2013 NC frequency by province.

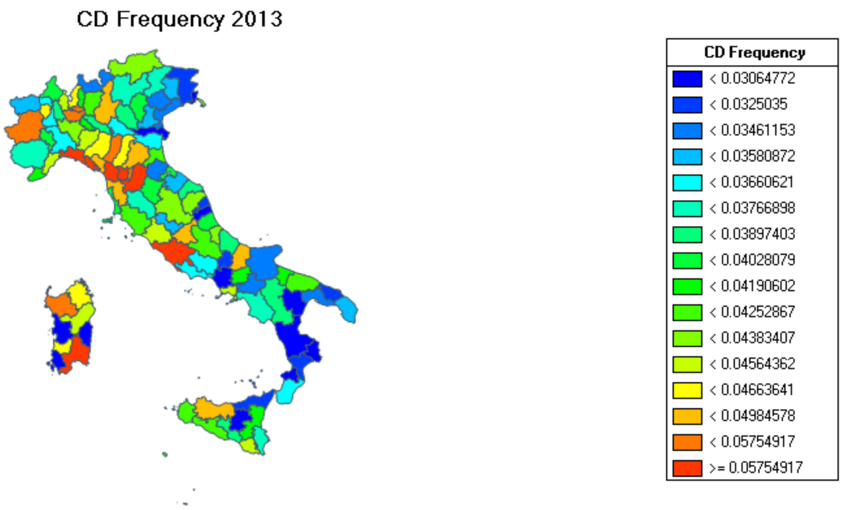

Figure 17: 2013 CD frequency by province.

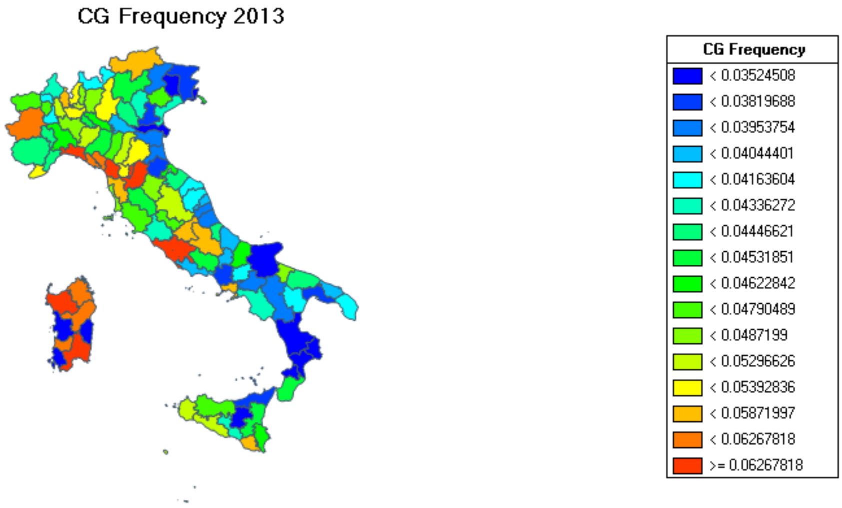

Figure 18: 2013 CG frequency by province.

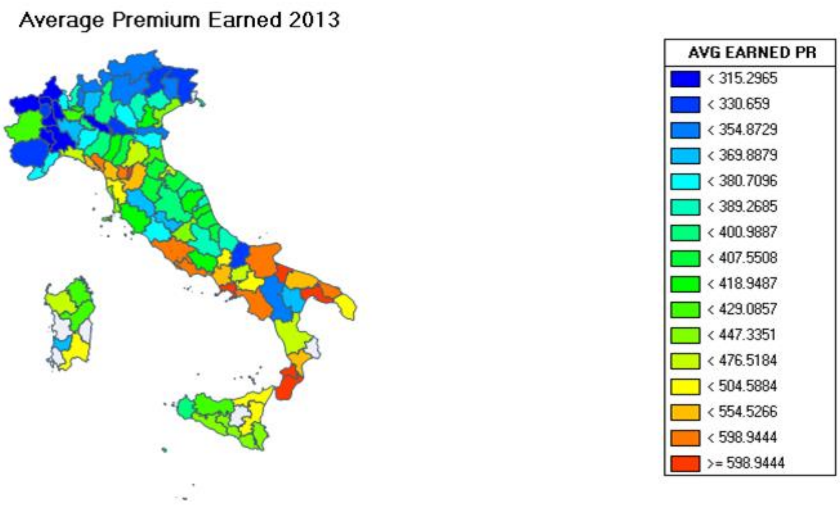

Figure 19: 2013 average premium earned by province.

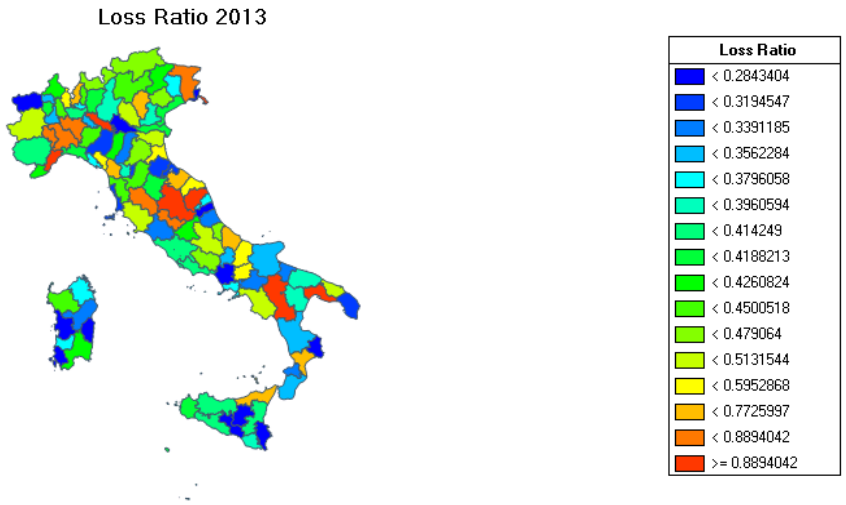

Figure 20: 2013 Loss Ratio by province.

Province	Portfolio Distribution	CD Frequency	CG Frequency	NC Frequency	Loss Ratio	Average Premium
BO	1.5%	4.8%	5.3%	0.9%	46.2%	€ 472.75
FC	0.3%	3.3%	3.6%	0.3%	30.5%	€ 404.64
FE	0.8%	3.6%	3.8%	0.6%	49.5%	€ 372.40
MO	1.1%	4.6%	5.0%	0.6%	33.9%	€ 404.54
PC	0.7%	4.5%	5.2%	0.9%	44.6%	€ 374.87
PR	1.2%	4.6%	4.5%	0.6%	30.6%	€ 395.94
RA	0.6%	4.2%	3.8%	0.7%	57.3%	€ 428.32
RE	0.6%	5.2%	4.6%	0.8%	42.3%	€ 415.16
RN	0.2%	3.9%	4.6%	0.8%	30.6%	€ 447.34
PN	0.4%	3.5%	3.2%	0.9%	37.1%	€ 330.66
TS	0.1%	4.3%	4.6%	0.7%	271.5%	€ 398.38
UD	2.1%	3.1%	3.7%	0.7%	85.3%	€ 327.00
GE	0.6%	6.8%	7.6%	0.8%	41.7%	€ 472.14
IM	0.2%	4.2%	5.3%	1.1%	42.6%	€ 369.89
SP	0.2%	5.8%	6.0%	1.1%	35.6%	€ 504.59
SV	0.3%	4.5%	4.4%	0.6%	93.6%	€ 376.72
BG	3.1%	4.2%	4.8%	0.7%	41.7%	€ 361.07
BS	2.3%	4.8%	5.3%	0.7%	38.0%	€ 391.69
CO	0.8%	4.7%	5.3%	1.2%	60.5%	€ 384.08
CR	0.8%	3.9%	4.6%	0.7%	129.2%	€ 296.21
LC	1.0%	4.1%	4.8%	0.6%	44.3%	€ 351.29
LO	0.4%	4.2%	4.5%	0.7%	34.1%	€ 389.27
MB	1.4%	5.3%	5.0%	0.7%	35.6%	€ 409.93
MI	3.0%	5.0%	5.4%	0.9%	40.0%	€ 425.49
MN	0.6%	3.6%	4.0%	0.3%	26.2%	€ 323.75
PV	1.1%	4.3%	4.8%	0.9%	85.6%	€ 362.77
SO	0.5%	3.3%	4.1%	0.9%	46.4%	€ 339.33
VA	1.9%	4.4%	5.4%	1.0%	54.7%	€ 371.07
AL	1.1%	3.6%	4.6%	0.9%	77.3%	€ 314.29
AT	1.0%	3.9%	4.4%	0.6%	85.3%	€ 295.99
BI	0.2%	4.6%	4.8%	0.7%	41.7%	€ 316.06
CN	3.0%	3.7%	4.4%	0.6%	40.8%	€ 315.30
NO	0.6%	3.9%	4.9%	0.7%	43.6%	€ 305.35
TO	11.8%	5.0%	5.9%	0.8%	48.2%	€ 419.91
VB	0.3%	3.9%	4.3%	0.6%	41.9%	€ 304.16
VC	1.7%	3.6%	4.2%	0.7%	34.7%	€ 310.05
BZ	0.2%	4.3%	5.7%	1.2%	45.0%	€ 338.29
TN	0.8%	3.7%	4.5%	0.7%	42.6%	€ 352.72
AO	0.4%	3.5%	4.7%	0.6%	28.4%	€ 288.38
BL	0.3%	3.7%	3.8%	0.8%	42.0%	€ 324.17
PD	1.6%	3.5%	3.6%	0.8%	39.5%	€ 409.74
RO	0.3%	2.8%	2.5%	0.6%	41.6%	€ 345.27
TV	0.5%	3.4%	4.8%	1.0%	45.5%	€ 388.79
VE	1.6%	3.4%	4.2%	0.7%	41.7%	€ 429.09
VI	1.7%	4.0%	4.3%	0.8%	62.5%	€ 380.71
VR	2.4%	3.9%	4.3%	0.6%	51.1%	€ 374.48
FR	0.6%	3.6%	4.5%	1.0%	45.1%	€ 416.15
LT	0.8%	3.6%	4.0%	1.0%	41.1%	€ 559.04
RI	0.1%	4.9%	5.8%	0.6%	41.9%	€ 445.27
RM	1.5%	6.5%	6.8%	1.2%	40.9%	€ 567.14
VT	0.2%	4.6%	4.2%	0.7%	32.1%	€ 373.31
AN	1.3%	3.8%	4.1%	0.8%	51.3%	€ 400.97

Province	Portfolio Distribution	CD Frequency	CG Frequency	NC Frequency	Loss Ratio	Average Premium
AP	0.5%	2.9%	3.9%	0.9%	25.2%	€ 407.55
FM	0.2%	3.2%	4.0%	0.8%	36.6%	€ 389.06
MC	0.6%	4.3%	4.1%	0.7%	180.9%	€ 412.36
PU	1.3%	3.5%	4.4%	0.8%	69.8%	€ 405.87
AR	0.9%	4.0%	4.8%	0.7%	41.5%	€ 403.40
FI	0.5%	5.9%	6.3%	1.0%	44.7%	€ 545.04
GR	0.4%	4.2%	4.8%	0.9%	49.7%	€ 410.42
LI	0.4%	4.0%	4.8%	0.4%	28.4%	€ 430.75
LU	0.7%	5.8%	6.6%	1.0%	61.1%	€ 520.32
MS	0.3%	4.9%	6.0%	1.6%	53.7%	€ 598.78
PI	0.5%	4.7%	5.4%	1.0%	44.2%	€ 476.52
PO	0.4%	5.7%	8.3%	0.7%	31.0%	€ 617.24
PT	0.5%	6.1%	5.3%	0.9%	39.6%	€ 583.22
SI	0.7%	3.7%	4.4%	0.6%	83.5%	€ 354.87
PG	2.0%	4.4%	4.9%	0.8%	88.9%	€ 396.18
TR	0.6%	3.5%	4.7%	0.6%	85.6%	€ 360.93
AQ	0.6%	4.2%	5.6%	0.8%	48.4%	€ 386.99
CH	1.0%	3.8%	4.0%	0.8%	62.5%	€ 387.08
PE	0.5%	4.4%	4.4%	1.3%	47.1%	€ 400.99
TE	0.4%	4.0%	3.8%	0.8%	32.3%	€ 404.41
MT	0.7%	2.8%	4.1%	0.7%	39.5%	€ 367.59
PZ	1.0%	3.8%	3.9%	0.6%	157.6%	€ 344.14
CS	1.0%	3.0%	3.3%	0.9%	34.5%	€ 460.41
CZ	0.8%	3.1%	3.4%	0.7%	59.5%	€ 513.06
RC	0.5%	3.6%	4.5%	1.5%	34.8%	€ 630.72
VV	0.2%	2.5%	3.2%	1.9%	33.5%	€ 599.57
AV	0.3%	3.4%	3.8%	0.6%	32.6%	€ 490.40
BN	0.4%	4.0%	4.0%	0.9%	54.2%	€ 465.02
CE	1.2%	3.0%	3.5%	0.9%	28.3%	€ 550.36
NA	4.1%	4.4%	5.8%	1.8%	36.1%	€ 749.68
SA	1.6%	3.7%	4.2%	1.1%	47.9%	€ 554.53
CB	0.2%	4.9%	4.6%	0.5%	52.7%	€ 323.45
IS	0.1%	3.2%	4.0%	0.5%	33.9%	€ 487.44
BA	2.1%	4.2%	4.4%	1.1%	38.0%	€ 538.95
BR	0.4%	3.2%	4.0%	0.9%	48.4%	€ 577.64
BT	0.5%	4.1%	4.9%	1.3%	31.9%	€ 598.94
FG	1.0%	3.3%	3.4%	1.2%	35.2%	€ 577.12
LE	1.0%	3.5%	4.1%	0.8%	31.7%	€ 491.65
TA	0.3%	3.3%	3.8%	1.1%	106.7%	€ 601.31
CA	0.3%	5.8%	7.9%	0.8%	42.3%	€ 480.17
NU	0.3%	4.5%	6.1%	0.7%	33.6%	€ 419.58
OT	0.3%	4.6%	6.0%	1.0%	35.9%	€ 421.16
SS	0.7%	5.1%	6.3%	1.0%	42.9%	€ 466.45
VS	0.8%	4.6%	5.9%	0.6%	35.6%	€ 360.41
AG	0.2%	4.2%	5.0%	1.0%	40.4%	€ 446.18
CL	0.7%	3.9%	4.2%	0.9%	28.4%	€ 439.03
CT	1.4%	4.0%	4.5%	1.5%	39.8%	€ 485.20
ME	1.4%	3.2%	3.7%	1.2%	60.7%	€ 483.36
PA	0.7%	4.7%	4.6%	1.1%	39.6%	€ 418.95
RG	0.6%	4.5%	5.5%	1.0%	38.6%	€ 442.53
SR	0.6%	3.7%	4.5%	0.7%	23.6%	€ 444.40
TP	0.3%	4.2%	5.2%	1.2%	41.4%	€ 396.77

Table 26: Summary of data by province.

We start from the policies in force on the 31 December 2013, and we must consider the renewal process of the Company and the expected new business. In order to simplify this projection we suppose that the features of the 2014 new business are equal to the ones of the previous year, even in terms of subscription date.

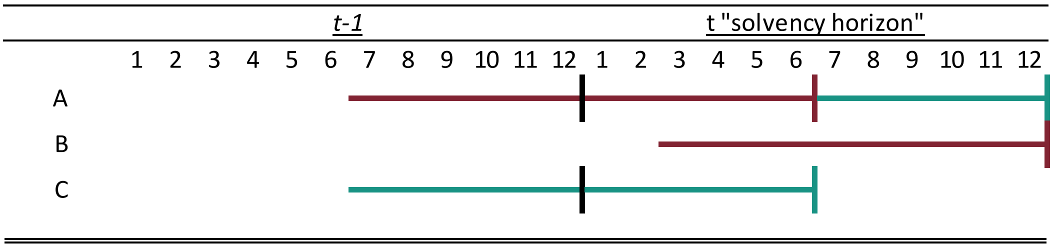

Figure 21: Types of policies.

In the previous graph are summarized all the types of policies that we must face.

Customer "A" bought a MTPL policy on July t-1 at the age of 23, and renews his policy on July t at the age of 24.

For Costumer A we can define the following data

$$exposure_{t-1} = \frac{31dec(t-1) - subscrition\ date}{365}$$

$$exposure_{t\ bfr} = \frac{renewal\ date - 31dec(t-1)}{365}$$

$$exposure_{t\ afr} = \frac{31dec(t) - renewal\ date}{365}$$

$$\lambda_{NC}^{2013} = \lambda_{NC-year\ exposure}^{2013} \cdot exposure_{t-1} = base_{NC} \prod_{d=1}^{n} C_d^{2013} \cdot exposure_{t-1}$$

$$\lambda_{NC}^{2014\,bfr} = \lambda_{NC-year\,exposure}^{2014\,bfr} \cdot exposure_{t\,bfr} = base_{NC} \prod_{d=1}^{n} C_d^{bfr} \cdot exposure_{t\,bfr}$$

$$\lambda_{NC}^{2014\,afr} = \lambda_{NC-year\,exposure}^{2014} \cdot exposure_{t\,afr} = base_{NC} \prod_{d=1}^{n} C_d^{afr} \cdot exposure_{t\,afr}$$

$$\lambda_{NC}^{solvency} = \lambda_{NC}^{2014\,bfr} + \lambda_{NC}^{2014\,afr}$$

Customer “B” bought a MTPL policy on March t, so he is a new business policy. For Costumer B the previous indicators are

$$exposure_{t-1} = 0$$

$$exposure_{t\,bfr} = 0$$

$$exposure_{t\,afr} = \frac{31dec(t) - renewal\ date}{365}$$

$$\lambda_{NC}^{2013} = 0$$

$$\lambda_{NC}^{2014\,bfr} = 0$$

$$\lambda_{NC}^{2014\,afr} = \lambda_{NC-year\,exposure}^{2014} \cdot exposure_{t\,afr} = base_{NC} \prod_{d=1}^{n} C_d^{afr} \cdot exposure_{t\,afr}$$

$$\lambda_{NC}^{solvency} = \lambda_{NC}^{2014\,bfr} + \lambda_{NC}^{2014\,afr}$$

Customer "C" bought a MTPL policy on July t-1 at the age of 23 , and he doesn't renew his policy on July t at the age of 24.

For Costumer C we can define the following data

$$exposure_{t-1} = \frac{31dec(t-1) - subscrition\ date}{365}$$

$$exposure_{t\,bfr} = \frac{renewal\ date - 31dec(t-1)}{365}$$

$$exposure_{t\,afr} = 0$$

$$\lambda_{NC}^{2013} = \lambda_{NC-year\,exposure}^{2013} \cdot exposure_{t-1} = base_{NC} \prod_{d=1}^{n} C_d^{2013} \cdot exposure_{t-1}$$

$$\lambda_{NC}^{2014\,bfr} = \lambda_{NC-year\,exposure}^{2014\,bfr} \cdot exposure_{t\,bfr} = base_{NC} \prod_{d=1}^{n} C_d^{bfr} \cdot exposure_{t\,bfr}$$

$$\lambda_{NC}^{2014\,afr} = 0$$

$$\lambda_{NC}^{solvency} = \lambda_{NC}^{2014\,bfr} + \lambda_{NC}^{2014\,afr}$$

The problem is that $\lambda_{NC}^{2014\,afr}$ depends to the fact the costumer has made or not a claim, with guilty, in the previous insurance year (PY). But what is this probability ? It's simply the probability to have or not a CD or/and a NC claim

$$\begin{aligned} P\{No\ Claims\} &= P\{\widetilde{N}_{NC-PY} = 0\} \cdot P\{\widetilde{N}_{CD-PY} = 0\} = \\ &= P\{Poisson(\lambda_{NC-PY}) = 0\} \cdot P\{Poisson(\lambda_{CD-PY}) = 0\} \end{aligned} \qquad 4.22$$

Where

$$\begin{aligned} \lambda_{j-PY} = exposure_{t-1} \cdot \lambda_{j-year\ exposure}^{2013} + (1+\vartheta_j)\lambda_{j-year\ exposure}^{2014\,bfr} \\ \cdot exposure_{t\ bfr} \end{aligned} \qquad 4.23$$

ϑ_j can be seen as an expected increase or decrease of the market frequency in the solvency year inside the j type of claim.

It follows

$$\begin{aligned} \lambda_{NC}^{2014\,afr} = \{\lambda_{NC}^{2014\,afr-no\ claims} \cdot P\{No\ Claims\} + (1 - P\{No\ Claims\}) \\ \cdot \lambda_{NC}^{2014\,afr-claims}\} \end{aligned} \qquad 4.24$$

The $\lambda_{NC}^{solvency}$ can be formally written as:

$$
\begin{aligned}
\lambda_{NC}^{solvency} &= \sum_{h=1}^{r} {}_{h}\lambda_{NC}^{solvency} \\
&= \sum_{h=1}^{r} (1+\vartheta_{NC})\{\lambda_{NC-year\ exposure}^{2014\ bfr} \cdot exposure_{t\ bfr} \\
&+ P\{renewal\}\{\lambda_{NC-year\ exposure}^{2014\ afr-no\ claims} \cdot P\{No\ Claims\} \\
&+ (1-P\{No\ Claims\}) \cdot \lambda_{NC-year\ exposure}^{2014\ afr-claims}\} \\
&\cdot exposure_{t\ afr}\}
\end{aligned}
\qquad 4.25
$$

Using that approach for each claim types we have

Variable	***New Business***	***Renewal***	***ToT***
COMPANY			ALFA
Scenario			Base
NC Increase			1,05
CD Increase			1,05
CG Increase			1,05
Retention			90%
Renewal Discount Increase			0
NB Discount			35,0%
NB Increase			0
Number NC	204	1630	1834
Number CD	750	7002	7752
Number CG	846	7806	8652
Exposure	13811	152198	166009
Frequency NC	1,48%	1,07%	1,10%
Frequency CD	5,43%	4,60%	4,67%
Frequency CG	6,13%	5,13%	5,21%

Table 27: Company ALFA, Scenario BASE, summary of frequency parameters.

Once estimated the expected number of claims for each models we can look at the severity.

Below the empirical distributions of each claim types and their principal charateristics are shown and summarized.

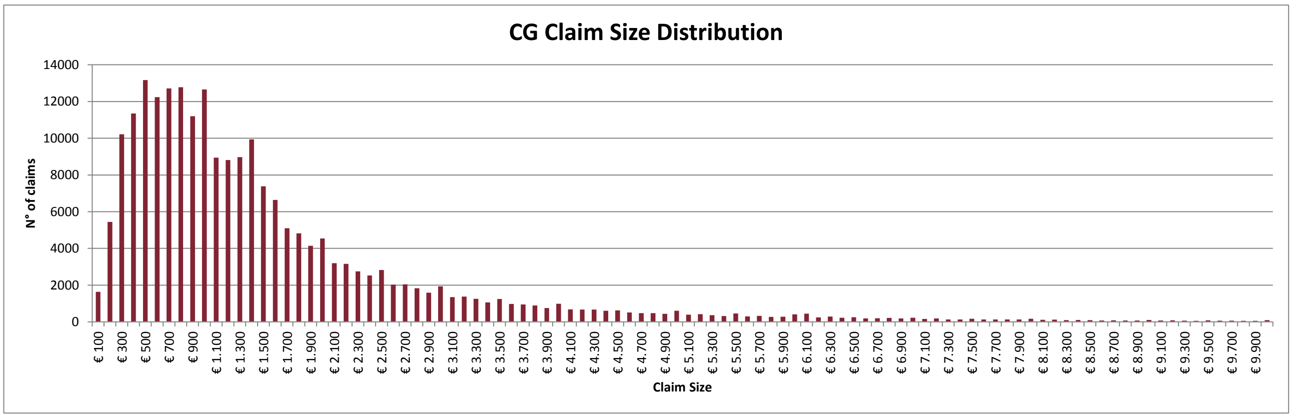

Figure 22: CG Claim Size empirical distribution.

Variables	***CG GROUP 1***	***CG GROUP 2***	***CG GROUP 3***
Mean	€ 1.942,47	€ 1.687,11	€ 1.407,86
CV	1,12	1,31	1,29

Table 28: CG Claim Size moments divided by provice groups.

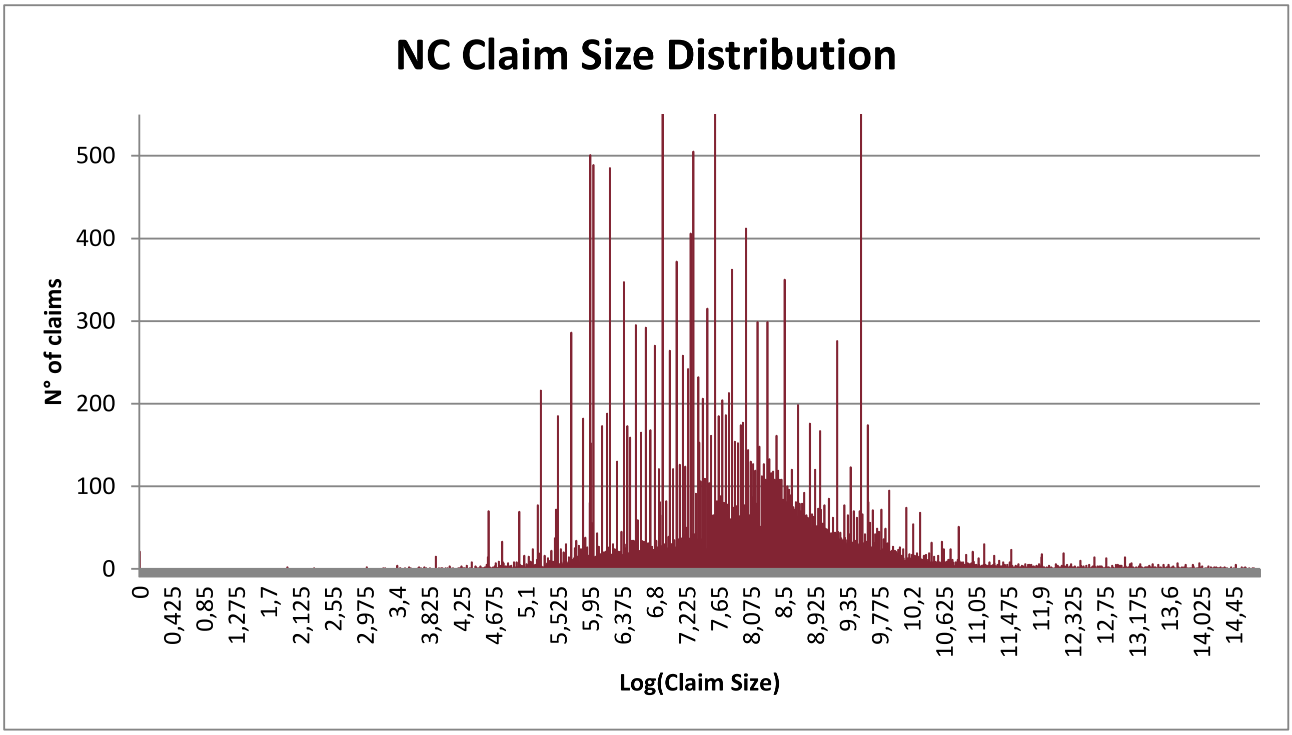

Figure 23: NC Claim Size empirical distribution (Log-scale).

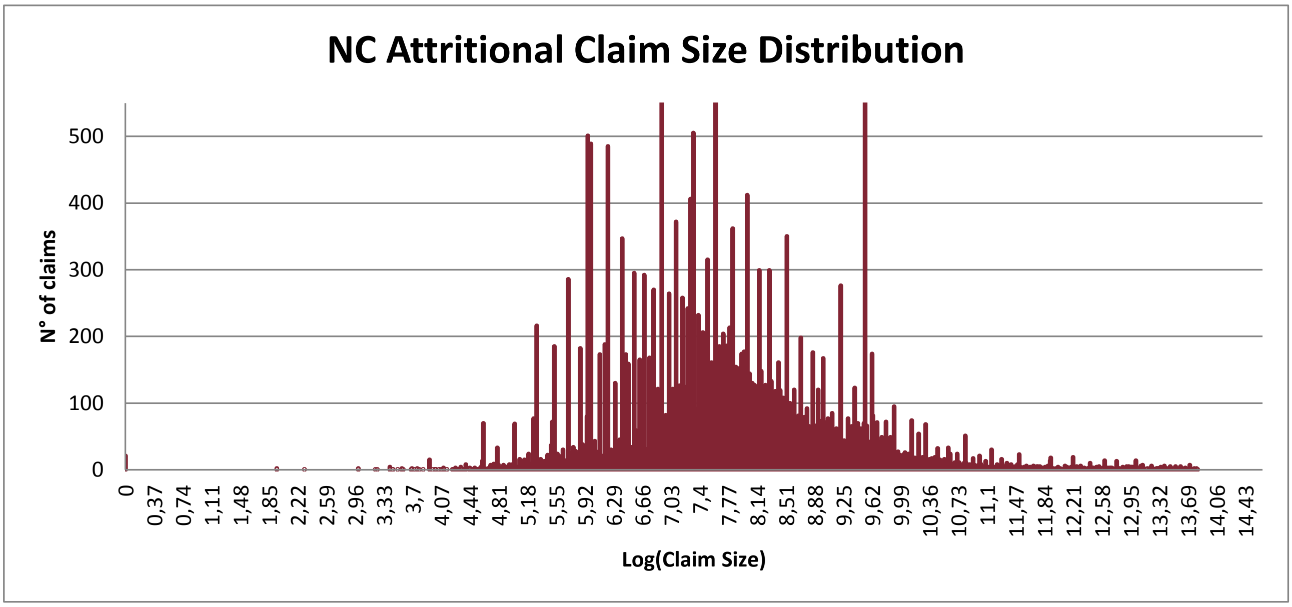

Figure 24: NC Attritional Claim Size empirical distribution (Log-scale, claims < 1.000.000€).

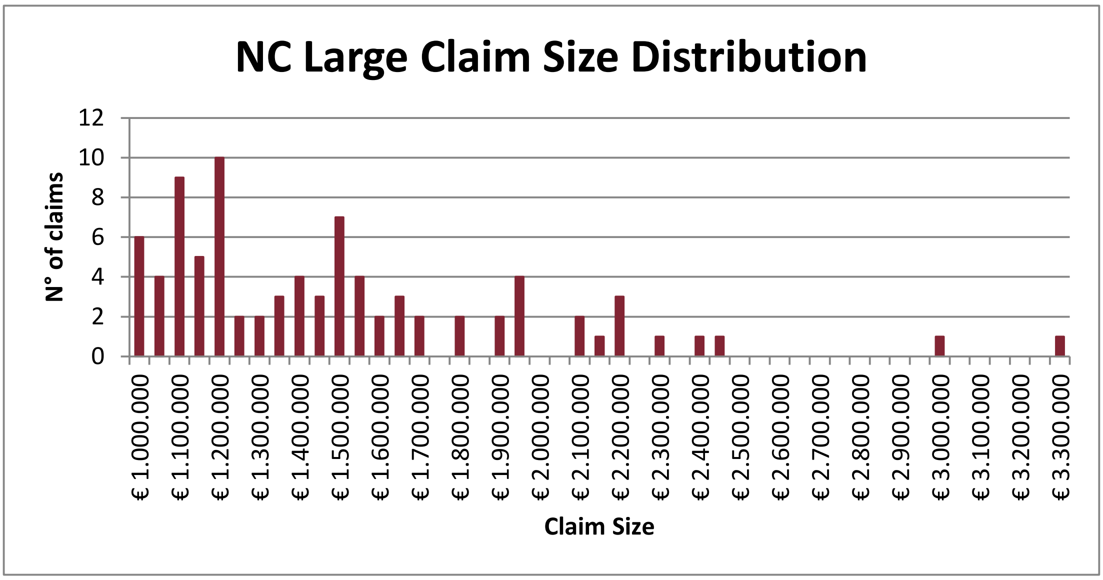

Figure 25: NC Large Claim Size empirical distribution (claims >= 1.000.000€).

Variables	***NC - TOT***	***NC - ATT***	***NC - LARGE***
Mean	€ 14.247,87	€ 11.361,67	€ 1.591.297,09
CV	5,98	4,45	0,30
Threshold	€ 1.000.000,00		

Table 29: NC Claim Size moments.

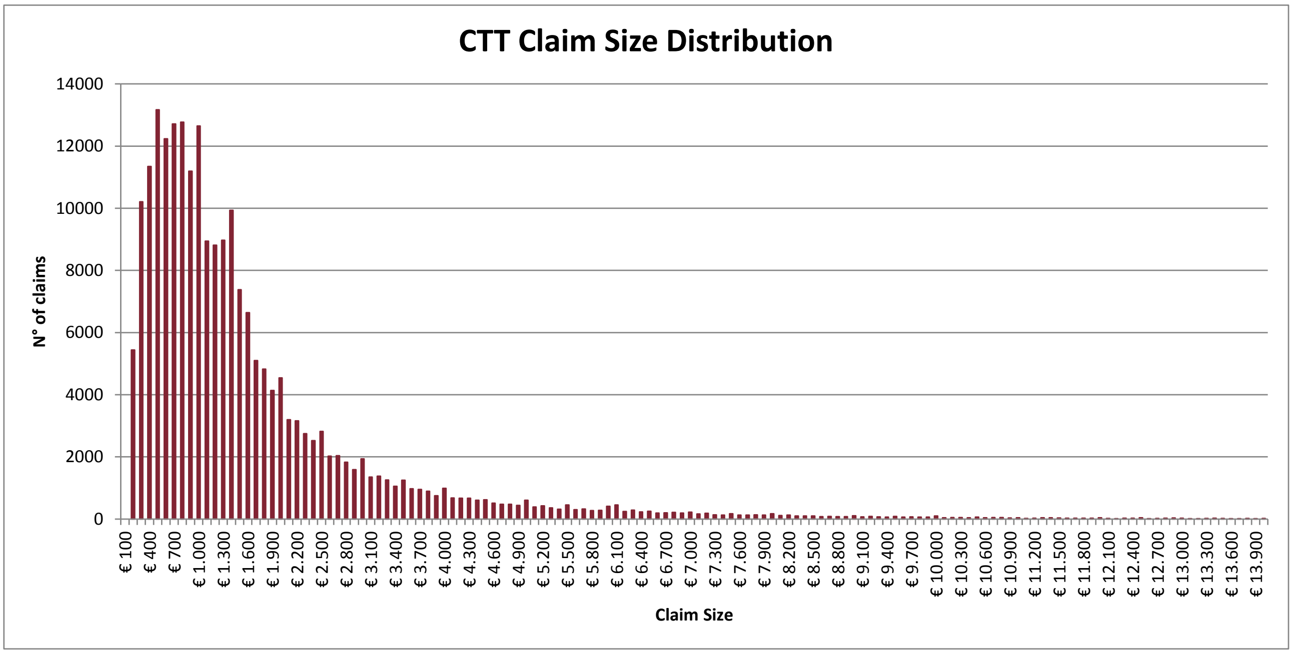

Figure 26: CTT Claim Size empirical distribution.

Variables	***CTT CLAIMS***
Mean	€ 11.801,83
CV	4,15

Table 30: CTT Claim Size moments.

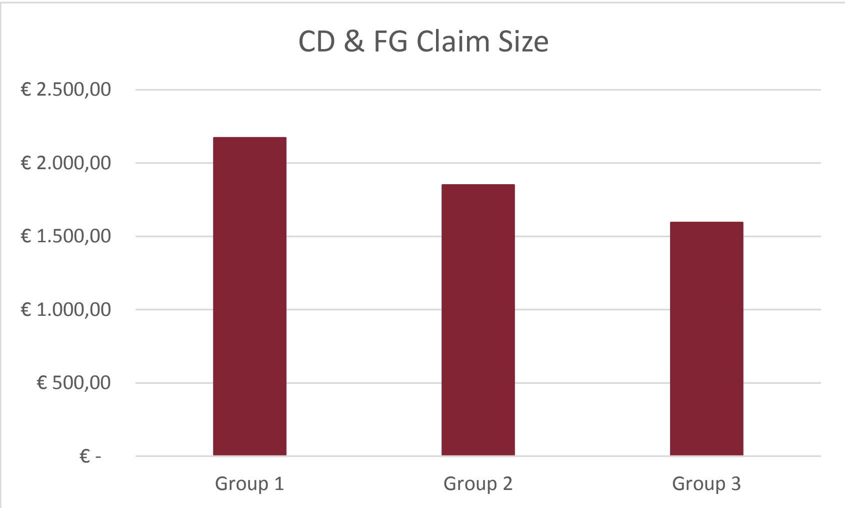

Figure 27: CD & FG forfait amount divided by province groups.

Variables	*FORFAIT*	
Group 1	€	2.171,00
Group 2	€	1.850,00
Group 3	€	1.593,00

Table 31: CD & FG forfait amount divided by province groups.

For the CD and FG Forfait we know that is a fixed amount that depends only on the province.

We will use the lognormal distribution to model the $\tilde{S}^{NC-att}$, a pareto distribution for $\tilde{Y}^{NC-large}$, a lognormal distribution for ${}_j\tilde{Y}^{CG-std}$ and ${}_j\tilde{Y}^{DN}$, and different functions from the same lognormal distribution for ${}_j\tilde{Y}^{CG-CTT}$, ${}_j\tilde{Y}^{FG-CTT}$ and $\tilde{Y}^{CD-CTT}$.

The parameters of the Log-Normal distributions are calculated with the "method of moments" as follow:

$$\sigma = \sqrt{ln(Coefficient\ of\ variation^2 + 1)} \quad 4.26$$

$$\mu = ln(Expected\ claim\ size) - \frac{\sigma^2}{2} \quad 4.27$$

While the parameter of the Pareto distribution is derived using the maximum likelihood estimator

$$\alpha = {}^{n}\!/\!\left(\sum ln(Claim\ Size_i) - nln(H)\right) \quad 4.28$$

Variable		*ToT*
COMPANY		ALFA
Scenario		Base
NC Claim Size Mean	€	14.248
NC Claim Size CV		5,98
NC Attritional Claim Size Mean	€	11.362
NC Attritional Claim Size CV		4,45
NC Large Claim Size Mean	€	1.591.297
NC Large Claim Size CV		0,30
NC Threshold	€	1.000.000
CD Claim Size Group 1	€	2.171
CD Claim Size Group 2	€	1.850
CD Claim Size Group 3	€	1.593
CTT Claim Size Mean	€	11.802
CTT Claim Size CV		4,15
CG Claim Size Group 1 Mean	€	1.942
CG Claim Size Group 2 Mean	€	1.687
CG Claim Size Group 3 Mean	€	1.408
CG Claim Size Group 1 CV		1,12
CG Claim Size Group 2 CV		1,31
CG Claim Size Group 3 CV		1,29

Table 32: Company ALFA, Scenario BASE, summary of severity parameters.

We now introduce the third connection with the Actuarial Pricing.

The SCR definition is

$$SCR_{PR} = VaR_{99.5}(\tilde{X}_{coll}) - E(\tilde{X}_{coll}) \quad 4.29$$

The underlying assumption of this approach is that the future Pure Premiums income will be completely "correct", so that the tariff is, on average, capable to face the expected cost of claims.

The second weak point is that if two Companies has the same portfolio, the same cost structure, so are completely equal except for the discount applied to tariff, they will obtain the same level of $\mathrm{SCR}_{\mathrm{PR}}$, while the Company with a high discount should have a higher $\mathrm{SCR}_{\mathrm{PR}}$.

For these reasons, instead of 4.29, we will use the following formula

$$SCR_{PR} = VaR_{99.5}(\tilde{X}_{coll}) - E(\tilde{P}_{t+1}) \qquad 4.30$$

Where $E(\tilde{P}_{t+1})$ is obtained applying the future tariff structure (yet defined at the moment of the Solvency calculation) to the projected portfolio and extrapolating from these premiums only the part relates to the payment of the claims.

Variable	Parameter
Loading factor	1,0%
Expenses coefficient	27,73%

Table 33: Loading and expenses factors.

For each costumer the future expected earned pure premium will be

$$E(\tilde{P}_{t+1}) = \left\{ {(B_t - cB_t)}\Big/{(1 + loading\ factor)} \right\} \cdot exposure_{bfr} + P\{renewal\} \cdot \left\{ \left[P\{No\ Claims\} \cdot {\left(B_{t+1}^{no\ claims} - cB_{t+1}^{no\ claims}\right)}\Big/{(1 + loading\ factor)} + P\{Claims\} \cdot {\left(B_{t+1}^{claims} - cB_{t+1}^{claims}\right)}\Big/{(1 + loading\ factor)} \right] \right\} \cdot exposure_{afr} \qquad 4.31$$

Where B is the paid premium, that depends even from the discount applied. In addition, as the expected number of claims, the "renewal" level of the premium depends to the claim history of the costumer during the last insurance year.

The parameters used in the simulation are

Variable	***New Business***	***Renewal***	***ToT***
COMPANY			ALFA
Scenario			Base
NC Increase			1.05
CD Increase			1.05
CG Increase			1.05
Retention			90%
Renewal Discount Increase			0
NB Discount			35.0%
NB Increase			0
Number NC	204	1630	1834
Number CD	750	7002	7752
Number CG	846	7806	8652
Exposure	13811	152198	166009
Frequency NC	1.48%	1.07%	1.10%
Frequency CD	5.43%	4.60%	4.67%
Frequency CG	6.13%	5.13%	5.21%
% CD Group 1			22.70%
% CD Group 2			62.45%
% CD Group 3			14.85%
% CG Group 1			23.72%
% CG Group 2			61.68%
% CG Group 3			14.60%
% DN			3.81%
% CTT			2.70%
NC Attritional Claim Size Mean			€ 11,362
NC Attritional Claim Size CV			4.45
% NC Large Claims			0.19%
NC Large Claim Size Mean			€ 1,591,297
NC Threshold			€ 1,000,000
CD Claim Size Group 1			€ 2,171
CD Claim Size Group 2			€ 1,850
CD Claim Size Group 3			€ 1,593
%CTT (CD)			8.16%
CTT Claim Size Mean			€ 11,802
CTT Claim Size CV			4.15
CG Claim Size Group 1 Mean			€ 1,942
CG Claim Size Group 2 Mean			€ 1,687
CG Claim Size Group 3 Mean			€ 1,408
CG Claim Size Group 1 CV			1.12
CG Claim Size Group 2 CV			1.31
CG Claim Size Group 3 CV			1.29

Table 34: Company ALFA, Scenario BASE, summary of simulation parameters.

Variable	New Business	Renewal	ToT
Average Premium Paid	€ 414	€ 330	€ 344
Expected Earned Premium Net Expenses	€ 278	€ 257	€ 259
Total Amount of Earned Premiums	€ 5.365.330	€ 54.694.232	€ 60.059.562
Total Amount of Earned Premiums Net Expenses	€ 3.839.133	€ 39.136.160	€ 42.975.293
Loading factor			1,0%
Expenses coefficient			27,73%

For what concern the Premiums

Table 35: Company ALFA, Scenario BASE, summary of premium parameters.

The graphical results of 100.000 simulations are

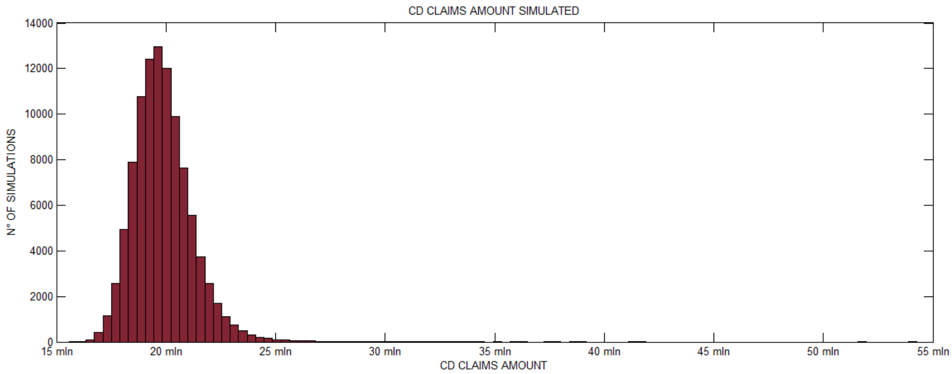

Figure 28: Simulated distribution of CD claims amount, Company ALFA, Scenario BASE.

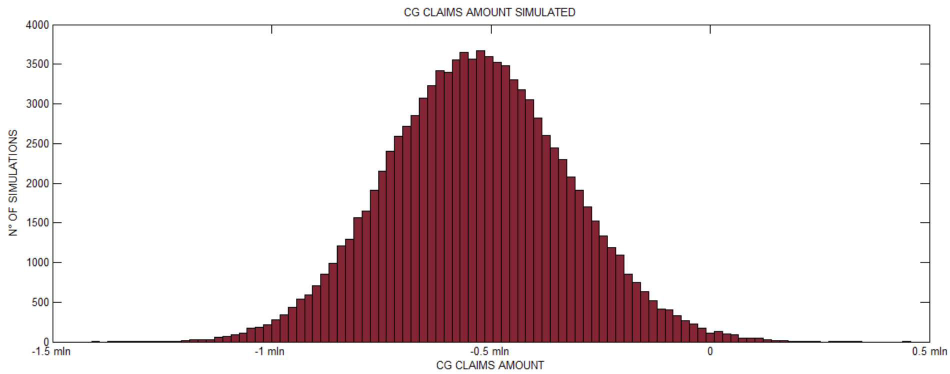

Figure 29: Simulated distribution of CG claims amount, Company ALFA, Scenario BASE.

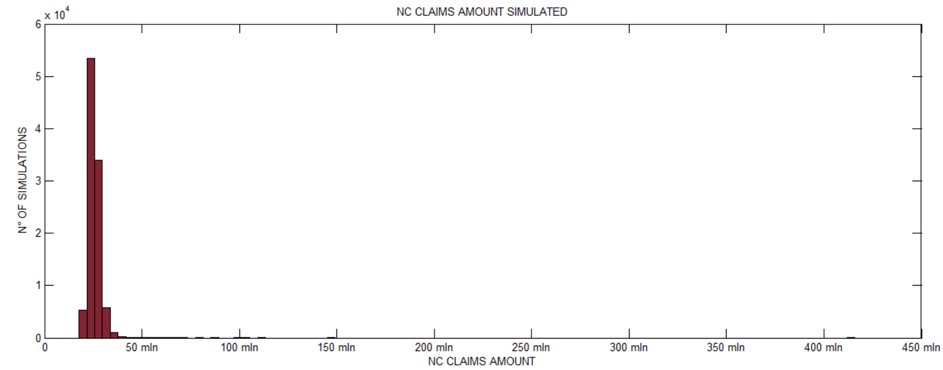

Figure 30: Simulated distribution of NC claims amount, Company ALFA, Scenario BASE.

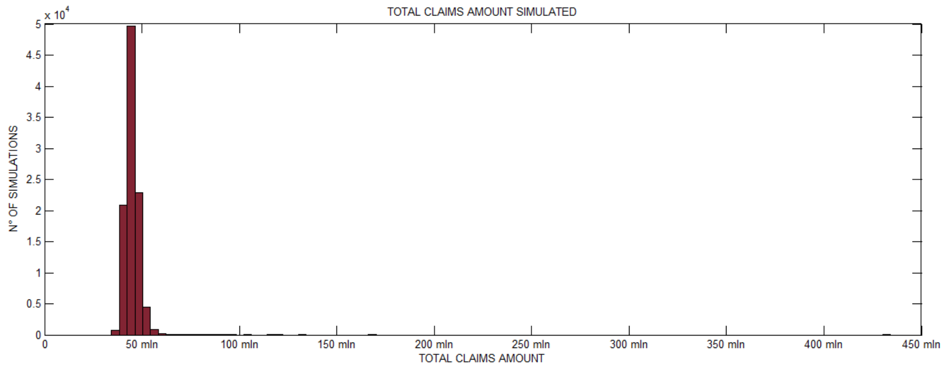

Figure 31: Simulated distribution of Total claims amount, Company ALFA, Scenario BASE.

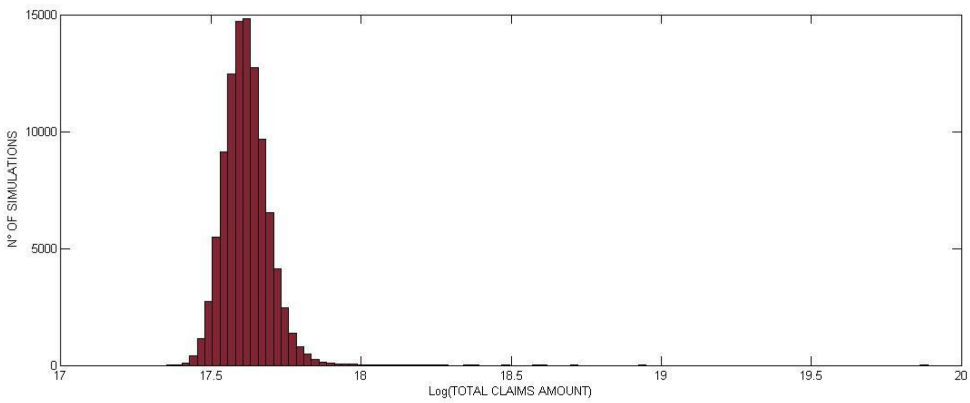

Figure 32: Simulated distribution of Total claims amount (Log Scale), Company ALFA, Scenario BASE.

In the following table some interesting values are reported

Variable		***Results***
COMPANY		ALFA
Scenario		Base
NC Claims Amount Mean	€	25.589.673
NC Claims Amount CV		12,7%
NC Claims Amount Skeweness		19,64
CG Claims Amount Mean	-€	531.120
CG Claims Amount CV		38,6%
CG Claims Amount Skeweness		0,06
CD Claims Amount Mean	€	19.873.050
CD Claims Amount CV		7,0%
CD Claims Amount Skeweness		1,71
TOTAL Claims Amount Mean	€	44.931.603
TOTAL Claims Amount CV		8,2%
TOTAL Claims Amount Skeweness		13,66
TVaR	€	60.044.648
VaR	€	55.033.148
Total Amount of Earned Premiums	€	60.059.562
Total Amount of Earned Premiums Net Expenses	€	42.975.293
RBC "Mean"/Premiums Earned Net		35,2%
SCR "Mean"/Premiums Earned Net		23,5%
RBC "Premiums"/Premiums Earned Net		39,7%
SCR "Premiums"/Premiums Earned Net		28,1%
Premiums Earned Net/Mean		95,65%

Table 36: Simulation results, Company ALFA, Scenario BASE.

First of all we have defined two levels of capital requirement depending on the risk measure used. The first one is the Tail Value at Risk which is used in the Swiss Solvency Test with a probability of 99%, while the second is the typical Value at Risk with a probability of 99.5% used in the Solvency II Framework.

In addition for each of these risk measures we have two requirements depending on the expected value used.

In the "Mean" Scenario the Capital requirements is calculated as

$$CR_{PR}(Mean) = RiskMeasure\left(\tilde{X}_{coll}\right) - E\left(\tilde{X}_{coll}\right) \quad 4.32$$

While in the "Premiums"

$$CR_{PR}(Premiums) = RiskMeasure\left(\tilde{X}_{coll}\right) - E\left(\tilde{P}_{t+1}\right) \quad 4.33$$

The last value reported in "Table 36" shows how the Company tariff seems to be not sufficient to face the future average of claims cost. Analyzing these kind of data should be clear that an increasing of tariff is needed in order to reach a nice level of Loss Ratio and not to put the company in bad situation.

In that context not only the percentile of the distribution is important, but even its mean compared with the future pure premiums. This kind of model should be seen as a test of the policy prices strength.

The underestimation of the tariff is clearer from the results of a second simulation, where the expected frequency is higher compared to "Base Scenario".

The parameters of the simulation are

Variable	***New Business***	***Renewal***	***ToT***
COMPANY			ALFA
Scenario			Two
NC Increase			1.15
CD Increase			1.15
CG Increase			1.15
Retention			90%
Renewal Discount Increase			0
NB Discount			35.0%
NB Increase			0
Number NC	223	1786	2009
Number CD	821	7672	8494
Number CG	927	8549	9476
Exposure	13811	152198	166009
Frequency NC	1.62%	1.17%	1.21%
Frequency CD	5.95%	5.04%	5.12%
Frequency CG	6.71%	5.62%	5.71%
% CD Group 1			22.70%
% CD Group 2			62.45%
% CD Group 3			14.85%
% CG Group 1			23.72%
% CG Group 2			61.68%
% CG Group 3			14.60%
% DN			3.81%
% CTT			2.70%
NC Attritional Claim Size Mean			€ 11,362
NC Attritional Claim Size CV			4.45
% NC Large Claims			0.19%
NC Large Claim Size Mean			€ 1,591,297
NC Threshold			€ 1,000,000
CD Claim Size Group 1			€ 2,171
CD Claim Size Group 2			€ 1,850
CD Claim Size Group 3			€ 1,593
%CTT (CD)			8.16%
CTT Claim Size Mean			€ 11,802
CTT Claim Size CV			4.15
CG Claim Size Group 1 Mean			€ 1,942
CG Claim Size Group 2 Mean			€ 1,687
CG Claim Size Group 3 Mean			€ 1,408
CG Claim Size Group 1 CV			1.12
CG Claim Size Group 2 CV			1.31
CG Claim Size Group 3 CV			1.29

Table 37: Company ALFA, Scenario TWO, summary of frequency and severity parameters.

And the premiums

Variable	*New Business*	*Renewal*	*ToT*
Average Premium Paid	€ 414	€ 330	€ 344
Expected Earned Premium Net Expenses	€ 278	€ 257	€ 259
Total Amount of Earned Premiums	€ 5.365.330	€ 54.722.857	€ 60.088.187
Total Amount of Earned Premiums Net Expenses	€ 3.839.133	€ 39.156.642	€ 42.995.775
Loading factor			1,0%
Expenses coefficient			27,73%

Table 38: Company ALFA, Scenario TWO, summary of premium parameters.

The results of 100.000 simulations are

Variable	*Results*
COMPANY	ALFA
Scenario	Two
NC Claims Amount Mean	€ 27.925.015
NC Claims Amount CV	12,2%
NC Claims Amount Skeweness	2,70
CG Claims Amount Mean	-€ 581.616
CG Claims Amount CV	36,9%
CG Claims Amount Skeweness	0,04
CD Claims Amount Mean	€ 21.771.954
CD Claims Amount CV	6,7%
CD Claims Amount Skeweness	1,63
TOTAL Claims Amount Mean	€ 49.115.353
TOTAL Claims Amount CV	8,0%
TOTAL Claims Amount Skeweness	1,88
TVaR	€ 65.430.715
VaR	€ 60.429.621
Total Amount of Earned Premiums	€ 60.088.187
Total Amount of Earned Premiums Net Expenses	€ 42.995.775
RBC "Mean"/Premiums Earned Net	37,9%
SCR "Mean"/Premiums Earned Net	26,3%
RBC "Premiums"/Premiums Earned Net	52,2%
SCR "Premiums"/Premiums Earned Net	40,5%
Premiums Earned Net/Mean	87,54%

Table 39: Simulation results, Company ALFA, Scenario TWO.

As expected the Capital Requirement increases in connection with the frequency movements, but probably, the most dangerous data is the ratio between the future Pure Premiums and the mean of the distribution. This ratio underlies the bad situation of the tariff, for the following year Company "ALFA" will be not able to pay all the claims using only the premiums but it would consume the Society Capital in order to face all its commitments.

A Third – and a Fourth- Scenario has been introduced in order to verify the importance of the New Business of the forthcoming year. In this Third Scenario the Simulated New Business is five times the one seen on 2013. The parameters of the simulation are the following

Variable	***New Business***	***Renewal***	***ToT***
COMPANY			ALFA
Scenario			Three
NC Increase			1,05
CD Increase			1,05
CG Increase			1,05
Retention			90%
Renewal Discount Increase			0
NB Discount			35,0%
NB Increase			5
Number NC	1223	1630	2852
Number CD	4500	7002	11502
Number CG	5078	7806	12883
Exposure	82867	152198	235065
Frequency NC	1,48%	1,07%	1,21%
Frequency CD	5,43%	4,60%	4,89%
Frequency CG	6,13%	5,13%	5,48%
% CD Group 1			21,66%
% CD Group 2			62,22%
% CD Group 3			16,12%
% CG Group 1			22,81%
% CG Group 2			61,53%
% CG Group 3			15,66%
% DN			3,81%
% CTT			2,70%
NC Attritional Claim Size Mean			€ 11.362
NC Attritional Claim Size CV			4,45
% NC Large Claims			0,19%
NC Large Claim Size Mean			€ 1.591.297
NC Threshold			€ 1.000.000
CD Claim Size Group 1			€ 2.171
CD Claim Size Group 2			€ 1.850
CD Claim Size Group 3			€ 1.593
%CTT (CD)			8,16%
CTT Claim Size Mean			€ 11.802
CTT Claim Size CV			4,15
CG Claim Size Group 1 Mean			€ 1.942
CG Claim Size Group 2 Mean			€ 1.687
CG Claim Size Group 3 Mean			€ 1.408
CG Claim Size Group 1 CV			1,12
CG Claim Size Group 2 CV			1,31
CG Claim Size Group 3 CV			1,29

Table 40: Company ALFA, Scenario THREE, summary of frequency and severity parameters.

And the premiums are

Variable	New Business	Renewal	ToT
Average Premium Paid	€ 414	€ 330	€ 375
Expected Earned Premium Net Expenses	€ 278	€ 257	€ 264
Total Amount of Earned Premiums	€ 32.191.979	€ 54.694.232	€ 86.886.211
Total Amount of Earned Premiums Net Expenses	€ 23.034.795	€ 39.136.160	€ 62.170.955
Loading factor			1,0%
Expenses coefficient			27,73%

Table 41: Company ALFA, Scenario THREE, summary of premium parameters.

The results of the simulation are summarized below

Variable	Results
COMPANY	ALFA
Scenario	Three
NC Claims Amount Mean	€ 40.322.455
NC Claims Amount CV	10,1%
NC Claims Amount Skeweness	7,90
CG Claims Amount Mean	-€ 791.197
CG Claims Amount CV	31,7%
CG Claims Amount Skeweness	0,03
CD Claims Amount Mean	€ 29.486.354
CD Claims Amount CV	6,1%
CD Claims Amount Skeweness	1,25
TOTAL Claims Amount Mean	€ 69.017.612
TOTAL Claims Amount CV	6,8%
TOTAL Claims Amount Skeweness	5,16
TVaR	€ 87.947.862
VaR	€ 82.074.430
Total Amount of Earned Premiums	€ 86.886.211
Total Amount of Earned Premiums Net Expenses	€ 62.170.955
RBC "Mean"/Premiums Earned Net	30,4%
SCR "Mean"/Premiums Earned Net	21,0%
RBC "Premiums"/Premiums Earned Net	41,5%
SCR "Premiums"/Premiums Earned Net	32,0%
Premiums Earned Net/Mean	90,08%

Table 42: Simulation results, Company ALFA, Scenario THREE.

If compared with the Base Scenario is easy to understand that New Business means higher expected frequency that implies more SCR, and more premiums. The problem is that the increasing of the premiums is not enough to face the increasing of the expected Claims Amount, in fact the ratio between Premiums Earned (net expenses and loading factor) and expected Claims Amount decreases until 90.08%.

Not only the number of the New Business is important when the Company defines a tariff and calculates the SCR. In the Fourth Scenario, the macro parameters of the simulation are equal to the Base Scenario, the difference is on the geographical area where the total New Business is underwritten. In that example, each new policy of the 2014 is located in Naples.

The parameters, the premiums and the results are summarized in the following three tables.

Variable	***New Business***	***Renewal***	***ToT***
COMPANY			ALFA
Scenario			Four
NC Increase			1,05
CD Increase			1,05
CG Increase			1,05
Retention			90%
Renewal Discount Increase			0
NB Discount			35,0%
NB Increase			0
Number NC	693	1630	2323
Number CD	947	7002	7948
Number CG	1630	7806	9435
Exposure	13811	152198	166009
Frequency NC	5,02%	1,07%	1,40%
Frequency CD	6,85%	4,60%	4,79%
Frequency CG	11,80%	5,13%	5,68%
% CD Group 1			22,55%
% CD Group 2			62,30%
% CD Group 3			15,15%
% CG Group 1			23,23%
% CG Group 2			61,51%
% CG Group 3			15,26%
% DN			3,81%
% CTT			2,70%
NC Attritional Claim Size Mean			€ 11.362
NC Attritional Claim Size CV			4,45
% NC Large Claims			0,19%
NC Large Claim Size Mean			€ 1.591.297
NC Threshold			€ 1.000.000
CD Claim Size Group 1			€ 2.171
CD Claim Size Group 2			€ 1.850
CD Claim Size Group 3			€ 1.593
%CTT (CD)			8,16%
CTT Claim Size Mean			€ 11.802
CTT Claim Size CV			4,15
CG Claim Size Group 1 Mean			€ 1.942
CG Claim Size Group 2 Mean			€ 1.687
CG Claim Size Group 3 Mean			€ 1.408
CG Claim Size Group 1 CV			1,12
CG Claim Size Group 2 CV			1,31
CG Claim Size Group 3 CV			1,29

Table 43: Company ALFA, Scenario FOUR, summary of frequency and severity parameters.

Variable	New Business	Renewal	ToT
Average Premium Paid	€ 788	€ 330	€ 406
Expected Earned Premium Net Expenses	€ 535	€ 257	€ 280
Total Amount of Earned Premiums	€ 10.322.344	€ 54.694.232	€ 65.016.576
Total Amount of Earned Premiums Net Expenses	€ 7.386.097	€ 39.136.160	€ 46.522.257
Loading factor			1,0%
Expenses coefficient			27,73%

Table 44: Company ALFA, Scenario FOUR, summary of premium parameters.

Variable	Results
COMPANY	ALFA
Scenario	Four
NC Claims Amount Mean	€ 32.717.238
NC Claims Amount CV	10,7%
NC Claims Amount Skeweness	3,06
CG Claims Amount Mean	-€ 577.846
CG Claims Amount CV	36,9%
CG Claims Amount Skeweness	0,05
CD Claims Amount Mean	€ 20.367.246
CD Claims Amount CV	6,8%
CD Claims Amount Skeweness	1,54
TOTAL Claims Amount Mean	€ 52.506.638
TOTAL Claims Amount CV	7,5%
TOTAL Claims Amount Skeweness	2,25
TVaR	€ 69.461.846
VaR	€ 64.042.952
Total Amount of Earned Premiums	€ 65.016.576
Total Amount of Earned Premiums Net Expenses	€ 46.522.257
RBC "Mean"/Premiums Earned Net	36,4%
SCR "Mean"/Premiums Earned Net	24,8%
RBC "Premiums"/Premiums Earned Net	49,3%
SCR "Premiums"/Premiums Earned Net	37,7%
Premiums Earned Net/Mean	88,60%

Table 45: Simulation results, Company ALFA, Scenario FOUR.

Comparing the Base Scenario to the last is clear that even the type of new business is important, adverse selection must be taken in to account and sustainable new business campaign must be planned. The SCR increases incredibly, as the frequency of each claim types. In addition, not only the risk of the portfolio increases, but even the expected profitability decreases.

In the following graphs the main results are summarized.

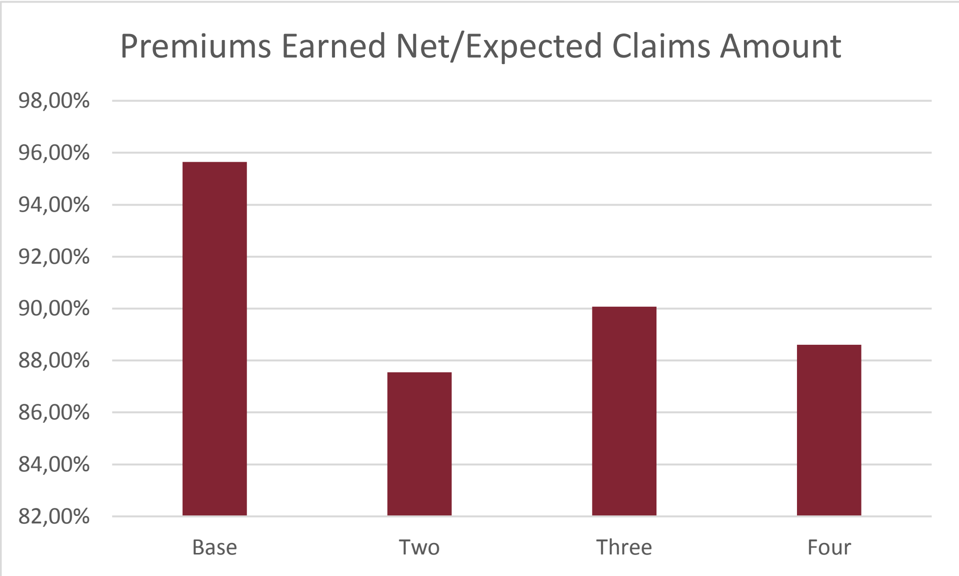

Figure 33: Premiums Earned Net vs Expected Claims Amount depending on the Scenario.

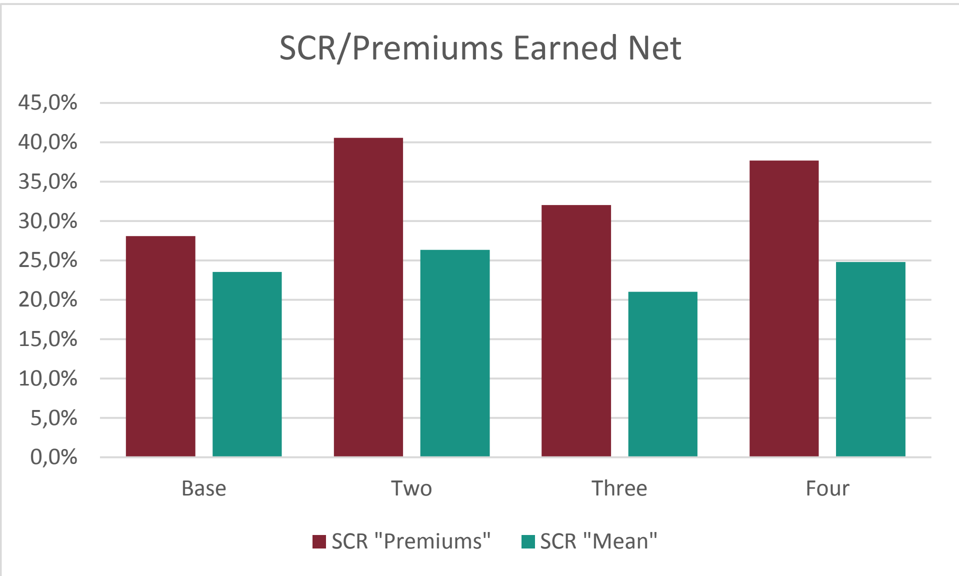

Figure 34: SCR/Premiums Earned Net depending on the Scenario.

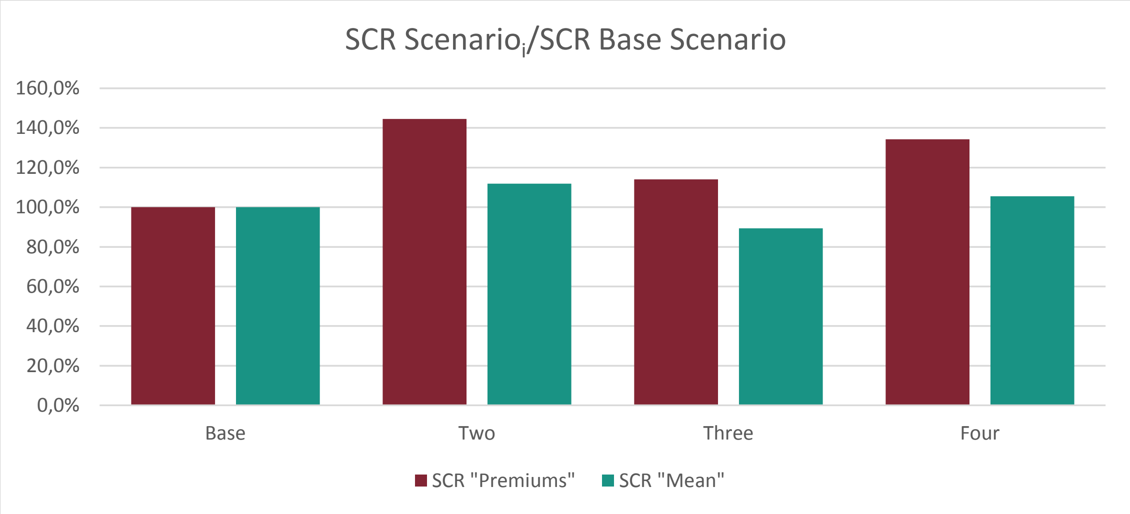

Figure 35: SCR Scenario(i)/ SCR Base Scenario.

Figure 36: Loss Ratio & Combined Ratio over different scenarios.

From the graphs above it's clear that in all the Scenarios analyzed the Company is exposed to an underestimation of the tariff. The ratio between the "Premiums Earned Net" and the "Expected Claims Amount" is always under 100%, so, even on average, the Company will suffer losses in the Solvency Period. This aspect is completely confirmed by the expected combined and loss ratio.

The most dangerous situations for the Company are represented by Scenario Two and Four, where not only the ratio between the "Premiums Earned Net" and the "Expected Claims Amount" is definitely under 100% (87.54% in Scenario Two and 88.60% in

Scenario Four) but even the SCR required is very high and equals to 40.5% for Scenario Two and 37.7% in Scenario Four.

If we compare the results of the SCR under the so called “Premiums” approach and the one of the “Mean” approach we can obtain some interesting informations. For example in the “Mean” approach the SCR goes from 21% to 26.3% while in the “Premiums” approach from 28.1% to 40.5% which suggests that the second one is able to catch in a better way the risk. In addition in the first approach the “Best” situation for the Company is represented from Scenario Three (SCR equals to 21%) where the portfolio increases. In the “Premiums” approach the results are different, the Scenario Three is not yet the best one due to the capability of this approach to take into account the tariff level of the New Business which is characterized by a high level of mutuality connected with discount percentage (35%).

4.2 SOLVENCY II STANDARD FORMULA

In this part of Chapter IV we will see the “journey” of the Solvency II premium risk standard formula. In particular we will calculate the SCR for Company ALFA under:

- QIS 2 Market Wide Approach
- QIS 2 Undertaking Approach
- QIS 4 Market Wide Approach
- QIS 4 Undertaking Approach
- QIS 5 Market Wide Approach
- LTGA Market Wide Approach.

We will use the same expected premiums for the Solvency Time horizon used in the internal model.

As explained in Chapter III, the standard Formula for the premium risk under the QIS2 is:

$$SCR_{pr} = p(\sigma)P \qquad 4.34$$

where

$$p(x) = \frac{0.99 - \phi\left(N_{0.99} - \sqrt{log(x^2+1)}\right)}{0.01} \qquad 4.35$$

Under the Market Wide approach sigma is put equal to

$$\sigma_{M,lob} = sf_{lob} \cdot f_{lob} \qquad 4.36$$

With $f_{lob} = 12.5\%$ and $sf_{lob} = 1.29$.

Instead of $\sigma_{M,lob}$ we can use the undertaking specific approach under which the sigma is

$$\sigma_{U,lob} = \sqrt{c_{lob} \cdot \sigma_{CR,lob}^2 + (1 - c_{lob}) \cdot \sigma_{M,lob}^2} \qquad 4.37$$

With

$$\sigma_{CR,lob} = \sqrt{\frac{1}{(J_{lob} - 1) \cdot P_{lob}} \sum_y P_{lob,y} \cdot \left(CR_{lob,y} - \mu_{lob}\right)^2} \qquad 4.38$$

The inputs, equal to the Base Scenario of Company “ALFA”, and the results are summarized below.

Variable		*ToT*
Total Amount of Earned Premiums	€	60.059.562
Total Amount of Earned Premiums Net Expenses (with λ)	€	43.405.045
Loading Factor		1,00%
Expenses Coefficient		27,73%
Size Factor		1,290
QIS II Volatility Factor		0,125
MW Sigma		0,161
$SCR_{Premium,MW}$/Earned Premiums		52,04%
Number Of Combined Ratios		11
c_{lob}		0,2
μ_{lob}		93%
US Sigma		0,147
$SCR_{Premium,US}$/Earned Premiums		46,59%

Table 46: QIS 2 Standard Formula results – Base Scenario parameters.

But what would change if instead of the Base Scenario parameters we would use the parameters of the forth?

The new results are the following.

Variable		*ToT*
Total Amount of Earned Premiums	€	65.016.576
Total Amount of Earned Premiums Net Expenses (with λ)	€	46.987.480
Loading Factor		1,00%
Expenses Coefficient		27,73%
Size Factor		1,240
QIS II Volatility Factor		0,125
MW Sigma		0,155
$SCR_{Premium,MW}$/Earned Premiums		49,68%
Number Of Combined Ratios		11
c_{lob}		0,2
μ_{lob}		93%
US Sigma		0,141
$SCR_{Premium,US}$/Earned Premiums		44,50%

Table 47: QIS 2 Standard Formula results – Scenario Four parameters.

Both the MW approach and the US bring to lower SCR Ratios compared to the one calculate under the Base Scenario. This is due only to the presence of the size factor, which decreases in connection on the increasing of the expected premiums.

The problem is that, in this particular case, having more premiums doesn't decrease the volatility but just increases the expected frequency (see results of the previous section).

We can now analyze the QIS 4 Standard Formula, under this the SCR is equal to

$$SCR_{pr} = p(\sigma) \cdot V \qquad 4.39$$

Where

$$p(x) = exp\left\{-\frac{\sigma^2}{2} + N_{0.995} \cdot \sqrt{log(\sigma^2+1)}\right\} - 1 \qquad 4.40$$

If we apply the MW approach, being the size factor eliminated, sigma is equal to 9%, while the estimation under the US follows the next formula

$$\sigma_{prem,lob} = \sqrt{c_{lob} \cdot \sigma^2_{U,prem,lob} + (1 - c_{lob}) \cdot \sigma^2_{M,prem,lob}} \qquad 4.41$$

Where

$$\sigma_{U,prem,lob} = \sqrt{\frac{1}{(n_{lob} - 1) \cdot V_{prem,lob}} \sum_y P_{lob,y} \cdot \left(LR_{lob,y} - \mu_{lob}\right)^2} \qquad 4.42$$

The results under the QIS 4 are

Variable	***BASE SCENARIO***	***SCENARIO FOUR***
Total Amount of Earned Premiums	€ 60.059.562	€ 65.016.576
Total Amount of Earned Premiums Net Expenses (with λ)	€ 43.405.045	€ 46.987.480
Loading Factor	1,00%	1,00%
Expenses Coefficient	27,73%	27,73%
MW Sigma	0,090	0,090
$SCR_{Premium,MW}$/Earned Premiums	25,78%	25,78%
Number Of Loss Ratios	11	11
c_{lob}	0,730	0,730
μ_{lob}	70%	70%
US Sigma	0,069	0,068
$SCR_{Premium,US}$/Earned Premiums	19,42%	18,98%

Table 48: QIS 4 Standard Formula results – Base Scenario & Scenario Four parameters.

Using the MW approach the results are equal in the two scenarios, while is lower in "Scenario Four" even if we know that is riskier than "Base Scenario".

We now apply the QIS5 and LTGA.

The QIS5 formula is equal to the QIS4 but with some modifications on the volatility factors. The results of the MW approach are

Variable	*BASE SCENARIO*	*SCENARIO FOUR*
Total Amount of Earned Premiums	€ 60.059.562	€ 65.016.576
Total Amount of Earned Premiums Net Expenses (with λ)	€ 43.405.045	€ 46.987.480
Loading Factor	1,00%	1,00%
Expenses Coefficient	27,73%	27,73%
MW Sigma	0,100	0,100
$SCR_{Premium,MW}$/Earned Premiums	28,94%	28,94%

Table 49: QIS 5 Standard Formula results – Base Scenario & Scenario Four parameters.

Even the QIS5 standard formula is not able to catch the real risk profile of the Company under the two different scenarios.

In the end the LTGA introduces some modifications on the formula to derive the volume measure which don't affect our examples. The only one that has an impact is the substitution of $p(\sigma)$ that brings to the new formula

$$SCR_{pr} = 3 \cdot \sigma \cdot V \qquad 4.43$$

The results are

Variable	*BASE SCENARIO*	*SCENARIO FOUR*
Total Amount of Earned Premiums	€ 60.059.562	€ 65.016.576
Total Amount of Earned Premiums Net Expenses	€ 43.405.045	€ 46.987.480
Loading Factor	1,00%	1,00%
Expenses Coefficient	27,73%	27,73%
MW Sigma	0,100	0,100
$SCR_{Premium,MW}$/Earned Premiums	30,30%	30,30%

Table 50: LTGA Standard Formula results – Base Scenario & Scenario Four parameters.

4.3 INCLUDING EXPENSES

Till now we have only talk about the pure risk coming from the technical part of the claims. But we must remember that we have to face even the uncertainty coming from the expenses volatility.

For that reason, and in order to have a better idea of the premium risk, we are going to calculate, in a very simple way, a capital requirement for that source of risk.

We suppose that the expense ratio

$$ExpenseRatio_t = {}^{Expenses_t}/_{Gross\ Premiums_t} \tag{4.44}$$

follows a lognormal distribution.

Using the method of moments we calculate the parameter of the distribution underlying the data, and then we derive a $SCR_{expenses}$ using the usual $VaR_{99.5\%}$ risk measure.

The historical series of data available is summarized in the following table

Year	Expense Ratio
2007	24,98%
2008	25,74%
2009	25,72%
2010	25,12%
2011	24,42%
2012	24,30%
2013	27,73%
Mean	25,43%
Variance	0,012%
CV	4,22%

Table 51: Historical series of expense ratio.

The results of this approach are

Varaible	***ToT***
Mean	25,43%
Variance	0,012%
CV	4,22%
$VaR_{99.5\%}$	28,44%
$SCR_{EXPENSES}$	3,01%

Table 52: Results of the expenses Capital Requirement.

This means that the $SCR_{expenses}$ for the forthcoming year will be equal to the 3.01% of the future Gross Premiums.

If applied to our Scenarios will yeld to

Scenario	***Gross Premiums***		***SCR*** EXPENSES	
BASE	€	60.059.562	€	1.805.400
TWO	€	60.088.187	€	1.806.260
THREE	€	86.886.211	€	2.611.813
FOUR	€	65.016.576	€	1.954.409

Table 53: $\boldsymbol{SCR_{expenses}}$ over different Scenarios.

Now, assuming incorrelation between pure "Premium Risk" and "Expenses Risk", we can easily calculate the $SCR_{premium\ tot}$

$$SCR_{premium\ tot} = SCR_{Premium} + SCR_{Expenses} \tag{4.45}$$

And a useful ratio

$$SCR\ RATIO_{premium\ tot} = {SCR_{premium\ tot}}\Big/{Gross\ Premiums_t} \qquad 4.46$$

Table 54 summarized the results obtained using both the $SCR_{\text{"}Premium\text{"}}$ (equation 4.30) and $SCR_{\text{"}Mean\text{"}}$ (equation 4.29)

Scenario	***SCR RATIO*** "Premium" TOT	***SCR RATIO*** "Mean" TOT
BASE	23,1%	19,8%
TWO	32,0%	21,8%
THREE	25,9%	18,0%
FOUR	30,0%	20,7%

Table 54: $\boldsymbol{SCR\ RATIO_{premium\ tot}}$ over different Scenarios.

For what concerns the results of the standard formula under the different QISs of Solvency II, the volatitily underlying the expenses is supposed to be equal to the one of the claims. For that reason the ratio between ${SCR_{premium\ tot}}\Big/{Gross\ Premiums_t}$ and ${SCR_{premium}}\Big/{Premiums\ Earned\ Net_t}$ will be the same.

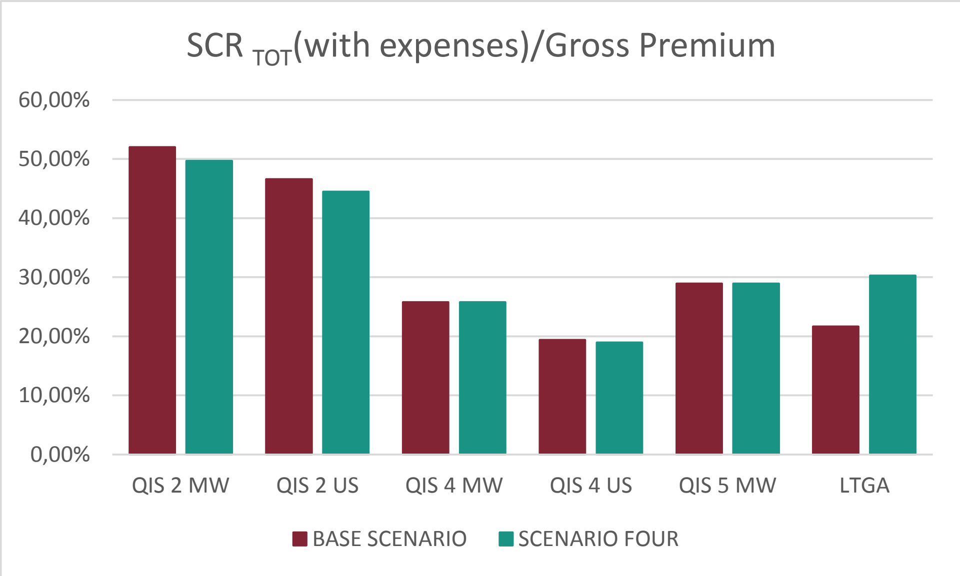

Figure 37: SCR/Gross Premium (including expenses) using Solvency II standard formulas.

CHAPTER FIVE

CONCLUSIONS

In the previous Chapters we have analyzed different scenarios and applied a lot of formulas and approaches in order to derive a consistent measure of the Premium Risk capital requirement.

The results are quickly summarized in the following table.

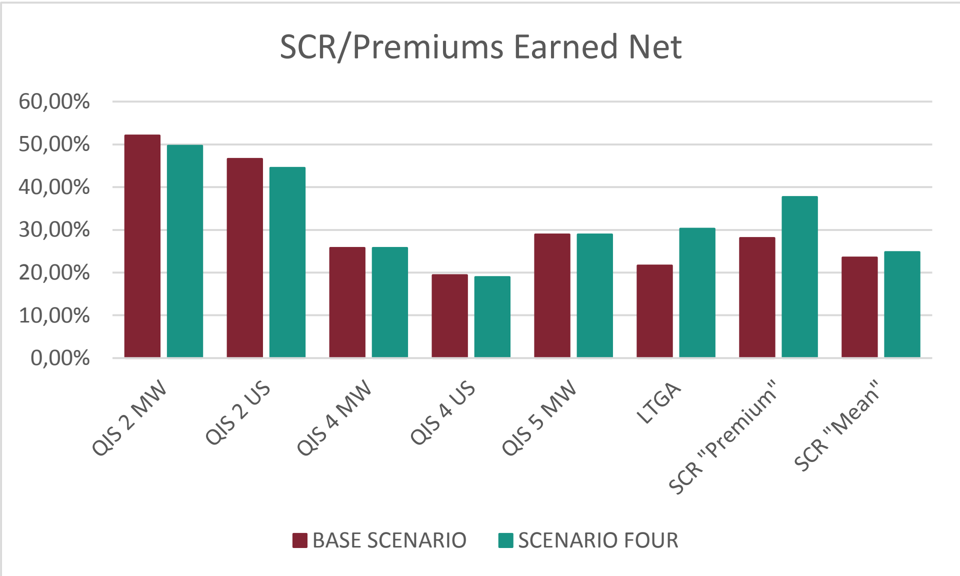

Figure 38: Comparisons between the different approaches followed.

The first important feature clearly rapresented by the graph is that only the two Internal model approaches (SCR "Premium" and SCR "Mean") are able to catch the increasing of risk going from "Base Scenario" to "Scenario Four", while the other approaches bring to a lower (or equal) capital requirement. We must remember that "Scenario Four" is carachterized (by construction) by a higher level of risk connected both to the higher frequency expected in Naples and the underpricing of that area.

A particular remark is connected with the effect of the size factor in QIS 2 standard formula. Theorically this parameter should takes into account the size of the company and brings bigger portfolio to a lower capital requirement compare to the premium income. The problem is that not always having a bigger portfolio means more stability, and the size factor is not able to catch the risk connected with the "premium surplus" of the bigger company.

The internal model approach, as well known, is a better approximation of the true risk profile of the portfolio, expecially if its parameters are estimated starting from the portfolio in force as done in that paper. In addition the usage of the "Premiums" method brings the company to have a better idea not only of the risk insured but even on the positioning of tariff defined, its expected profitability, and to define the possible actions to improve the situation (if necessary).

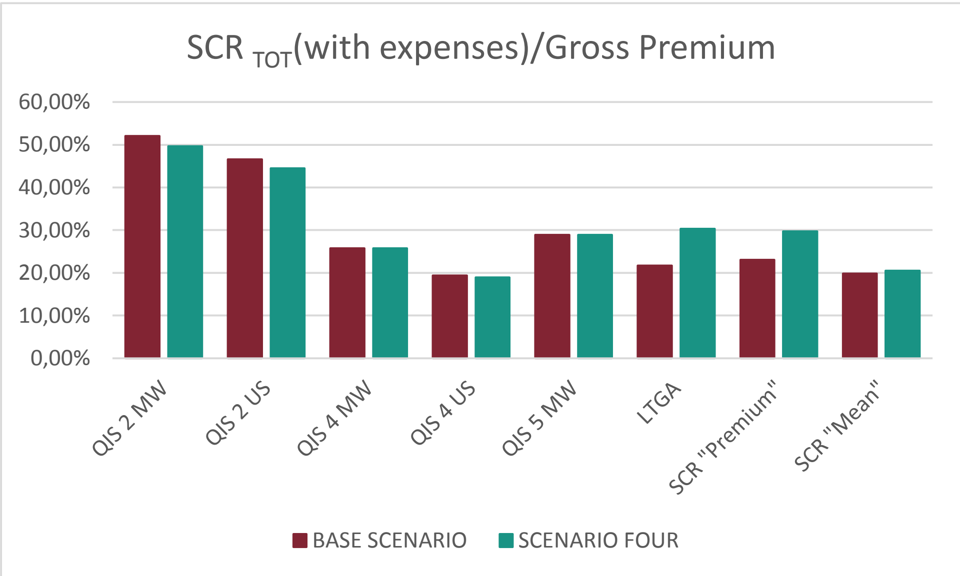

Figure 39: Comparisons between the different approaches followed including $SCR_{Expenses}$.

The inclusion of the expenses in the calculation of the SCR confirms what we have said in the previous paragraph. In addition, supposing the same volatility between the expenses and the claims (as in the results of the QISs standard formula), seems a very strong hypothesis and very conservative. The data analyzed (even if they are referred only to 7 years) support this point of view and for that reason the results of the internal model approaches are more reliable.

The model explained is able to be completed consistent with the company strategy and the business plan, the market environment, the Solvency II framework and, applying the actuarial best practices, is strictly connected with the tariff definition. Finally the approach proposed is very flexible and is able to help the Company board to take different decision knowing their implications in terms of risk and expected profitability.

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E come al solito l'ultimo pensiero è per te….

"And now you live in heaven
But I know they let you out
To take care of me"

R.W.